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# Computer-aided durability analysis of complex structures under multiaxial random loadings through the Projection-by-Projection spectral method

Computer-aided durability by PbP method

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## **Conflict of interest**

The authors do confirm that they have no conflict of interest to declare.

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## **Abstract**

This letter deals with a set of numerical case studies that were devised as mean to help users in implementing the Projection-by-Projection (PbP) multiaxial spectral criterion. A first case study scrutinised the PbP method applied to different types of biaxial random stress and material fatigue properties. A second case study investigated a structure under random excitations and provided practical guidelines on how to implement the PbP method for a computer-aided durability analysis.

## **Keywords:**

Multiaxial fatigue; random loading; frequency domain analysis; Power Spectral Density; finite element method

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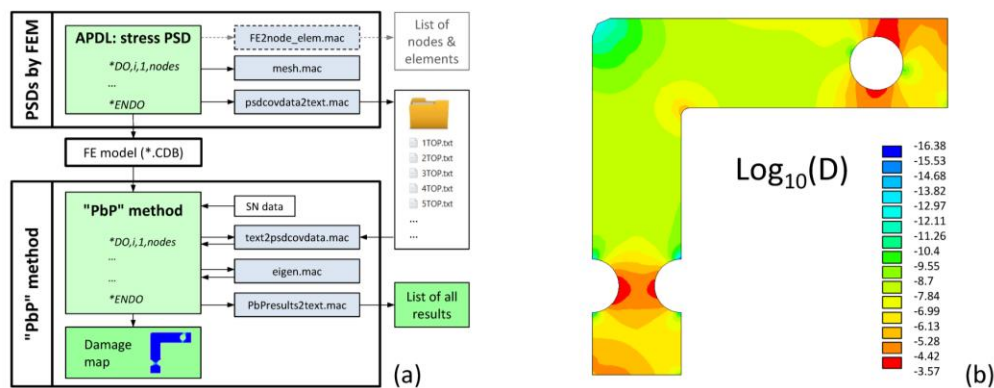
## **Introduction**

Computer-aided analysis has become a valuable tool in the durability assessment of complex structures subjected to multiaxial random loadings. This is especially true when the fatigue life is estimated through a finite element (FE) analysis, in which stress power spectral density data are processed by multiaxial spectral methods defined in the frequency-domain. Writing routines to implement a spectral method, to embed them within a FE software and to check if they provide correct output, is a task often not trivial, but which can be made easier if examples are available for comparison.

This goal has been pursued in <sup>1</sup> for the Projection-by-Projection (PbP) criterion – a stress invariant based criterion for multiaxial random loadings<sup>2</sup>. Numerical test cases were investigated, in which different types of biaxial stress were applied to various materials. Results of each analysis step were tabulated to allow an easy understanding of the PbP criterion, as well as to highlight some of its main features (as, for example, the material sensitivity to hydrostatic stress, and the role of stress correlation). A computer-aided durability analysis of a L-shaped structure under random excitations was also presented with the purpose to give practical details on how to implement the PbP method in FE software Ansys.

## **Numerical test cases and results**

The work reported in <sup>1</sup> presented some numerical examples, whose results can be used as a basis for comparison if one is about to implement the PbP spectral method. The first case study investigated three material types (with different sensitivity to hydrostatic stress) subjected to four types of biaxial stress states (i.e. tension-tension and tension-torsion loading). Each stress is defined by its frequency spectrum (Power Spectral Density, PSD). Proportional and not-proportional stresses were considered. The results of each analysis step were tabulated to permit anyone to compare them with own results, making easier to learn the PbP method. The article also pointed out some features of the method, which emerged from the material/stress combinations analysed (e.g. material sensitivity to hydrostatic stress, effect of stress correlation on fatigue damage).



**Figure 1.** (a) Flowchart of PbP method in Ansys; (b) Damage map (logarithmic scale).

The second case study dealt with a computer-aided durability analysis of a thin L-shaped structure subjected to random accelerations (although the analysis procedure is applicable to structures of whatsoever complexity). The case study described the flowchart and APDL routines by which the PbP criterion was embedded into the finite element software Ansys (Figure 1(a))<sup>1</sup>. The analysis returned the fatigue damage in every point of the structure (Figure 1(b))<sup>1</sup>. Once the stress PSDs are computed, the damage map for any other material can be calculated with no great computational effort. This result is of great relevance in early design phase, when one often needs to scrutinise the fatigue behaviour of several materials with the purpose to find out the most suitable for a given component or structure subjected to vibratory input; or when one needs to find that component geometry which best exploits the fatigue properties of a given material.

### Recent Trends in Research

The development of spectral methods (or frequency-domain criteria) dates back to the 1950s and it has shown a considerable growth over the last decades<sup>3</sup>. Spectral methods have now widely recognised as being reliable tools for structural durability. They exploit a frequency-domain approach in which a random loading (or stress) is characterised through its frequency spectrum – called Power Spectral Density (PSD). Through the theory of random processes, analytical expressions can be derived by which the fatigue damage can be estimated directly from the stress PSD<sup>3</sup>. To date, analytical solutions are available for stationary random loadings with normally distributed values, although corrections for non-Gaussian case, or solutions for special classes of non-stationary processes, were also proposed<sup>3</sup>.

Developed for uniaxial random loadings, spectral methods were later extended to multiaxial loadings<sup>4-6</sup>. Most of the classical multiaxial criteria were thus reformulated in a frequency-domain fashion, which is more suitable for random loadings. Noteworthy examples are criteria based on the critical plane (Matake, Carpinteri-Spagnoli), on stress invariants (Crossland, Sines, PbP), on equivalent stress (normal or shear stress, von Mises stress, multiaxial rainflow counting)<sup>4-6</sup>. General principles, analogies and differences among criteria, advantages and drawbacks, were emphasised<sup>4</sup>.

Laboratory tests were also performed with different types of uniaxial and multiaxial random loadings, thus providing a set of experimental data which – though not as numerous as those of constant amplitude loading – allowed checking the accuracy of theoretical proposals, with very promising outcomes<sup>6,7</sup>. Meanwhile, case studies were also presented in which spectral methods were used to process results from finite element analysis, so taking full advantage of a low computational effort needed to perform a durability analysis in frequency-domain. The task, as shown in<sup>1</sup>, is also that of writing routines and programs (or adapt existing ones) with the purpose to implement spectral methods directly into commercial finite element software, thus making the overall analysis easier.

### **Future Developments and Challenges**

In the authors' opinion, the research on spectral methods is likely to progress on two main directions in the near future. From a more theoretical point of view, new models will be developed to address issues only partly investigated to date, with the purpose to widen the hypotheses and the range of applicability of existing methods. For example, the topic of non-stationary loadings for which only few cases were solved, or the variability in the estimated fatigue damage that comes from the uncertainty in estimated power spectral density data. In multiaxial fatigue criteria, also the role of tension-torsion material parameters, or of stress correlation, deserves attention. It has finally a certain relevance to perform tests with various types of multiaxial random loadings, with the purpose of widening the existing (rather scarce) experimental database.

From a more practical point of view, there is the need to make the spectral methods more accessible to engineers using computer-aided design. To this end, the work in<sup>1</sup> illustrated an example for the PbP criterion, showing how it could be a valid tool in the durability analysis of complex structures under random loading. The authors' aim was to write a contribution that provided engineers engaged in vibration fatigue a sort of practical guide to learn the PbP

method. It is the authors' belief that following the same approach for other fatigue criteria could help users interested in multiaxial spectral methods.

## References

1. Benasciutti D, Zanellati D, Tovo R. The “Projection-by-Projection” (PbP) criterion for multiaxial random fatigue loadings: Guidelines to practical implementation. *Frattura e Integrità Strutturale*. 2019;13(47): 348–366.
2. Cristofori, A., Benasciutti, D., Tovo, R. A stress invariant based spectral method to estimate fatigue life under multiaxial random loading. *Int J Fatigue* 2011;33(7): 887–899.
3. Benasciutti D. *Fatigue analysis of random loadings. A frequency-domain approach*. LAP Lambert Academic Publishing; 2012.
4. Benasciutti D, Sherratt F, Cristofori A. Recent developments in frequency domain multi-axial fatigue analysis. *Int J Fatigue*. 2016;91: 397–413.
5. Carpinteri A., Spagnoli A., Vantadori S. A review of multiaxial fatigue criteria for random variable amplitude loads. *Fatigue Fract Eng Mater Struct*. 2017;40(7): 1007–1036.
6. Niesłony A, Macha E. *Spectral method in multiaxial random fatigue*. Germany: Springer-Verlag; 2007.
7. Mršnik M, Slavič J, Boltežar M. Multiaxial vibration fatigue – A theoretical and experimental comparison. *Mech Syst Signal Proc*. 2016;76-77: 409–423.