2-D VERTICAL MODELLING FOR SHEARING FLOW OF A GRANULAR-LIQUID MIXTURE

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ABSTRACT

We propose a rheological model for a dense granular flow (e.g. debris-flow). It is based on the simultaneous presence of quasi static an dynamic stress components.

The model leads to rather simple relationships, suitable for implementation in mathematical debris-flow models. The relative simplicity of the scheme is obtained uncoupling the problem of determining the concentration values from the rheological problem. Dynamic stress components are represented according to the relations originally proposed by Bagnold (1954) and the quasi static component are derived from more recent experimental works (Angeloni, Lamberti, Schippa 1997[1]). The ratio between the quasi static stress components is assumed to be constant and independent on the interstitial fluid. The model is tested considering the velocity profiles measured in a set of experiments in a rheometer with glass spheres shearing in newtonian viscous fluids (Angeloni, Lamberti, Schippa 1997[1]). In this experiments the mean bulk concentration was measured, and a granular concentration distribution along the vertical has been assumed to be of exponential type, with a given maximum at the fixed bottom.

Rather satisfactory results have been obtained.

Keywords: dense granular flow, debris-flow, rheology, 2-D model

INTRODUCTION

The problem of the rheological characterization of a dense granular-fluid mixture under shear has been largely studied in the last decades, mainly from the pioneering work of Bagnold (1954). Despite of the large efforts from both theoretical and experimental points of view, the problem is still far from a clear solution. Nevertheless the strong impact of natural phenomena like debris flows, on the environment of mountain areas, requires the development of suitable tools dedicated to a reasonable estimation of the risk in prone areas. As the risk is connected with the velocity and extension of the debris flow, a rheological model is necessary (among several other information) in order to obtain a suitable knowledge of a risk scenario, via mathematical simulations.

As already stressed by Bagnold, the dynamic of the mixture is strongly dependent on the particle concentration, therefore an accurate evaluation of this parameter should be provided by any model. To this aim several recent theories introduced the granular pseudo-temperature, and added to the mechanical balance (mass and momentum) equations the suitable pseudo-thermodynamic equations which are necessary for a well posed problem. For a review see Hutter et al (1996). Results of numerical simulations seem to be very promising, but unfortunately the complexity of such models does not allow until now a practical use in the technical applications.

For this reason we have tried to separate the problem of the rheological characterization of the mixture from the problem of the determination of its concentration. We refer to a simple rheological model derived from the Bagnold's original one and to a set of experiments carried out by Angeloni et al. (1997[1]) on a mixture of glass spheres and a newtonian liquid (water or water and glycerol); flow can be assumed a pure horizontal shear flow. Since during the experiments the mean granular concentration of the shearing mixture had been measured, we assumed a simple hypothesis on its vertical distribution. The comparison of measured and computed velocity profiles, show a rather good agreement.

THE EXPERIMENTAL APPARATUS AND THE TEST PROCEDURE

A small circular channel has been set up at the laboratory of the University of Bologna (Angeloni, Lamberti, Schippa 1997[1]) in order to measure velocity profiles and rheological characteristics of dense granular fluid mixtures. It consists of two concentric horizontal aluminium circular disk assemblies mounted on a vertical shaft fixed on the basement of the apparatus. The lower disk assembly is mounted on the shaft allowing rotation but no vertical motion. It is restrained from rotating by a torque arm connected to a load cell. This lower part has an annular trough 100 mm wide and 35 mm deep and has a mean radius of 200 mm; it is therefore called channel. The channel is closed above by a heavy rotating upper disk assembly mounted on the same vertical shaft allowing both rotation and vertical movements. avoiding any contacts with the side walls. Clearance is 0.4 mm and prevents the used grains, having mean diameter greater than 1 mm, from entering the meatus between the vertical walls of the shear cell and the outer edge of the upper plate, but allows fluid passing trough the gap. On the outer part of the top disk assembly there is a cogged wheel coupled to the gear of a d.c. variable speed servomotor that can induce any angular velocity of the upper disk in the range 0-100 rpm.

The channel is also provided in the external sidewall with glass windows in order to allow visual observations, video recording and velocity measurements of the shearing layer, by a Laser Doppler Anemometer (LDA) equipment. The optic unit was driven manually in order to perform measurements at different point inside the shearing channel. Measures along different verticals located at different positions apart from the wall were repeated as many time as possible depending on the optical characteristics of the mixture (usually we measured 3-4 verticals). The most remote measuring point was usually located 3÷4 mm apart from the sidewall towards the core of the channel. The observed velocity profiles along different verticals were self-similar. Assuming no-slip condition of the grains in contact with the upper rotating disk, extrapolated velocity profiles (avoiding any boundary effects) were derived .

The stress (τ) transmitted to the lower disk, the velocity (V), the vertical displacement of the upper disk and channel height (δ) are monitored during a test, while the preassigned vertical pressure (which is obtained via a system of counterweights balancing the upper disk) remains constant. Granular mean concentration (ν_m) inside the channel was derived accounting for the mass of material, for the surface of the channel and for the displacement of the upper disk. The tested glass spheres have density (ρ_s) equal to 2520 kg/m^3, uniform diameter (d_s) of 2 mm. The maximum concentration by volume actually reachable by the spheres are 0.63 or 0.65, accounting for natural or vibrated deposition respectively. The static friction angle in air and water are 22.5° and 23° respectively; all of those parameters were evaluated separately (Angeloni, Lamberti, Schippa 1997[1]). The main experimental parameters of the performed runs are summarized in the following tab.1.

run	ρs (kg/m^3)	μ x10^{-2} (Pa*s)	V (m/s)	σ_o x10^2 (Pa)	τ x10^2 (Pa)	δ (mm)	ν_m
L28	1095	39.4	0.524	5.4	1.6	11.3	0.58
L29	1095	39.4	0.785	5.4	1.8	11.3	0.58
L32	1223	54.0	0.524	3.2	1.5	10.4	0.62
L33	1223	54.0	0.785	3.2	1.7	10.4	0.62
L34	1223	54.0	0.524	5.8	2.3	10.3	0.62
L35	1223	54.0	0.785	5.8	2.7	10.3	0.62
L42	1083	10.2	0.262	2.3	1.2	10.0	0.57
L43	1083	10.2	0.785	2.3	0.76	10.4	0.55
L44	1083	10.2	0.524	1.5	0.42	10.5	0.54
L48	1000	0.10	0.785	3.9	1.6	10.9	0.60
L55	1223	59.8	0.262	5.5	1.9	10.4	0.61
L56	1223	59.8	0.524	5.5	2.2	10.4	0.61
L57	1223	59.8	0.785	5.5	2.5	10.4	0.60
L58	1223	59.8	1.047	5.5	2.8	10.5	0.60
L61	1223	59.8	0.262	4.5	1.6	10.4	0.61
L62	1223	59.8	0.524	4.5	1.9	10.4	0.61
L63	1223	59.8	0.785	4.5	2.1	10.5	0.60
L64	1223	59.8	1.047	4.5	2.3	10.6	0.60

Tab.1: Summary of experiments

THE THEORETICAL SCHEME

We assume the granular-fluid mixture as a continuum (i. e. we consider conveniently averaged values for density, velocity, etc.), and the flow as a pure shear uniform flow between two horizontal plates, the lower one being fixed and the upper one moving with a constant velocity. The continuity equation simply states that the vertical component of the mixture velocity is null, and the momentum equation leads:

$$\frac{\partial \tau}{\partial y} = f_x = 0 \qquad \frac{\partial \sigma}{\partial y} = f_y = -\rho_m g = -g\{\rho_s \nu(y) + \rho_l [1 - \nu(y)]\}$$

in which y indicates the vertical axis (positive upwards), σ and τ the normal and tangential stress respectively, $\nu(y)$ the volumetric granular concentration and ρ_m , ρ_s , ρ_l the density of the mixture, of the solid and of the fluid respectively. Accounting for the effective normal stress acting on the grains, and integrating upward from a generic vertical position (y) to the ceiling of the channel (h) where the known stress are τ_0 and σ_0 , we have:

$$\tau(y) = \tau_0 \qquad\qquad \sigma(y) = \sigma_0 + g\int_y^h \left\{ \left(\rho_s - \rho_l \right) \cdot \nu(y) \right\} \cdot dy$$

A simple rheological model has been proposed for the analysis of the experimental results of granular in shear flow. It is derived from the Bagnold original model, according to some recent suggestions (Johnson & Jackson 1987, Hutter et al. 1996, Ancey 1998, Schippa et al.1998). We assumed that normal and tangential stresses can be considered as the sum (in case, weighted) of a quasi-static component and of a dynamic component:

$$\sigma = p_1(\nu)\sigma_{qs} + p_2(\nu)\sigma_d \qquad\qquad \tau = p_1(\nu)\tau_{qs} + p_2(\nu)\tau_d$$

where p_1 and p_2 are the weighting coefficients (in case functions of the granular concentration ν). In the following we shall assume both the weights to be constant and equal to 1.

We assumed the following relation between the quasi-static component of the stress is assumed to hold:

$$\tau_{qs} = -\sigma_{qs} \tan\phi_{qs}$$

and being:

$$\tau = \tau_0 = \tau_{qs} + \tau_d = -\sigma_{qs} \tan\phi_{qs} + \tau_d$$

with:

$$\sigma_{qs} = \sigma - \sigma_d$$

we have:

$$\tau_0 = -\tan\phi_{qs}\left(\sigma - \sigma_d\right) + \tau_d$$

Several Authors have proposed the following form for the dynamic components:

$$\tau_d = \alpha \frac{dv_x}{dy} + \beta \left|\frac{dv_x}{dy}\right| \left(\frac{dv_x}{dy}\right) \qquad \sigma_d = \gamma \frac{dv_x}{dy} + \delta \left(\frac{dv_x}{dy}\right)^2$$

where α, β, γ, δ are functions of few characteristics (including the concentration) of the granular and of the interstitial fluid.

Let us write for simplicity:

$$u' = \frac{dv_x}{dy}$$

and we obtain:

$$\left(\beta + \delta \tan\phi_{qs}\right) u'^2 + \left(\alpha + \gamma \tan\phi_{qs}\right) u' - \left(\tau_0 + \sigma \tan\phi_{qs}\right) = 0$$

From this algebraic equation we can calculate the value of u', and after integration, the vertical distribution of the velocity. We need to know α, β, γ, δ, $\tan\phi_{qs}$, τ_0, and σ(y), that requires the knowledge of the granular concentration distribution ν(y).

Therefore we assumed an exponential function (typical of diffusive phenomena) based on the measured mean granular concentration and on a limiting value (i.e. 0.65) at the very proximity of the channel bed. Thus also σ(y) can be easily calculated.

About coefficients α, β, γ, δ, we have adopted the expressions suggested by Bagnold for the macro-viscous and the grain-inertial regimes, only modifying the numerical coefficients of asymptotic macro-viscous regime, according to the introduction of the quasi-static component which was not taken into account by Bagnold:

$$\alpha = -0.5 \mu \lambda^{3/2} \qquad \beta = -0.013 \rho_s d_s^2 \lambda^2 \qquad \gamma = 0.65 \mu \lambda^{3/2} \qquad \delta = 0.04 \rho_s \lambda^2 d_s^2$$

where: $\lambda = \left[\left(\frac{\nu_0}{\nu}\right)^{1/3} - 1\right]^{-1}$ where ν_0=0.74 and $\tan\phi_{qs}$ = 0.24 (Angeloni et al. 1997)

Fig. 1 shows some comparisons between experimental results and theoretical ones. They appear to be rather satisfactory, but it must said that very small adjustments have been done in some cases for the mean concentration value, anyway largely within the estimated measurement error (few units percent).

CONCLUSIONS

We have here tested the possibility of using a simple rheological model based on the simultaneous presence of quasi static an dynamic components in normal and tangential stresses. The model leads to rather simple relationships and appears to be suitable for implementation in mathematical models of debris flow. The problem of determining the concentration values remains anyway unresolved and very important. In the considered experimental cases the mean bulk concentration was measured and then available, and its vertical distribution, reconstructed on the hypothesis of an exponential distribution (accounting for some diffusion phenomena) starting from a given concentration (0.65 in our cases) at the fixed bottom, proved to be an effective choice. It is anyway critical the mean bulk concentration value: any possible error in its determination is strongly amplified in computing the maximum velocity and the discharge values.

Further, the quasi static and dynamic weighting functions are herein assumed equal to 1 and the ratio between quasi static stress component has been assumed constant and independent from fluid characteristics. Some refinement of this choice could be suggested from future analysis.

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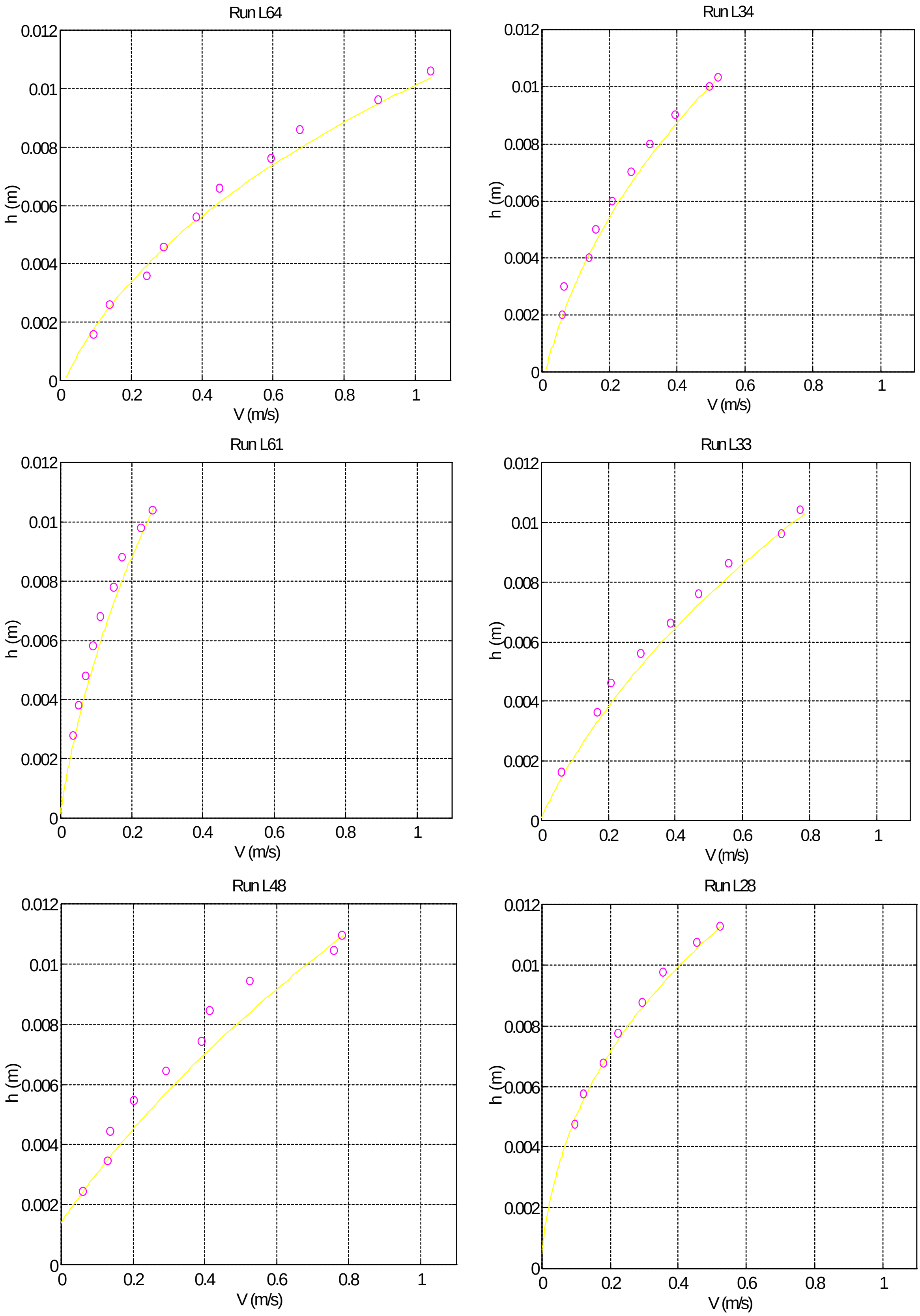

Fig.1: Representative runs. Measured (o) -vs- computed (--) velocity profiles.