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Mini-Workshop: Felix Klein's Foreign Students: Opening Up the Way for Transnational Mathematics

Organized by
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ABSTRACT. Extending existing analyses of the topic, the workshop aimed to investigate the influence of Felix Klein on the development of mathematics (especially number theory, algebra, geometry, analysis, applications of mathematics in scientific and technical fields as well as in mathematics education) in countries other than Germany. The goal of the workshop was to take a look at mathematicians of foreign origin who studied with Klein that have received little attention so far (including Czech, Greek, Hungarian, Japanese, Polish, Russian, and Ukrainian mathematicians) and uncover how Klein guided them through his lectures and seminars. The protocols of the lectures held in Klein's seminars (from 1872 to 1912 in Göttingen, Erlangen, and Leipzig), which are a unique and so far largely unexplored source, were the basis for the workshop.

Mathematics Subject Classification (2020): Primary: 01A55, 01A60, 01A65, 01A70; secondary: 01A72, 01A73, 97A30.

Introduction by the Organizers

The mini-workshop *Felix Klein's Foreign Students: Opening Up the Way for Transnational Mathematics*, organised by Danuta Ciesielska (Warsaw) and Renate Tobies (Jena), aimed to investigate (extending existing analyses of the topic) the influence of Felix Klein on the development of mathematics (especially number theory, algebra, geometry, analysis, and applications of mathematics in scientific and technical fields as well as in mathematics education) in countries other than Germany.

The idea for this international collaborative project came from Danuta Ciesielska, who for several years has been researching (together with two Polish colleagues) how Polish mathematicians studied in Göttingen with Klein and Hilbert, resulting in a recently published Polish monograph [1].

The mini-workshop *Felix Klein's Foreign Students: Opening Up the Way for Transnational Mathematics* joined together 17 researchers from different countries, familiar not only with their own mathematical traditions, but also with the development of national identities as well as political and cultural histories of the various regions.

Previous research showed that Klein did not have to “court” students from abroad throughout his career. Rather, they were sent to him from Scandinavian countries, from Italy, France, Great Britain, America, the Netherlands, Russia, Switzerland, Austria-Hungary, Greece, etc. We now have a good overview of Klein’s first international students and we also have a good analysis of all the women who studied under Klein (from 1893).

The mini-workshop aimed to examine the causes of Klein’s international success. Before the workshop, we had arrived at the following hypotheses, partially based on ([3]):

- (1) Klein deliberately aimed to found a *mathematical school* as early as 1872. In a letter to Gaston Darboux, Klein spoke of recreating a “school of geometrical production” as he had come to know it under Alfred Clebsch, who had just died. This was later to be considered as “a style of mathematical life that promised colossal successes for the future” ([2]).
- (2) This goal required that Klein readily share his own ideas and seek to advance them through *cooperative work*, but now, unlike Clebsch, on an international level – increasingly incorporating new methods into his practice.
- (3) Klein’s early efforts to become acquainted with various mathematical schools at home and abroad led to good personal contacts with mathematicians of numerous countries, who recommended their own students to Klein. Even when Klein was still in Erlangen in the early 1870s, Sophus Lie recommended Scandinavian students to go to Klein because they *would be encouraged* there (which would not happen if they went to Berlin).

In order to test these hypotheses, it was necessary to look deeper into the sources, especially into the protocol books containing handwritten records of the talks given at Klein’s seminars from 1872 to 1912. These 29 volumes are available online:

- <https://www.uni-math.gwdg.de/aufzeichnungen/klein-scans/klein/>
- <https://page.mi.fu-berlin.de/moritz/klein/>

In the case of (b) an attempt has been made to identify the complete names of the presenters. There are numerous errors, however, especially with foreign persons. Therefore, we also want to correct the sources with the help of our experts in the future. In addition to Klein himself, the speakers in the seminars were his students or distinguished visitors, many of them foreigners.

During the workshop the state of current knowledge about students with different languages, ethnicities and traditions at Klein's various career stations, especially in Göttingen, were discussed. Participants investigated the similarities and differences between them, while trying to identify all foreign participants in Klein's seminars and analyzing their contributions. Because of that, it was possible to achieve a better understanding of the socio-geographic profiles of the students coming to Klein, their own professional development and their subsequent impact on mathematics and mathematical life in their homelands.

The work on the records of the foreign participants in Klein's seminars allowed for addressing more detailed questions about these participants, including the following:

- Why did they want to work with Klein?
- What was their mathematical preparation before they arrived?
- What topics did Klein assign to them for their own seminar presentation?
- How did Klein further encourage them to work on these or related topics?
- Did this encouragement lead them to creating results of their own in the field?
- Were their results published, e.g. in the *Mathematische Annalen* (which was edited by Klein)?
- Were they later involved in other projects of Klein (e.g. *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*)?

The mini-workshop also addressed some questions of a general character, in particular these:

- To what extent did Klein influence (directly or indirectly) persons who later achieved outstanding results in individual mathematical fields?
- In what ways did former students of Klein impact the organisation of mathematical life in their homelands (university education, publishing, mathematical societies)?

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From Naples to Pavia, passing from Göttingen. The scientific trajectory of Ernesto Pascal and his relationship with Felix Klein

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The Italian mathematician Ernesto Pascal (1865–1940), born in Naples in 1865, completed his primary and secondary education in his hometown. Attracted by the mathematical teachings of Nicola Trudi, Emanuele Fergola, Achille Sannia and, most of all, Giuseppe Battaglini, he graduated in mathematics at the University of Naples in 1887. Soon after his degree, Pascal obtained a training scholarship for the academic year 1887–88 at the University of Pisa, where he had the opportunity to attend the lessons of Enrico Betti, Ulisse Dini, Luigi Bianchi and the young Vito Volterra. In the next academic year Pascal, encouraged by Eugenio Beltrami – who at that time was Professor at the University of Pavia – decided to go to the University of Göttingen to improve his studies.

From November 1888 to August 1889 Pascal was in Göttingen where he could meet and study with Hermann Amandus Schwarz and, most of all, Felix Klein, who contributed to orient Pascal's research towards Sigma abelian functions. Soon after his return to Italy, Pascal was appointed “Extraordinary Professor” (1890–95) and then “Full Professor” (1895–1907) at the University of Pavia, after the death of Felice Casorati. In 1907 Pascal was called at the University of Naples, where he remained until his retirement in 1935. He kept the chair of Higher Analysis, in 1910 he moved to the chair of Complementary Algebra and maintained for assignment the chair of Higher Analysis. In the same year he became editor in chief of the journal “Giornale di Matematiche di Battaglini”, replacing Alfredo Capelli. Pascal died in Naples in 1940.

The main episodes of Pascal's academic life and his scientific trajectory of research can be better understood through the reading of the letters he wrote during all his professional life. As of 1889, Pascal was in correspondence with Klein. The Göttingen State and University Library preserved eleven manuscripts (ten letters and one draft) that Pascal sent to Klein between October 1889 and August 1913

([12]). The correspondence, even if composed only by eleven manuscripts, offers useful pieces to enrich Pascal's academic life. The correspondence began soon after Pascal's return to Italy. The letters proved Pascal's positive memory of his German experience. He recalled with enthusiasm, but also with nostalgia his stimulating meetings with Klein. The Italian mathematician took part in Klein's course in Summer Semester 1889, that were devoted to the theory of Abelian functions.

During his stay in Göttingen Pascal met and could work with many German and foreign mathematicians who came to Göttingen to study under Klein. In the letters to Klein Pascal referred to some of these mathematicians with whom he remained in touch after his return to Italy. Among the mathematicians who were in Göttingen in the same period he quoted Heinrich Burkhardt, Henry White, Mellen Woodman Haskell. Klein presented the research of his students in the sessions of the Göttingen Academy of Science. The first results of Pascal's studies about Abelian sigma functions appeared in two short articles, presented by Klein, and were published in the volume of 1889 of the "Nachrichten von der K. Gesellschaft der Wissenschaften und der Georg-Augusts-Universität": *Zur Theorie der ungeraden Abel'schen Sigmafunktionen* (pp. 416-423); *Zur Theorie der geraden sigma-Funktionen* (pp. 547-553). The two articles were republished in a longer version in the volume 18 of the *Annali* of Brioschi in 1889 ([3]; [4]).

Between 1889 and 1895 Pascal's research dealt with Abelian, hyperelliptic and elliptic functions. This wide field of research was developed by Pascal taking inspiration from Klein's lectures and publications and gave birth to seven articles that appeared in the volumes 17-19 of the *Annali* of Brioschi ([3]; [4];[5];[6];[7];[8];[9]). Other influences of Klein can be found in Pascal's works about sigma elliptical functions, that were published in 1895 ([10]).

The correspondence with Klein was interrupted between March 1895 and February 1901. In this period Pascal's research continued, but his publications were mainly oriented towards handbooks for university teaching. Many monographs, prepared for his university courses, appeared between 1895 and 1897, first in a litographic version and then they were printed in paperback size by the editor Hoepli in Milan: *Esercizi e note critiche di calcolo infinitesimale* (1895); *Teoria delle funzioni ellittiche* (1896); *I determinanti: teoria ed applicazione con tutte le più recenti ricerche* (1897); *Calcolo delle variazioni e calcolo delle differenze finite* (1897); *Repertorio di matematiche superiori* (1897-1900, 2 volumes). Some years later these books were translated into German by the mathematicians Hermann Leitzmann and Adolf Schepp. These translations contributed to spread Pascal's works outside Italy. The main mathematical handbooks of Pascal had also a Polish translation thanks to the editorial work of the Polish mathematician Samuel Dickstein.

The *Repertorio* constituted an excellent contribution to a significant assessment of nineteenth-century mathematical production. It responded to the way in which studies were organised in Germany, providing an overall vision of a single discipline (analysis or geometry), in opposition to the extremely sectorial approach of Germany. Themes related to the development and the teaching of mathematics

were particularly important for Pascal. In Naples he gave a great stimulus to the teaching of mathematics thanks to the creation of mathematical seminars and laboratories, the so-called “mathematical cabinets” (*gabinetti scientifici*). Pascal treasured his German experience when he decided to realise in Naples these scientific places and he talked about them in a letter to Klein (1913, August 14th).

In my paper I will give an overview of the content of Pascal's letters preserved in Klein's archive in Göttingen in order to reconstruct the development of Pascal's research following suit Klein. The letters represented also a proof of Pascal's devotion and respect for Klein. The Italian mathematician strongly supported two scientific and celebrating ventures in Naples: a prize for summarising Klein's results about hyperelliptic and Abelian functions and the appointment of Klein as a foreign member of the Royal Academy of Sciences of Naples.

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Wilhelm Wirtinger (1865–1945) and his publications on Abelian functions, in particular theta functions

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Even though Wilhelm Wirtinger himself saw a strong cultural, in particular scientific, connection between the German-speaking parts of the Austro-Hungarian Empire and the Deutsche Reich, which had been formed between 1867 and 1871 under Prussian leadership, he can be counted among Klein's “foreign students”: