The Slovak Society for Operations Research
Department of Operations Research and Econometrics
Faculty of Economic Informatics
University of Economics in Bratislava

Proceedings of the International Scientific Conference

QUANTITATIVE METHODS IN ECONOMICS
Multiple Criteria Decision Making XVII

28th – 30th May 2014

Virt, Slovakia

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Web: http://www.fhi.sk/en/katedry-fakulty/kove/ssov/papers//

Publisher: Vydavateľstvo EKONÓM ISBN 978-80-225-3868-8
Number of copies: 80

CONTENT

PARTIAL SERVICE ROUTES WITH MULTIPLE VISITS TO EDGES

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Abstract

This paper deals with searching for a "best" circular service route passing through a given road network represented by an undirected graph. Each edge is characterized by a generalized cost, e.g. the length or travel time, and by a benefit associated to the importance of serving that edge. All these parameters are positive. The sought service route should meet a cost limit constraint and maximize the route benefit. The route does not need to be elementary, since edges can be traversed multiple times. However, the benefit associated to each traversal of an edge decreases as the number of traversals increases. The total benefit of a service route is defined as the sum of the benefits provided by each individual traversal of each edge of the route.

Keywords: *circular service route, multiple edge traversals, optimization, depth-first-search*

JEL Classification: C44
AMS Classification: 05C38, 90B10

1 INTRODUCTION

A given road network is modeled as an undirected graph $G = (V, E, c, g_n)$ where c and g are defined on the edge set E; $c(e) > 0$ is the cost of traversing edge $e \in E$ (e.g. modeling time, length, genuine cost, fuel or water consumption and so on, depending on the application) and $g_n(e) \geq 0$ is the "importance" of providing service (e.g. by a watering truck, etc.) to edge e for the n^{th} time and it will be referred to as the n^{th} *benefit* of the edge. If r is a route on G then the cost $c(r)$ of route r is the sum over all edges of the individual cost of the edge time the number of traversals along r, while the route benefit, denoted as $g(r)$, is the sum over all edges of the benefit provided by each traversal of an edge along r.

The **basic problem** (BP) is the following: Given a threshold $c_o>0$ and a vertex $v_o \in V$. The problem is to find a circular route r from v_o to v_o in the graph G meeting the constraint $c(r) \leq c_o$ and maximizing the route benefit $g(r)$.
BP was studied by [7] and [8] with application to a bus service route, where the importance $g_n(e)$ is related to the number of passengers boarding or alighting the bus when traveling along edge e for the n^{th} time, while the cost $c(e)$ models the bus traveling time along e. Both papers present heuristic methods for the solution of BP, in particular a Simulated Annealing based algorithm was proposed in [8].

The goal of the current paper is to introduce a method based on a depth first search type visit on G and show that it can improve literature results.

2 SIMILAR PROBLEMS IN THE LITERATURE

BP belongs to the family of *circular service route problems*, where a set of clients located on the edges (service on edges, SoE), or at the vertices (service on vertices, SoV) of a graph must be serviced by an agent traveling along a circular route, usually leaving from and returning to a given node v. A further distinction can be made depending on the requirement that the whole set of clients has to be serviced or only a subset, due to the limited availability of one (or more) resource(s) related to the route, such as duration, length, budget and so on. In the latter case, a sort of priority (reward, benefit, prize) is associated to the clients and the feasible route visiting the most valuable subset is searched. In the former case, usually a cost function associated to the route must be minimized. The most well known representative of the class where all vertices must be visited is the *Traveling Salesman problem*, while the *Chinese Postman problem* is among the most studied problems in the service-on-edges class.

Although BP belongs to SoE class, even in the SoV class some hints can be found for its solution, based on the fact that a vertex u can be expanded into an edge [u',u'']. Think for example to the orienteering problem [10], [6], which shares with BP the constraint on the maximum cost of the route as well as the objective function.

In the SoE class, the *Rural Postman problem* [9] requires that only a subset of edges must be visited, while the others can be visited, and this feature can be modeled by a two-values priority function: $c(e) = 1$ if service on e is compulsory, while $c(e) = 0$ if e is optional. In the *Windy Postman problem* [2], [3] the cost function varies according to the direction the edge is traversed. In the *Prize-Collecting Rural Postman problem* [1] edges have a cost and can be traversed several times but only the first traversal provides a reward; the objective function maximizes the total profit minus the route cost.

BP is also related to the *Elementary Resource Constrained Shortest Path Problem* tackled in [4] where resources are accumulated along the path, in case the problem is reformulated on an expanded network in which there is a copy of each edge for each traversal.

However, none of the abovementioned problems considers the case of having the edge benefit associated to multiple traversals of the same edge and not being constant but being a function of the number of times the edge is traversed. This feature is analyzed in [8], whose results the current paper strives to improve, and also in [5] which deals with the design of a cyclo-tourist route of maximum attractiveness, connecting two vertices u and v, with bounded duration, and subject to a budget constraint. When $u = v$, BP is a relaxation of the previous problem, so that the mixed integer linear programming (MILP) formulation developed in [5] can be straightforwardly adapted for the solution of BP. Nevertheless, as an alternative to the use of a MILP solver, this paper presents a very simple exact method of the primary type, i.e. starting from a feasible solution and iteratively improving the value of the objective function until an optimal solution is reached. For large size instances this method can be used as a heuristic, by setting a time out to the running time.

3 COMBINED EXACT-HEURISTIC METHOD AND ITS VERIFICATION

The proposed exact method is based on the combinatorial inspection of all possible routes. The method is based on a "depth first search" type visit of the graph. Clearly, its computational complexity is very high, so that it can be practically used only for small instances, but the optimal solutions reached by the exact method in those cases can be used as a "quality etalons" for heuristic methods. In some cases, if the search is wisely guided, the suboptimal feasible solutions reached by the exact method within a limited time can improve upon those reached by other heuristic methods.

At the beginning of the exact method, the set of the incident edges of each vertex $u \in V$ (the so called star of u denoted by $S(u)$) are ordered according to the value of the edge attributes. We implemented and tested the following two approaches:

- The first variant V1 uses "importance" $g_1(e)$ of the edge e as a descending ordering criterion.
- The second variant V2 uses the ratio $g_1(e)/c(e)$ of the edge e as a descending ordering criterion.

The procedure starts from the initial vertex v and selects the first incident edge in $S(u)$ according to the chosen criterion. Such edge is appended to the current route. If the total length of the new route does not exceed the maximum duration c_0, the procedure iterates, resuming from the ending vertex of the added edge. Otherwise, the procedure backtracks, the inspected edge is discarded, and the next one in $S(u)$ according to the chosen order is considered. When the route goes back to the initial vertex v, the total demand is computed and compared to that of the incumbent solution. In case of improvement, the current route is saved as the new incumbent solution and the procedure resumes the search, backtracking from the second-to-last node.

3.1 Experimental results

Both the above mentioned variants of the proposed exact method were implemented in relatively slow environment of Visual Basic for Application in MS Access on standard hardware configuration (PC with Windows 7, Intel® Core™ i5 CPU 750@ 2,67 GHz, RAM 4 GB). In order to compare the efficacy, we tested our method on the same instance as in [8], which in turn is the same used in [7], since it is the only one for which all data are reported. The test instance is a network with 25 vertices and 40 edges. For each edge both attributes, i.e., costs $c(e)$ and "importance" $g_1(e)$, are given, as well as the fading strategy for benefit at successive traversals, at which the benefit goes down to one fifth of the initial one, i.e., $g_n(e)=0.2g_1(e)$ for $n>1$. The maximum duration of the route c_o is set to 100, again accordingly to [7] and [8].

Figures 1 and 2 depict the test network, on which the best route found in [8] (Figure 1) and the route corresponding to our best solution (Figure 2) are highlighted in bold. On each edge, duration and benefit at first traversal are depicted as the pair $c(e).g_1(e)$.

Regarding running time, remind that we are comparing a prematurely terminated exact approach to a meta heuristic. Moreover, since the application concerns a design problem which does not need to be solved in real time, and taking into account the limited performance of the computational environment, we considered about 100 minutes as a reasonable timeout after which the search is interrupted and the incumbent solution is returned. Both variants of the proposed exact method (EM) could not converge within this time limit, so that the global optimality of their solution could not be certified. However, both variants yielded the same solution, and this one improves upon those reached by the heuristics based on Simulated Annealing (SA) reported in [8]. In addition, it is well known that the performance of SA based procedures is quite sensitive to parameter setting. In fact, an intensive and thorough calibration phase was performed in [8] to obtain the parameter setting used to find their best solution. Since such values can be instance dependent, we believe that a fair comparison of computing time can not be done without taking into account the time needed for calibration in addition to the solution time, which the author in [8] report to be less than 1 second. Actually, in [8] the maximum number of iterations of the SA is set to 10.000 and the authors say that this limit has been chosen in order to guarantee that the SA procedure could reach its best possible solution (extra time would yield no improvement). However, it is not specified if this limit is reached or some other stopping criteria, such as stagnation behavior, was also used. Furthermore, no information is provided regarding the computing environment used for the experiments. For all these reasons we believe that there is a lack of information for doing a meaningful comparison of running times between our methods and the one in [8].

Table 1 summarizes the results. The computational time reported in table 1 is the time at which the best solution has been found. The total running time has been the same and limited by time-out for both variants of EM. It is worth noticing that our solution uses all the time allowed as its solution reaches the maximum duration c_0, while the solution in [8] uses only 99% of it.

Table1: Comparison of reached results

	EM – V1	**EM – V2**	**SA ([8])**
Total route importance *g*(*r*)	157,4	157,4	154,4
Total route costs *c*(*r*)	100	100	99
Computational time [h:m:s]	1:40:22	0:05:28	unknown

Table 1 shows a substantial difference between EM variants V1 and V2 in computational time. As follows from the description of EM variants above, the only difference between the method variants is in the order of route inspections. This ordering strategy strongly influence the method efficacy depending on the configuration of the network topology and edge attributes. We have tested the both variants of EM on 5 other network instances with randomly generated edge attributes. In 3 cases the V1 was more effective, in one case the computational time was nearly the same and only in one case the V2 was more effective.

Table 2 summarizes the progress of feasible solutions for both variants of proposed exact method. The abbreviation NIR represents the Number of Inspected Routes. Computational time is in h:mm:ss format and represents the time at which a new incumbent solution is found. Solution process was interrupted after above mentioned time limit.

Table2: Comparison of EM solution progress

	EM – V1			**EM – V2**		
Feasible solution Nr.	**Time**	**g(r)**	**NIR**	**Time**	**g(r)**	**NIR**
1	0:00:01	110,2	1	< 1s	109,2	1
2	0:00:01	112,4	4	< 1s	112,0	2
3	0:00:01	120,8	115	< 1s	115,4	10
4	0:00:01	124,6	491	< 1s	119,6	11
5	0:00:03	128,4	1468	< 1s	120,6	39
6	0:00:03	129,6	1513	< 1s	120,8	61
7	0:00:05	133,0	2319	< 1s	122,8	81
8	0:00:06	134,4	2837	< 1s	123,4	118
9	0:00:06	137,6	2978	< 1s	129,0	150
10	0:00:07	139,2	3092	0:00:02	134,4	1175
11	0:00:07	140,4	3100	0:00:08	134,8	4331
12	0:00:07	141,6	3102	0:00:08	137,8	4337
13	0:00:07	145,2	3280	0:00:14	138,8	7604
14	0:16:04	145,6	494522	0:00:16	141,8	8256
15	0:16:04	146,8	494524	0:00:20	143,8	10207
16	1:35:08	147,8	3491448	0:00:20	147,8	10208
17	1:35:09	150,4	3492222	0:00:21	150,4	11030
18	1:35:11	151,4	3492858	0:00:23	151,4	11618
19	1:35:59	156,8	3517788	0:01:04	156,8	32727
20	1:40:22	157,4	3651982	0:05:28	157,4	167675

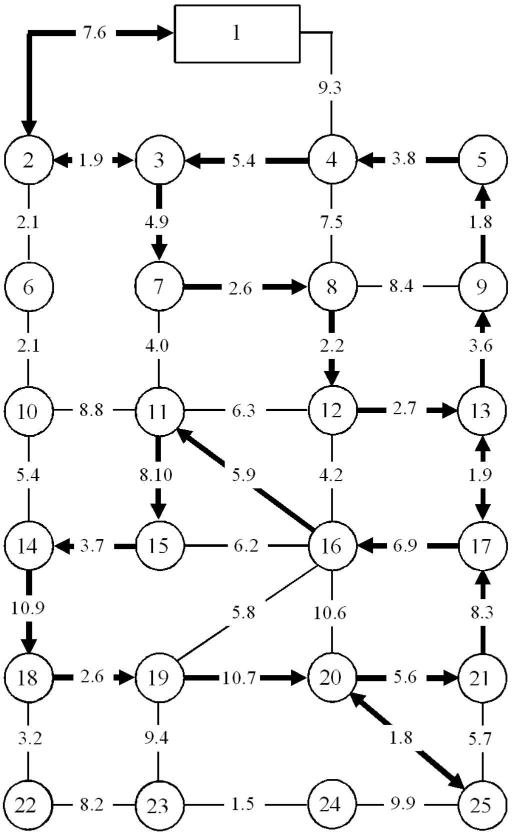

Figure 1: The best solution presented in [8]

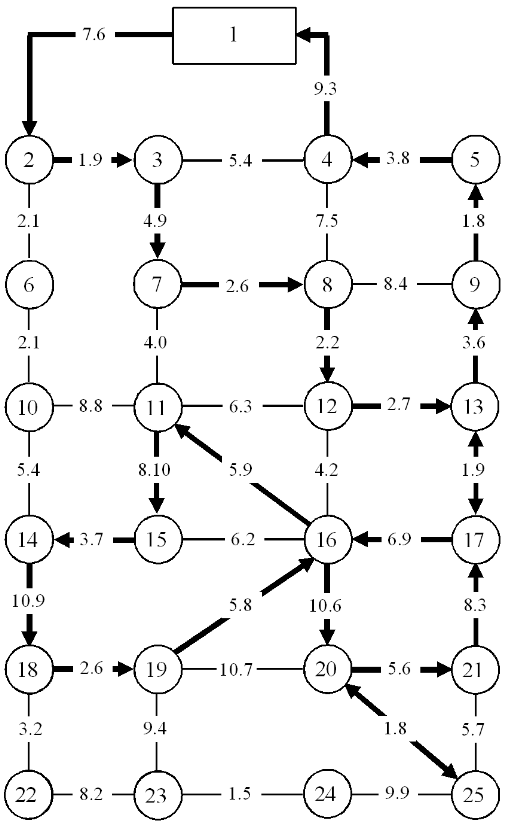

Figure 2: The best solution reached by EM

4 CONCLUSION AND OUTLINE OF FUTURE RESEARCH

This paper deals with a problem, denoted BP, of finding a circular service route in an undirected graph. The constraint requires that the "cost" of the resulting route does not exceed a given limit and the objective i.e. the total benefit of traversed edges is maximized. BP is characterized by the special feature that edges can be traversed several times, and each time they yield a (potentially different) contribution to the objective function.

Moreover, the relations of BP to other types of circular service routes is outlined and it is shown that, from the mathematical point of view, BP is a relaxation of the problem of designing optimal routes for cyclo-tourists. A combined exact-heuristic method, based on dept-first-search technique is proposed and the results are compared to those provided by Kuo et al. in [8]. The new method reached better solutions within a time that is reasonable for a design problem which is solved off line.

The future research will address the following issues:

- improving the convergence speed of the proposed method, for example by computing for each vertex u the minimum cost path to destination v, so that we can detect at an earlier stage that a given partial route from v to u can not return back to v within the limit c_0;
- enlarging the test bed to include other network instances;
- exploring the potentials of applying local search to the suboptimal solutions found by our method, yielding a hybrid math-heuristic approach, to be used to tackle larger instances;
- finding alternative MILP models, different from the one related to the problem in [5], which take advantage of the simpler structure of BP with respect to the problem in [5];

- studying how the method can be customized to a variety of practical applications, such as routing watering trucks equipped with a limited amount of water and due to clean streets with different priorities.

Acknowledgements: This work was supported by the Czech Science Foundation Grant No. P 402/12/2147, "*Economically Optimal Processes on Networks*"

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