High-efficiency deflection of high energy protons due to channeling along the ⟨110⟩ axis of a bent silicon crystal

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Abstract

A deflection efficiency of about 61 % was observed for 400 GeV/c protons due to channeling, most strongly along the ⟨110⟩ axis of a bent silicon crystal. It is comparable with the deflection efficiency in planar channeling and considerably larger than in the case of the ⟨111⟩ axis. The measured probability of inelastic nuclear interactions of protons in channeling along the ⟨110⟩ axis is only about 10 % of its amorphous level whereas in channeling along the (110) planes it is about 25 %. High efficiency deflection and small beam losses make this axial orientation of a silicon crystal a useful tool for the beam steering of high energy charged particles.

Keywords: Crystal; channeling; beam deflection

1. Introduction

In the last twenty years, channeling has been exploited exploited for steering [1], collimation [2, 3, 4] and extraction [2, 5, 6, 7] of relativistic beams in circular accelerators, as well as splitting and focusing of extracted beams [8]. In the last decade, a significant boost to the research on particle-crystal interactions was provided by the fabrication of uniformly bent crystals with thickness along the beam direction suitable for experiments at high-energy. The novel generation of crystals has demonstrated the capability of efficiently steering positively charged [9, 10] particle beams and to observe the deflection of negatively charged particle beams [11, 12]. As well as efficient deflection, channeling has been shown to modify the probability of incoherent interactions with atomic nuclei with respect to an amorphous material of the same length [13]. In particular, the use of bent crystals as a primary collimator has been demonstrated to reduce the beam losses in the SPS proton synchrotron at CERN [14, 15, 16], leading to the installation of two bent crystals in the LHC collider [17]. The crystals installed in the LHC were successfully tested and shown to reduce the beam losses in the LHC ring with 6.5 TeV/c protons [18].

As a charged particle traverses a crystal with a small angle with respect to one of the main crystallographic axes, the particle motion is governed by the potential of the atomic strings averaged along the axis [19], i.e. the particle is subject to axial channeling (AC). The maximum angle between the particle incident angle and the crystal axis, for which channeling occurs is called critical angle for channeling $\psi_{ac} = (4Ze^2/p\beta d)$ [19], where d is the distance between neighboring atoms in the atomic string, $p\beta$ the particle momentum-velocity, Z the atomic number of the crystal atoms and e the elementary charge. For a silicon crystal, ψ_{ac} is almost three times larger than the critical angle for channeling between atomic planes, i.e. planar channeling (PC) [20]. However, because the potential barrier separating the axial channels formed by neighboring atomic strings is rather low, the fraction of particles confined within a single axial channel is limited. As an example, for a silicon crystal, such a potential barrier is $\sim$ 6 eV for the ⟨110⟩ direction and only 1 eV for the ⟨111⟩. If a particle under AC has a transverse energy slightly higher than the potential well barrier, particles interacting with a bent crystal may be deflected by the multiple scattering on the atomic strings toward the bending direction, increasing the efficiency of AC. Indeed, as soon as a particle penetrates the crystal, the position of scattering centers shifts,

causing the subsequent random scattering with atomic strings to acquire a preferential direction. Such an effect was experimentally demonstrated for positively [10, 21] and negatively [12] charged particles.

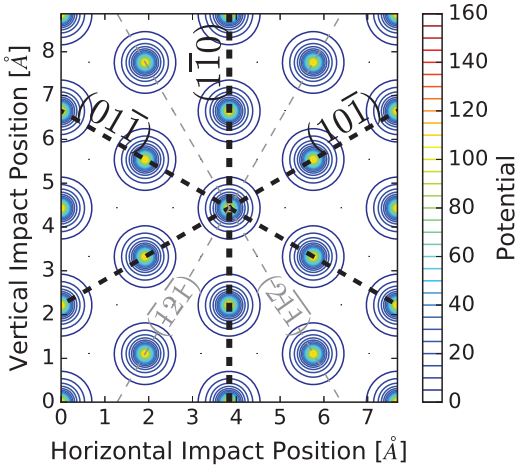

Figure 1: Potential averaged along the ⟨111⟩ axis of a silicon crystal. The directions of the (110) and (211) planes are shown.

As predicted in [22] high deflection efficiency and small beam losses due to inelastic interactions may be achieved using the axial channeling, most strongly along the ⟨110⟩ axis of a silicon crystal. Despite that fact, experimental measurements on AC are limited to studies of the deflection efficiency of 400 GeV/c protons interacting with a ⟨111⟩ silicon crystal [10, 12, 21], and measurement of the inelastic interaction frequency (INI) under AC was observed only for 15 GeV/c pions interacting with germanium crystals [23, 24].

In this paper, we experimentally investigate the deflection efficiency and the inelastic interaction frequency of 400 GeV/c protons interacting with ⟨111⟩ and ⟨110⟩ bent silicon crystals under axial channeling at the H8 external line of the SPS at CERN.

2. AC mechanism

Charged particles interacting with an aligned crystal under AC are mainly deflected by two phenomena, hyperchanneling [25] and the randomization of transverse momenta of the particles because of multiple scattering by atomic strings [26]. Hyperchanneling consists in the confinement of the particle trajectory between the potential well barriers separating neighboring atomic planes. Due to the low potential barrier, such a phenomenon is possible only for a few percent of the incident particles. On the contrary, when particles scatter on atomic strings, the deflection due to the randomization of transverse particle momentum affects all the particles not under hyperchanneling and is possible if the crystal bending angle (α) is lower than a characteristic value α_{ts} [27]

$$\alpha < \alpha_{ts} = \frac{2R\psi_{ac}^2}{l_0} \tag{1}$$

where R is crystal bending radius, $l_0 = 4/(\pi^2 n d R_a \psi_{ac})$ the minimum encounter length between the incident particle and a nucleus, n being the concentration of atoms in the crystal and R_a the atomic screening radius.

Although Eq. 1 proved to be a good condition for the observation of the particle deflection due to incoherent scattering [10, 12, 21], it does not furnish an estimate of the best crystal length at a fixed bending angle. A useful parameter introduced to account for the maximum crystal length for efficient particle steering at the full bending angle $\alpha = L/R$ is the relaxation length l_r [21], i.e. the length within which the fraction of particles in the AC regime are reduced to $1/e$ and escape from AC to skew planes. In order to efficiently deflect under AC a crystal has to fulfill the inequality [21]

$$L < l_r \tag{2}$$

Up to now, an analytic equation for the calculation of l_r does not exist. The dependence of the l_r on R was worked out only for a 400 GeV/c proton beam interacting with a ⟨111⟩ Si crystal [21] and included the incoherent scattering on nuclei and electrons.

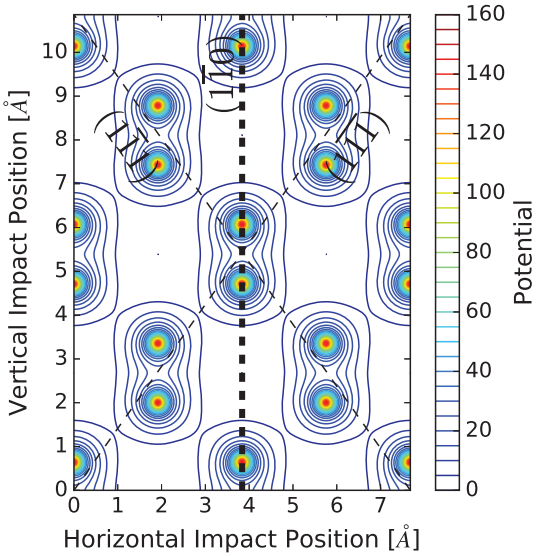

Figure 2: Potential averaged along the ⟨110⟩ axis of a silicon crystal. The directions of the (111) planes are shown.

3. Experimental measurements

The experimental setup was based on a particle telescope [28], consisting of ten planes of silicon microstrip sensors, arranged as five pairs, each measuring two orthogonal coordinates, with an active area of 3.8 x 3.8 cm^2. The telescope provided excellent angular and spatial resolution for measuring the trajectories of incident and outgoing particles. The apparatus had a long baseline, of approximately 10 m in each arm, and achieved an angular resolution in the incoming arm of 2.5 μrad

Table 1: Parameters of the ⟨111⟩ and ⟨110⟩ bent Si crystals, with R the crystal bending radius, L the crystal length along the beam direction, α_{pl} the channeling mean deflection angle.

	⟨111⟩	⟨110⟩
Plane	$(1\bar{1}0)$	$(1\bar{1}0)$
Axis	⟨111⟩	⟨110⟩
L (mm)	1.941 ± 0.002	1.881 ± 0.002
R (m)	32 ± 2	35 ± 2
α_{pl} (μrad)	63 ± 1	54 ± 1

and a total angular resolution on the difference of the two arms of 5.2 μrad, with performance limited by multiple scattering in the sensor layers. The crystal was mounted on a high-precision goniometer with an angular resolution of about 1 μrad. This instrument allowed three degrees of freedom, one linear and two rotational movements, to align the crystal along either the horizontal or vertical directions. Two pairs of scintillators were placed after the target outside the beampipe in order to measure the secondary particles produced by the nuclear interactions of protons with the silicon crystals. Pre-alignment of the samples was achieved by means of a laser system parallel to the beam direction.

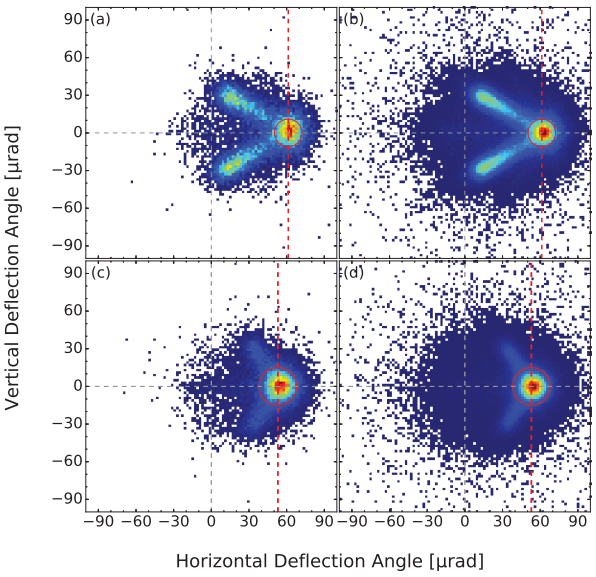

Figure 3: Experimental (a. and c.) and simulated (b. and d.) distributions of the horizontal and vertical deflection angles of 400 GeV/c protons interacting with the ⟨111⟩ (a. and b.) and ⟨110⟩ (c. and d.) crystals.

The targets were two silicon crystals produced according to the method described in Ref. [29, 30, 31]. The crystals were mounted on mechanical holders that impart a controlled deformation to the primary and the anticlastic curvatures [5, 32]. The crystal parameters are reported in Table 1. Both the crystals have a ∼ 2 mm length and the same bent ⟨1$\bar{1}$0⟩ axis orthogonal to the beam direction. Therefore, both crystals make use of a $(1\bar{1}0)$ bent plane for PC. However, the bent axes parallel to the beam direction are different, being respectively ⟨111⟩ and ⟨110⟩. As a consequence the use of such crystals allows comparison of the AC features for two different crystal axes under the same experimental condition. The crystal torsion due to the mechanical holder was compensated in the data analysis for both the crystals [33].

The INI for a particle interacting with a crystal under AC (n_{ac}) can be expressed in units of the INI for a particle interacting with an amorphous material (n_{am}). In the experiment, we tagged as nuclear interacting particles (N_{ini}) the incoming particles that interact with the crystals and produced a signal in both the scintillators. Subtracting the background events that produce a coincidence in the scintillators ($N_{bg,ini}$) from the measurement (N_{ini}) and dividing by the events under misaligned condition minus the background events ($N_{am,ini} - N_{bg,ini}$), we obtain the INI (n) for the particles under AC:

$$n = \frac{N_{ini} - N_{bg}}{N_{am} - N_{bg}} \tag{3}$$

Table 2: The experimental and simulated deflection efficiency (ϵ_{ac}) axial channeling along the ⟨111⟩ and ⟨110⟩ silicon crystal axes.

	⟨111⟩		⟨110⟩	
	Experiment	Simulation	Experiment	Simulation
ϵ_{ac}	22 ± 2	24 ± 2	61 ± 2	67 ± 2

Firstly, both the crystals were oriented under PC in order to measure the efficiency (ϵ_{pl}) and the mean deflection angle (α_{pl}). The particles with an incident angle below 2 μrad with respect to the $(1\bar{1}0)$ bent plane were analyzed. The maximum deflection efficiency and the average deflection angle under PC are 83 ± 1 % and 63 ± 1 μrad for the ⟨111⟩ crystal, 82 ± 1 % and 54 ± 1 μrad for the ⟨110⟩ crystal. Since the bending radii (R) for the ⟨111⟩ (32 ± 2 m) and ⟨110⟩ (35 ± 2 m) crystals are much greater than the 0.7 m critical bending radius (R_c) at 400 GeV/c [20], the minimal bending radius for which channeling is possible, the deflection efficiency under PC is expected to be the same for the two crystals [12]. The deflection efficiency under PC was evaluated as described in Ref. [34].

After the channeling was demonstrated under PC, the crystal was oriented in order to observe AC, i.e., with the crystal axis parallel to the particle incoming direction. In order to investigate AC, both the crystals have to fulfill the inequalities in Eqs. 1 and 2. The parameter l_r was estimated as ∼ 2.5 mm for R = 32 and ∼ 3.7 for R = 35 [21]. Although the ⟨111⟩ axis was used for the estimation, we expect a similar or better behavior for the ⟨110⟩ axis, since the potential well depth is similar and the potential well is wider (see Figs. 1 and 2). The parameter α_{ts} is ∼ 600 μrad for the ⟨111⟩ crystal and ∼ 800 μrad for the ⟨110⟩. Because $\alpha_{ts} \gg \alpha_{pl}$ and $L < l_r$ for both the crystals, we expect a high-efficiency beam deflection through AC for both of them.

The distribution of particle deflection angle after the interaction with the two crystals is shown in Fig. 3. Table 3 shows the deflection efficiency under AC. The particles with an incident angle below 3 μrad with respect to the bent axis of the two crystals were analyzed, i.e. with $\sqrt{\theta_{x0}^2 + \theta_{y0}^2} < \Theta$ where θ_{xo} and θ_{y0} are the horizontal and vertical particle incident angle with respect to the crystal axis orientation. The deflection efficiency ϵ_{ac} was evaluated as the fraction of particles in the circle centered on the mean horizontal deflection with radius 11 μrad for the $\langle 110 \rangle$ crystal and 15 μrad for the $\langle 110 \rangle$ crystal. The deflection efficiency was 22 ± 2 % for the $\langle 111 \rangle$ and 61 ± 2 % for the $\langle 110 \rangle$.

Figure 4 shows the distribution of the horizontal deflection angle for a 400 GeV/c proton beam interacting with the $\langle 111 \rangle$ and $\langle 110 \rangle$ silicon crystals. The particle with an incident angle below 3 μrad are analyzed. The efficiency of one side deflection is 97 % for the $\langle 111 \rangle$ and 98 % for the $\langle 110 \rangle$ axis orientations. For both crystals the particle distributions are symmetric on the vertical plane and skewed to positive values on the horizontal plane, i.e. the bent plane. Although all the particles are initially subject to AC a consistent fraction is captured by the skew planes that intersect the main axis. For the crystal offering the $\langle 111 \rangle$ axis the two symmetric tails that belong to the (110) family of planes at 60° from the main $(1\bar{1}0)$ plane (see Figs. 1 and 4), leading the particles captured by them to be deflected at an angle much lower than the full bending angle. For the crystal offering the $\langle 110 \rangle$ axis the angle between the two symmetric (111) planes and the main $(1\bar{1}1)$) plane is only 35.26°, causing the captured particles to be deflected at an angle similar to the main bending angle (see Figs. 2 and 4). As a consequence the deflection efficiency ϵ_{ac} is higher for the $\langle 110 \rangle$ crystal than for the $\langle 111 \rangle$.

The INI probability was measured for the axial and for amorphous (AM) orientations of the crystals. Fig. 5 shows the INI normalized to the AM condition as a function of an angular range $\pm\Theta$ around around the best AC position for both the crystals, i.e. $\sqrt{\theta_{xo}^2 + \theta_{yo}^2} < \Theta$. Particles under AM experience the same INI independently of Θ, because in such a case the crystal acts as an amorphous material. On the contrary, the INI for AC strongly depends on Θ. Indeed, as soon as the integration angle grows, the fraction of particles under AC diminishes in the beam, leading to the growing of the average INI.

The interaction frequency under PC is always higher than for AC. Indeed, as predicted in [22], the particles under AC impinge on the strong electromagnetic field of the atomic rows without approaching close to the nuclei until they are aligned with respect to the crystal axis within one ψ_{ac}. Since $\psi_{ac} \gg \psi_{pc}$ for both $\langle 111 \rangle$ and $\langle 110 \rangle$ axes ($U_{0,ac} \sim 100$ eV for $\langle 111 \rangle$ and $\langle 110 \rangle$ axes and $U_{0,pc} \sim 20$ for $(1\bar{1}0)$ plane), the INI under AC remains lower than the INI under PC for higher incident angles, resulting in a net reduction of the INI for the whole incident beam.

Whilst the potential well depth for $\langle 111 \rangle$ and $\langle 110 \rangle$ axes are similar, the shapes of the potential well are different. Indeed, the potential well for the $\langle 110 \rangle$ axis is much wider than the potential well for the $\langle 111 \rangle$ axis (see Fig. 2 and 1). As consequence, the fraction of particles captured in the skew planes is ~ 45 % for $\langle 111 \rangle$ and ~ 15 % for the $\langle 110 \rangle$. The INI under AC is ~ 0.30 for the $\langle 111 \rangle$ axis and ~ 0.10 for the $\langle 110 \rangle$ axis for particles with $\sqrt{\theta_{xo}^2 + \theta_{yo}^2} < 5$ μrad (see Fig. 5).

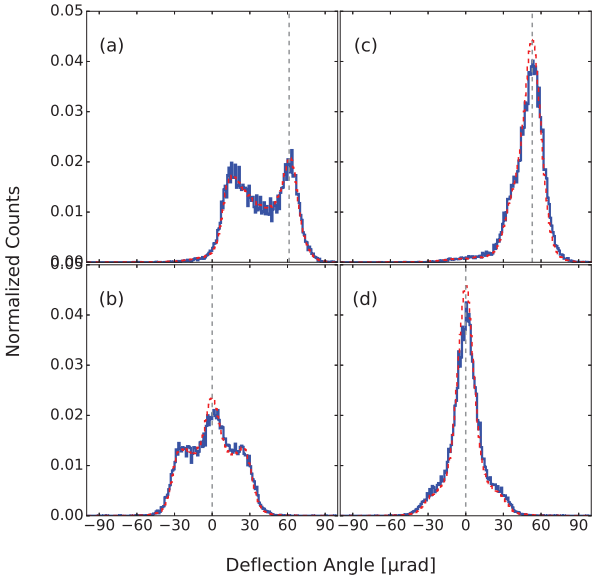

Figure 4: Distributions of horizontal (a. and c.) and vertical (b. and d.) deflection angles for a narrow fraction of the 400 GeV/c proton beam after the passage through the $\langle 111 \rangle$ (a. and b.) and $\langle 110 \rangle$ (c. and d.) silicon crystals. Continuous blue lines are the experimental data, red dashed lines are the Geant4 simulations.

4. Monte Carlo simulations

The investigation of the trajectories inside the crystals was carried out by means of the Geant4 Monte Carlo toolkit [35]. The experimental setup at the H8-SPS area is reproduced in the simulation in order to take into account the error due to the finite resolution of the telescope. Channeling is implemented by including DYNECHARM++ [36] and ECHARM [37] into the Geant4 channeling package [38]. Simulations of the INI are shown in Fig. 5 and of the distribution of the deflection angle in Figs. 3 and 4.

The distribution of the deflection angles reproduce the experimental results. Indeed, the spot of the particles deflected by the AC and the tails filled by the particles captured by skew planes are visible. Table 3 reports the simulation results for the deflection efficiency under AC. The efficiency is slightly higher than the experimental results for both the crystals. As a consequence, the INI of the simulation is slightly lower than the experimental results.

The good agreement between the Geant4 simulations and the experimental data allows investigation of the dynamics of the interaction with the Monte Carlo approach. Since the simulations with Geant4 takes into consideration the whole experimental setup and the contribution of the multiple scattering on

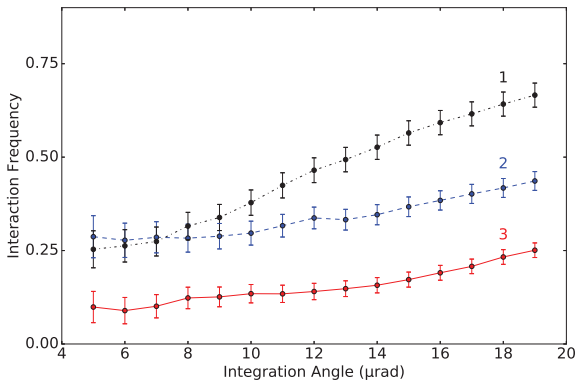

Figure 5: Measured inelastic nuclear interaction frequency (INI) of 400 GeV/c protons interacting with the ⟨111⟩ and ⟨110⟩ crystals as a function of the angular region around the (110) planar channeling (black dash-dotted line, 1), the ⟨111⟩ axial channeling (blue dashed line, 2) and ⟨110⟩ (red continuous line, 3) orientations. The values are normalized to the INI frequencies for the amorphous crystal orientation.

the beam line elements to the final results, a different Monte Carlo approach can be used to evaluate the deflection efficiency under AC under ideal conditions. Therefore, Monte Carlo simulations using the binary collision model were carried out. Simulations show that the deflection efficiencies may reach values of about 50 % and 80 % for the ⟨111⟩ and ⟨110⟩ axial orientations of the silicon crystals, respectively, with a narrow beam.

5. Conclusions

In summary, we compared the deflection efficiency and INI under AC of ⟨111⟩ and ⟨110⟩ axes. The experiment confirms the theoretical predictions proposed in [22] and paves the way to the use of AC as an efficient manipulator of charged particle beams.

The AC with ⟨110⟩ and ⟨111⟩ silicon crystals causes 90 % of the particles to be one side deflected. The particles captured by the skew planes (∼ 15 %) for the ⟨110⟩ axis acquire a deflection angle similar to α, while while the two most strongly acting (110) planes of the ⟨111⟩ axis cause a considerable fraction (∼ 45 %) of the particles to be deflected at an angle lower than α but > 0. Moreover, binary collision simulations show an 80 % deflection efficiency for the ⟨110⟩ axis if we consider only the particle-crystal interaction.

The measurement allowed to clearly establish that the INI probability for particles under AC is significantly lower than for particles under PC. Moreover, we observed that the INI probability for particles under AC for a ⟨110⟩ crystal is three times lower than for a ⟨111⟩ crystal. The experimental measurements were systematically compared with Monte Carlo simulations worked out via the Geant4 toolkit. The simulations reproduced the distributions of deflection angle for both crystals, confirming the understanding of the physics behind AC. Thanks to its deflection efficiency and the low INI probability, we experimentally demonstrated that AC along the most effective ⟨110⟩ axis of a bent silicon crystal is a good candidate for beam manipulation at high energy.

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