

Glue ball masses and string tension in lattice Q.C.D.

by

The Ape Collaboration

M. Albanese, F. Costantini\*, G. Fiorentini\*, F. Fiore,  
M.P. Lombardo and R. Tripiccone

Infn - Sezione di Pisa

\*: and Dipartimento di Fisica, Università di Pisa

†: and Dipartimento di Fisica, Università di Cagliari

P. Bacilieri, L. Fonti, P. Giacomelli‡ and E. Remiddi\*\*

Cnaf - Infn, Bologna

\*\* : and Dipartimento di Fisica, Università di Bologna, via Iriero, Bologna

‡: and Rockefeller University, New York.

M. Bernaschi, N. Cabibbo, E. Marinari, G. Parisi and G. Salina

Dipartimento di Fisica, Università di Roma 'Tor Vergata'

and

Infn - Sezione di Roma

S. Cabasino, F. Marzano, P. Paolucci, S. Petrarca and F. Rapuano

Dipartimento di Fisica, Università di Roma 'La Sapienza'

and

Infn - Sezione di Roma

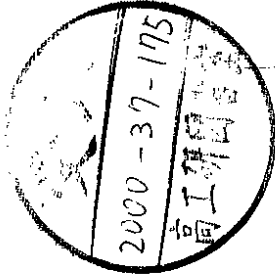
P. Marchesini and R. Rusack§

Cern - Geneve

§: and Rockefeller University, New York.

**Abstract**

We study glue ball masses in lattice QCD. We present the first numerical determination of the mass of the lowest spin 2 state in the scaling region, and find that it is close to the lowest spin 0 state. We present very precise results for the string tension and for the spin 0 state, obtained by analyzing a large set of operators. We find that finite size effects are significant.



ROMZF/87/014

R6-87/4

Numerical simulations of lattice QCD allow a determination of glue ball masses. These masses are determined by the large time behaviour of the connected correlation function

$$G(t) = \langle O(t) \cdot O(0) \rangle - \langle O(t) \rangle \cdot \langle O(0) \rangle \approx e^{-mt}$$

where  $O(t)$  is a suitable gauge invariant operator. The problem is in the fact that  $G(t)$  decreases rapidly, and is obtained as a difference of two fluctuating quantities of comparable magnitude. This difficulty, already challenging in the case of the spin 0 state, increases in the case of higher spin states. As a consequence the determination of the mass of  $2^{++}$  state is the object of a heated dispute. In this paper we try to solve this question, by both brute force (using our newly built Ape computer) and by methodological refinements - we propose a new method for choosing useful operators. The result is very satisfactory, and we succeed in measuring the mass of the  $2^{++}$  state, that turns out to be comparable to that of the  $0^{++}$ . We also give very precise estimates of the  $0^{++}$  state mass and of the string tension, carefully studying the finite size effects, and the convergence to the infinite volume result. We study large lattices, but also small ones. We think this is the only way to define what a large lattice is.

We study a pure gauge lattice  $SU(3)$  theory, with a standard Wilson action. We use a Metropolis updating scheme; we sweep sequentially through the lattice, and we try 5 updates of each link (5 hits, in the usual jargon). We keep the single hit acceptance close to 50 percent. We renormalize the  $SU(3)$  matrices and we measure all our observables once each 20 sweeps of the full lattice, except for the average energy, which is measured at every sweep.

The results of this paper have been obtained at  $\beta = 6/g^2 = 5.9$ . We have chosen this value in order to achieve a good control on the finite size and finite time effects, working as deep as we can in the scaling region. The subject of the scaling behaviour of our data will be studied in a forthcoming paper<sup>(1)</sup>. We work on lattices with periodic boundary conditions; we keep the time length of the lattices equal to 32 (the Euclidian time label  $t$  will run in the following from 1 to 32), and we consider spatial volumes of  $10^3$ ,  $12^3$  and  $16^3$  sites. In this way we can study finite size effects with no contamination from finite time effects, which should be negligible. We start from a  $\beta = \infty$  gauge field configuration,

discarding at least 1500 configurations (4000 for the  $10^3$  lattice) for thermalization. We have then performed 64000 iterations on the  $10^3$  lattice, 56000 on the  $12^3$  and 24000 on the  $16^3$ .

This computation has been performed on a 4 unit Ape computer<sup>(2)</sup>, a general purpose parallel processor, whose design has been optimized for the solution of homogeneous differential equations, and more specifically for dealing with lattice QCD. This 4 unit Ape is the first running prototype of the machine. It has a peak speed of 256 Mflops in complex arithmetics, and a 32 Mbyte memory. The whole Monte Carlo program is written in a high level language which, especially designed for Ape, is cross-compiled on a Vax. The program updates a link in 50  $\mu$ sec (we estimate that with some programming effort we could improve the performance of our code by a significant factor). The results described in this paper have been obtained in about one month of continuous runs. The market cost of performing the same computation on a commercial supercomputer would have exceeded the entire cost of the Ape-INFN project, including the construction of two 4 unit Apes and a 16 unit computer, a one Gflops machine.

We measure the decay of correlations from a cold source<sup>(3)</sup> at  $t=1$ . This gives a stronger signal than in the case of operator - operator correlation functions, but also introduces a strong distortion at short distance from the source. We choose this method because it seems to be very effective on large lattices. Work done by using operator correlations (and the smearing technique we introduce below) is in progress<sup>(4)</sup>. We construct the source by setting to the identity all the  $t=1$  links pointing in the  $\hat{x}$  and in the  $\hat{y}$  directions. With such a source we can monitor the behaviour of the  $0^{++}$  and the  $2^{++}$  gluonic states, and of the string tension.

We estimate the energy of the system in absence of the source by averaging over time slices that are at least 8 lattice spacings away from the cold wall. We get results that are compatible for the 3 lattice sizes, that is 0.58187(6) at  $10^3$ , 0.58182(4) at  $12^3$  and 0.58187(3) at  $16^3$ . We have checked that these values are really asymptotic, in the sense that the mean value does not systematically change if we discard more time slices close to the source.

As we already mentioned we consider different operators, that we define by using a smearing procedure. The general point of view is the one proposed in references 3 and 5, that is we try to eliminate as much as

possible the unphysical short distance fluctuations. We use a sequence of operators  $O^{(s)}$ , such that  $O^{(0)}$  is the elementary space like plaquette (the four links that form it are pointing in 2 of the 3 space directions). The operator  $O^{(s)}$  is recursively defined from the operator  $O^{(s-1)}$ , by applying a smearing procedure (see fig.1):

$$1) \quad U_i(n) \longrightarrow U_i(n) + \epsilon \sum_{\eta=+1,-1} \sum_{j \neq i} P^\eta_{i,j}$$

where the r.h.s. of (1) is projected onto the  $SU(3)$  group, and  $P^\eta_{i,j}$  is the oriented incomplete plaquette operator, product of 3 gauge matrices. It lies in the  $(i,j)$  plane, excluding the link  $U_i(n)$ . For  $\eta = +1,-1$   $P^\eta_{i,j}$  has  $U_i(n)$  as its left or right edge. In fig.1 we show this transformation. We use the analogous construction for the Polyakov loops.

This transformation averages the gauge fields with their neighbours, suppressing in this way the short wavelength fluctuations in the gauge invariant sector. To clarify the meaning of this procedure we can consider the effect of a similar rule applied to a scalar field:

$$\phi^{(s)}(n) \longrightarrow \phi^{(s+1)}(n) = \phi^{(s)}(n) + \epsilon \sum' \phi^{(s)}(n')$$

where the sum runs over the first spacelike neighbours of  $n$ . For large  $s$  and small  $\epsilon$ , we can define a proper time for the smearing dynamics,  $\tau = s\epsilon$ , so that

$$2a) \quad \partial\phi/\partial\tau = \nabla^2\phi$$

In momentum space, the effect of the smearing operator  $S$  is the multiplication by  $\exp(-\tau k^2)$ . The same argument can be applied to our case. In the continuum limit, with proper time  $\tau = s\epsilon$ , eq. 1 is equivalent to

$$2b) \quad \partial A_i/\partial\tau = D_i F_{ik}$$

and, for gauge invariant quantities, the smearing operator is easily seen to correspond, neglecting the non linearities of equation 2a, to multiplication by  $\exp(-\tau k^2)$ . If we assume that the wave function of the  $0^{**}$  glueball is gaussian, (i. e.  $\propto \exp(-\eta k^2)$ ), and we measure the expectation

value of the smeared plaquette (that in the continuum limit corresponds to  $F_{ij}^2 \approx k^2$ ), we get that, for  $\tau \rightarrow \infty$ ,

$$3) \quad \langle 0^{**} | O_{ik} | 0^{**} \rangle = \langle 0^{**} | S_{ik} O_{ik} | 0^{**} \rangle \approx (\eta\tau)^{-5/2}$$

One can see some analogies between this method and the Monte Carlo Renormalization Group ideas. We like in our method the idea that we do not have to choose a priori some few operators (small in practice) on which to perform the matching. We consider instead a 'continuous' set of operators that become very large after many smearing steps. If we get (as we do) results that are independent from the operator (at least for large smearing) we can be sure that these results are not influenced from a scale (the dimension of the matching operator) that we have chosen by hand.

We compute the values of the glue ball masses (of quantum numbers  $J^P, C$ )  $m(0^{**})$  and  $m(2^{**})$ . We also compute the value of the string tension, by monitoring the behaviour of the Polyakov loops at large separation from the source. The  $0^{**}$  Green function is obtained by summing

$$4) \quad G_0^{(s)}(t) = \sigma^{(s)}(x,y) + \sigma^{(s)}(y,z) + \sigma^{(s)}(x,z)$$

where the plaquette  $O$  are evaluated at time  $t$ , and  $s$  indicates the different operators corresponding to different smearing. The source is at  $t=1$  in the  $x$ - $y$  plane). For the  $2^{**}$  state

$$5) \quad G_2^{(s)}(t) = -2 \sigma^{(s)}(x,y) + \sigma^{(s)}(y,z) + \sigma^{(s)}(x,z)$$

The two gluonic channels are contaminated by a state of mass  $2L\sigma$ , where  $\sigma$  is the string tension and  $L$  is the spatial extent of the lattice (see ref. 11 and references therein for discussions about this point). From what is known in the current literature<sup>(6,7,8)</sup> it is clear a priori, then, that for  $\beta=5.9$  the situation is very dangerous on a  $10^3$  lattice. Indeed in this case  $20\sigma \approx m(0^{**})$ . The situation is analogous, if not worse, in the  $2^{**}$  channels. In such conditions when we look at the ground state in the glueball channel we do not really know what we are looking at.

Our data analysis tries to use as few prejudices as it is possible. The effective masses are a very useful tool. They are defined as

the log of the ratio of the correlation function at time  $t$  with the one at  $t+1$ , but for the correction of the hyperbolic cosine effect coming from the boundary (In most of our data this correction is negligible). We obtain a reliable estimate when the effective mass does not depend on the time at which it is estimated, and on the operator used in the measurement. We stress that we are working with a very large set of operators, and that all the results we give here do not depend on the operator we consider.

In figs. 2a and 2b we give a typical example of what we mean by stability of the result. For the  $12^3$  lattice we plot the effective string tensions at different times versus the operator we consider. The points are plotted as a function of the inverse smearing number. The points at smearing $^{-1} = 1$  are the ones one would get by considering the usual Polyakov loops. The results from  $G(1)/G(2)$  are strongly dependent on the choice of operator. Already at time 2 over 3 the result is less dependent on the operator, and at this scale the results of  $G(3)/G(4)$ ,  $G(4)/G(5)$ ,  $G(5)/G(6)$  and  $G(6)/G(7)$  are identical. In fig. 2b we just show the largest operators (higher smearing), by magnifying fig. 2a, to evidentiate the residual 10% difference between times  $G(3)/G(4)$  and  $G(4)/G(5)$ ,  $G(5)/G(6)$  and  $G(6)/G(7)$  coincide with  $G(4)/G(5)$ , but with a larger statistical uncertainty. The crossing of two different curves for operators with  $s \approx 10$  can be interpreted as a change of sign of the coefficient of the first excited state. This interpretation is confirmed by a two state fit of the  $t$  dependence at different smearing, and offers a possible way to estimate the mass of the lowest state by using the point where two curves at different Euclidian time cross ( effective mass versus the smearing number). That will be a point in which the effect of the first excited state is very suppressed. We have verified that such an estimate, given for example by using the crossing of  $G(2)/G(3)$  versus  $G(3)/G(4)$ , which can be determined accurately, is systematically lower by 5% than the infinite time plateau.

We also use standard fitting routines. Our data are good enough to allow precise fits, which use the information coming from very large Euclidian times (up to a distance of order 10). We have found that the best approach was to use the simplest fit ( $G \approx A \exp(-mt)$ , with a single mass), discarding enough points close to the origin. This approach works, and we obtain fitted results which are independent from the operator and from the lowest time used in the fit. We have also performed two mass fits including all points but one. The value of the low mass obtained in these fits

agrees with that obtained in the simpler fits. The value of the high mass fluctuates too much when we divide our data in independent subsamples, and cannot be considered meaningful.

The last note about the analysis is that one can try to use results such as the one obtained in eq. 3 in order to get, for example, informations about the shape of the wave function. The results of the global fits are compatible with those from the fits described above.

Quoting our numerical results we will always quote what we judge to be true errors, evaluated over uncorrelated measurements. This is true both for effective masses and mass ratios, and for the results of fits done by using standard minimization programs. In the case of fitted results, due to the high non linearity of the observables, we never trust the standard error given by the fitting routine, but we always repeat the fit over  $n$  independent subsamples, and quote as standard error the dispersion over the results of different fits. As central value we quote the result of the single fit performed over the whole statistical sample. For the  $10^3$  lattice we consider five subsamples of 12000 consecutive iterations each, for  $12^3$  seven groups of 8000 iterations, and for  $16^3$  six groups of 4000 iterations.

We now present our results. We start from the  $0^{++}$  gluonic state. For the  $10^3$  lattice we estimate from the effective masses

$$6) \quad m(0^{++}, L=10) = 0.65(3).$$

A one mass fit with  $t_{\min} = 4$  gives a slightly higher result, independent from the operator, but with a larger statistical error. On the  $12^3$  lattice from the effective masses we get

$$7) \quad m(0^{++}, L=12) = 0.76(4).$$

A one mass fit with  $t_{\min}=4$  gives a very accurate result (with a mass that turns out to be independent from the operator) of 0.75(5). In fig. 3 we plot the result of the fit versus the operator we have used in the fit. In this case, like in all the other ones, a 2 mass fit is not very useful.

For the  $16^3$  lattice we get from effective masses

$$8) \quad m(0^{++}, L=16) = 0.82(5).$$

The same fit we used for  $12^3$  gives a very nice, operator independent result, with a slightly higher error, 0.83(7). The results seem to exhibit a possible residual finite size effect, of order of 5%, between  $L=12$  and  $L=16$ . We will discuss later the physical meaning of such results.

The amplitudes one gets from the one mass fits are very reasonable, and qualitatively consistent with the behaviour predicted in eq.3, and with the global fits, in which we were using as inputs all the operators.

For the  $2^{**}$  state we find useful results up to  $t=4$ . The signal on the fifth time slice is already too noisy to be really useful. The results obtained on the  $10^3$  lattice have a shape that is different from the other two lattice sizes. This is not a surprise as we know from the larger lattices that  $m(2^{**})$  is close to  $m(0^{**})$ . In this circumstance we expect a large contamination from the string tension, which on a  $16^3$  lattice corresponds to a state of  $2L\sigma \approx 1.0$ . If we want to read a mass from the  $10^3$  lattice we have to quote, from the crossing of  $G(2)/G(3)$  and  $G(3)/G(4)$  that  $m(2^{**}, L=10) = 0.6(3)$ . For the  $12^3$  and  $16^3$  lattice we get a nicer signal from  $G(3)/G(4)$ . The result is still dependent on the operator we choose, but from the crossing of  $G(2)/G(3)$  and  $G(3)/G(4)$  we get

$$9) \quad m(2^{**}, L=12) = 0.80(20), \quad m(2^{**}, L=16) = 1.00(15).$$

We estimate that the systematic error on these results is by far smaller than the statistical error (previous considerations on the  $0^{**}$  glue ball may suggest a systematic positive shift of .05). If we average the Green functions over the  $12^3$  and the  $16^3$  lattices we find

$$10) \quad m(2^{**}, L=12 \text{ and } 16) = 0.85(12).$$

In this case we also have a signal for  $G(4)/G(5)$  for large operators, consistent with the previous estimate. That makes our claim about the systematic error stronger.

The string tension is measured very precisely. The results are given in table 1, together with the ones for the glue ball states. In this case a very clear operator and time independence gives a result compatible with the one coming from the fit, and with that coming from the crossing

point.

Our data are in reasonable agreement with a  $\pi/(3L^2)^{12,7}$  contribution to the string tension, coming from the gaussian string. The difference in the value of the string tension we would expect between  $12^3$  and  $16^3$ , due to the  $\pi/3$ , would be 0.003. We have a difference of 0.001 in the wrong direction, but with an error of 0.002. Instead the deviation of the  $\sigma$  measured on the  $10^3$  lattice from the asymptotic value can be explained by such an effect (we do agree with the numerical result obtained in ref.7 on the  $10^3$  lattice at the same  $\beta$ ). If we include the  $\pi/3$  we get (in the following we assume  $\sqrt{\sigma} = 420$  MeV):

$$11) \quad a^{-1} = 1700(50) \text{ MeV}.$$

that would be raised by 100 MeV neglecting the  $\pi/3$  term. This is equivalent to:

$$12) \quad \Lambda_L/\sqrt{\sigma} = 10.7(2) \cdot 10^{-3}.$$

a value that is 10 percent higher than that obtained in ref.6 from the asymptotic behaviour of large Wilson loops at  $\beta=6.2$ .

We have also measured directly  $m(0^{**})/\sqrt{\sigma}$ . We give these results in table 1. At the 10 percent level we are not able to exhibit any statistically significant lattice dependence of this ratio. We thus agree with the result obtained with a different action in ref. 10, and our  $0^{**}$  glueball has a mass of 1400(100) MeV.

The main result of this paper is that the  $2^{**}$  state has a low mass, with a ratio  $m(2^{**})/m(0^{**})$  of order one (with an error of order 20 percent). This is a reasonable result, in general agreement with what we usually learn about spin forces for quarks. With our statistics we cannot say which state is the lighter one (we have indications, if any, for the  $0^{**}$  being lighter than the  $2^{**}$ ), but the point of view assumed in ref. 11 seems to lead to a roughly correct picture. This finding is in contrast with the claims of ref.9, asserting asymptotic scaling of the  $2^{**}$  state with  $m(2^{**})/m(0^{**}) \approx 3$ .

Our result for the  $0^{**}$  mass on the  $10^3$  lattice agrees with the one given in ref.8. On the contrary the results on larger lattices are very

different from the result of extrapolating from the  $10^3$  lattice by means of the small volume results<sup>(13)</sup>; the dimensionless 3 glueball coupling estimated in 7 was already very high (i.e.  $O(10^2)$ ). The same procedure applied to our data would give a 3 glueball coupling of the order of  $10^3$ , but for such large couplings the inclusion of multiple exchanges cannot be avoided. We rather believe that the finite volume effects on the glue ball masses originate from the mixing with states of mass  $2L\sigma$ .

These results would not have been obtained without the crucial contributions given to the Ape project from A. Fucci, S. Galeotti, A. Miotto, D. Pascoli, D. Passuello, R. Petronzio. We warmly acknowledge the precious contributions given to us, at various stages of our work, from C. Giovannella, A. Frighi, M. Matone, F. Pagani, P. Rossi, B. Tirozzi, G.M. Todesco. We especially thank F. Cesaroni and the whole electronics workshop in Roma. We have received a wonderful support from the Dipartimento di Fisica of Universita' di Roma 'La Sapienza', and we warmly thank its director R. Bizzarri. The logistic, organizative, administrative and technical help and support we have received have been so intense that we cannot try to list all the people who helped us.

## REFERENCES

- 1) Work in progress.
- 2) The Ape Collaboration: P. Bacilieri, S. Cabesino, F. Marzano, P. Penlucci, S. Petrarca, G. Salline, M. Cabibbo, C. Giovannella, E. Marinari, G. Parisi, F. Costantini, G. Fiorentini, S. Galeotti, D. Passuello, R. Tripliccione, A. Fucci, R. Petronzio, F. Rapuano, D. Pascoli, P. Rossi, E. Remiddi and R. Ruseck, at the conference "Computing in High Energy Physics" (Amsterdam, The Netherlands 1985).;
 

The Ape Collaboration: M. Albanese, P. Bacilieri, S. Cabesino, M. Cabibbo, F. Costantini, G. Fiorentini, F. Fiore, L. Fonti, A. Fucci, M. P. Lombardo, S. Galeotti, P. Giacomelli, P. Marchesini, E. Marinari, F. Marzano, A. Miotto, P. Penlucci, G. Parisi, D. Pascoli, D. Passuello, S. Petrarca, F. Rapuano, E. Remiddi, R. Ruseck, G. Salline and R. Tripliccione, at the conference "Computing in High Energy Physics" (Asilomar, California, USA 1987).
- 3) M. Falicioni, E. Marinari, M.L. Paciello, G. Parisi, B. Toglietti and Zhong Yi-Cheng, Nucl. Phys. **B215** [FS7] (1983) 265;  
K. Hutter and K. Schilling, Phys. Lett. **117B** (1982) 75.
- 4) Work in progress.
- 5) G. Parisi, Phys. Rep. **103** (1984) 203;  
G. Parisi, R. Petronzio and F. Rapuano, Phys. Lett. **128B**(1983)416;  
G. Parisi, in "Cargèse 1984" (Plenum, New York 1985);  
M. Falicioni, M.L. Paciello, G. Parisi and B. Toglietti, Nucl. Phys. **B251** [FS13] (1985) 624;  
E. Marinari, "Gif-sur-Yvette 1986" (Les editions de physique, Paris 1987).
- 6) D. Barkai, K.J.H. Moriarty and C. Rebbi, Phys. Rev. **D30**(1984)1293.
- 7) Ph. de Forcrand, G. Schierholz, H. Schneider and M. Teper, Phys. Lett. **160B**(1985)137.
- 8) Ph. de Forcrand, G. Schierholz, H. Schneider and M. Teper, Phys. Lett. **152B**(1985)107.

- 9) Ph. de Forcrand, G. Schierholz, H. Schneider and M. Teper, preprint Cern TH. 4167/85 (april 1985).
- 10) A. Patel, R. Gupta, G. Guralnik, G.W. Kilcup and S.R. Sharpe, Phys. Rev. Lett. **57**(1986)1288.
- 11) B. Berg and A. Billoire, Phys. Lett. **166B** (1986) 203, and erratum, to be published;  
B. Berg, A. Billoire and C. Vohwinkel, Phys. Rev. Lett. **57**(1986)400; preprint FSU-SCRI-86-70 ( Tallahassee, Florida, october 1986).
- 12) M. Lüscher, Nucl. Phys. **B100**(1981) 317.
- 13) M. Lüscher, in Progress in Gauge Field Theory, ed. by G. 't Hooft et al. (Plenum, New York 1984) 451.

Table 1

L	$m(0^{**})$	$m(2^{**})$	$\sigma(\infty)$	$m(0^{**})/\sqrt{\sigma(L)}$
10	0.65(3)	0.6(3)	0.058(3)	3.0(2)
12	0.76(4)	0.8(2)	0.062(1)	3.2(2)
16	0.82(5)	1.00(15)	0.058(2)	3.4(4)

The observables we have considered versus the lattice size.  $m(0^{**})/\sqrt{\sigma(L)}$  is computed directly from an effective ratio.  $\sigma(\infty) = \sigma(L) + \pi/(3L^2)$ .

### Figure captions

- 1) The smearing procedure. We substitute a link with itself plus  $\epsilon$  times the sum of the incomplete neighbouring space like plaquettes, and we project it back onto  $SU(3)$ .
- 2) The effective string tension as a function of the smearing.
  - a) At  $(\text{smearing})^{-1} = 1$ , starting from the top, we have the curve (1) derived from  $G(1)/G(2)$ , next (2) is the one from  $G(2)/G(3)$ . The curves (3) for  $G(3)/G(4)$ ,  $G(4)/G(5)$  and  $G(5)/G(6)$  are undistinguishable on this scale.
  - b) On a larger scale one can see the effect obtained when going from  $G(3)/G(4)$  to  $G(4)/G(5)$  (respectively curves 3 and 4). The effective masses at higher times would coincide with  $G(4)/G(5)$ .
- 3) Result of a fit for the  $0^{++}$  mass.  $12^3$  lattice,  $t_{\min}=4$ .

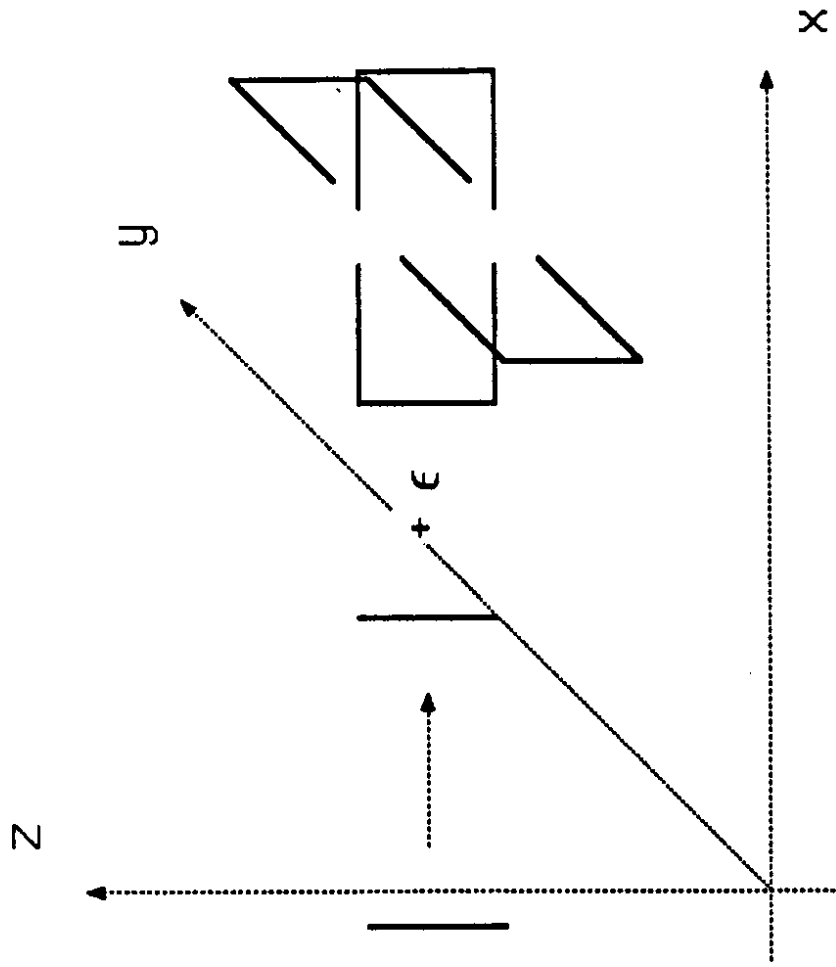


Fig. 1



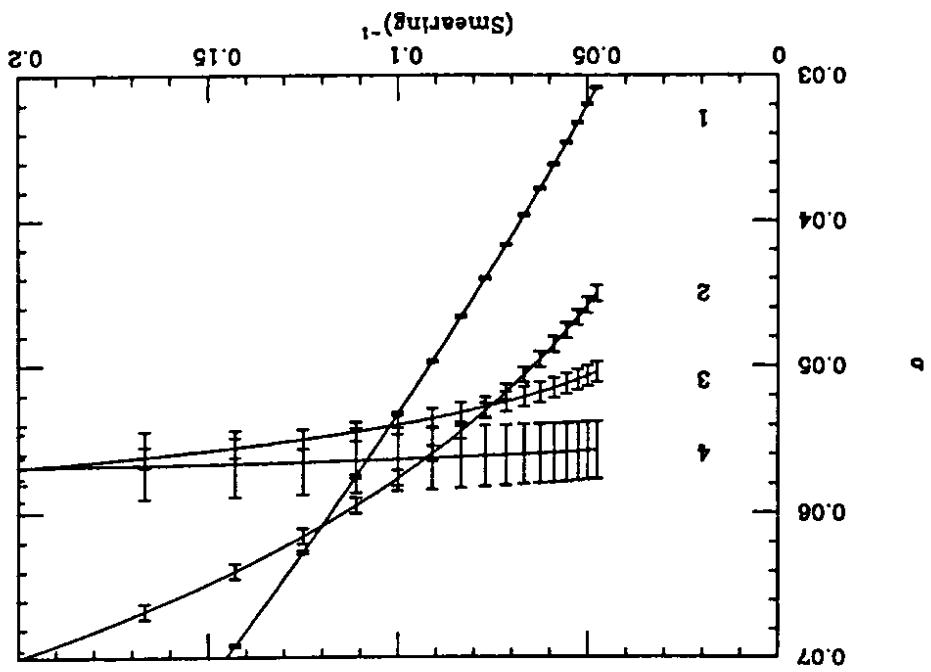


Fig. 2b

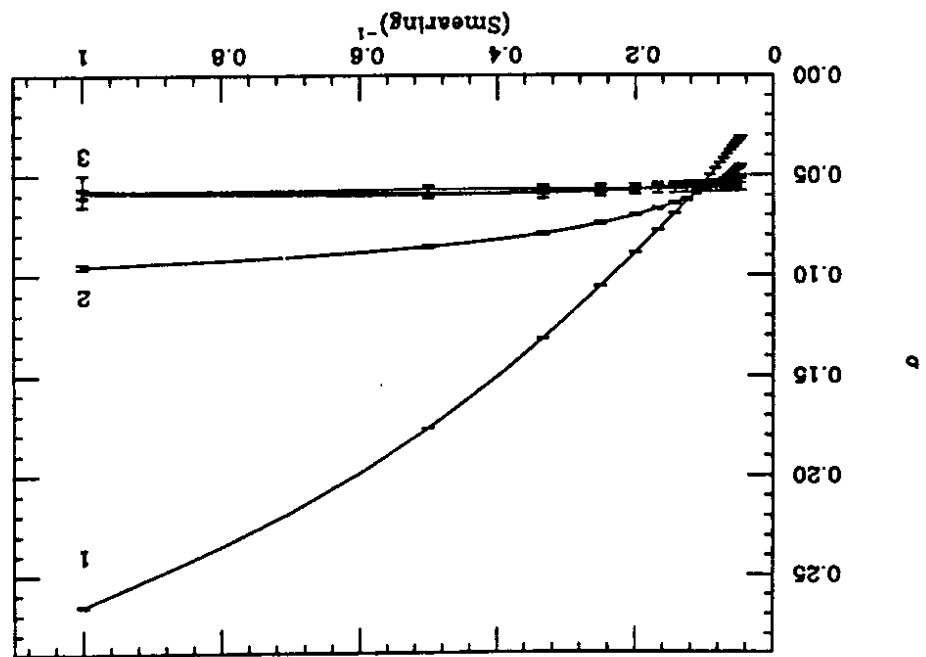


Fig. 2a

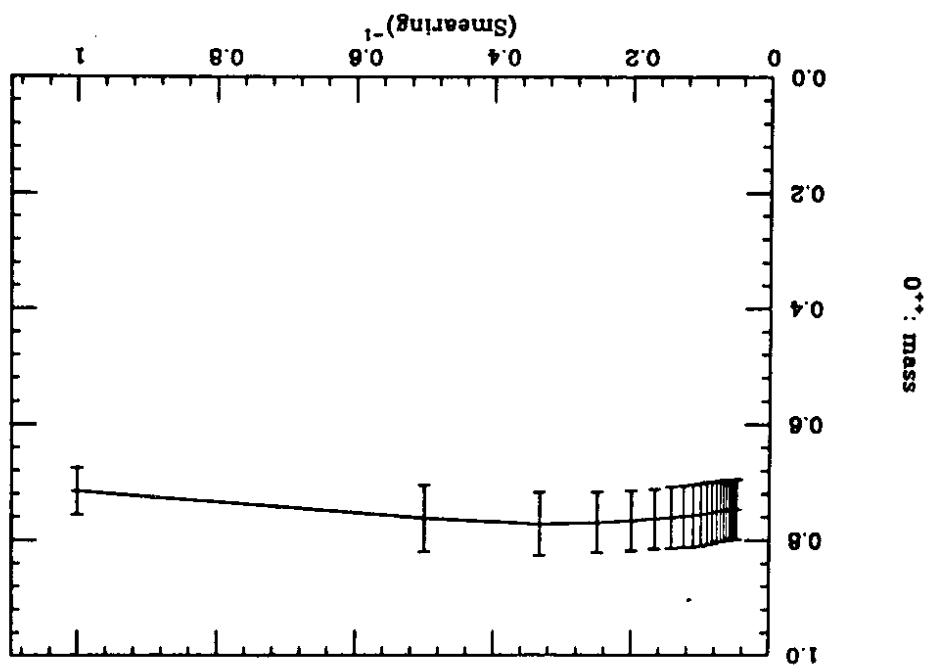


Fig. 3