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Mixed Spatial Duopoly, Consumers' Distribution and Efficiency

Corrado Benassi*

Alessandra Chirco#

Caterina Colombo§

* Dipartimento di Scienze Economiche, Alma Mater Studiorum - Università di Bologna, Piazza Scaravilli 2, 40122 Bologna, Italy. corrado.benassi@unibo.it

Corresponding Author. Dipartimento di Scienze dell'Economia, Università del Salento, Ecotekne, Via per Monteroni, 73100 Lecce, Italy. alessandra.chirco@unisalento.it

§ Dipartimento di Economia e Management, Università degli Studi di Ferrara, via Voltapaletto 11, 44121 Ferrara, Italy. caterina.colombo@unife.it

Abstract. We solve a mixed spatial duopoly with a generic log-concave consumers' distribution. We show that the sub-game perfect equilibrium in prices and locations exists and is generally inefficient, so that the efficiency in the standard uniform distribution case stands out as an exception. Notable examples show that the inefficiency may increase as the distribution becomes more concentrated.

Keywords. Mixed duopoly, spatial competition, consumers' distribution, efficiency of equilibrium.

JEL Codes: L13, L32, H44

1 Introduction

In the analysis of mixed duopoly, the mixed spatial duopoly model exhibits a distinctive feature: when the strategic interaction between a public (welfare-maximizing) firm and a private (profit-maximizing) firm is modeled according to a Hotelling-type framework, the market outcome is efficient (Cremer *et al*, 1991). Under the usual assumptions of quadratic transportation costs, constant production costs, unit demand, full market coverage and uniform consumers' distribution, a public and a private firm competing over prices and locations along a linear city end up choosing the locations at which the transportation costs are minimized and welfare is maximized. This is in sharp contrast with the results of other mixed duopoly set-ups, where under both quantity and price competition the existence of a public firm is not sufficient to guarantee that the market outcome be efficient. Indeed, one of the main normative implications of the efficiency result is that, in contrast to common findings in mixed markets analysis, in mixed spatial duopoly there is no advantage for the government to optimally manipulate the public firm objective function, e.g. through a partial privatization policy (Lu and Poddar, 2007).

The efficiency property has been shown to be robust to the existence of cost differentials (Matsumura and Matsushima, 2004) and to the hypothesis of sequential choice of locations, provided the public firm be the leader (Matsumura and Matsushima, 2003), while it vanishes when the assumption of unit demand is replaced with that of price-elastic demand at each location (Kitahara and Matsumura, 2013). In this paper we assess its robustness with respect to a fundamental element of any spatial model, namely the shape of the consumers' distribution. By solving a mixed spatial duopoly for generic log-concave distributions, we are able to show that the market outcome is typically inefficient, and that the well-known efficiency property is strictly related to the distribution being uniform.

In particular, in section 2 we discuss the solution of a mixed spatial duopoly for a large set of consumers' distributions, and we compare the outcome of the strategic interaction between the public and the private firm with the socially efficient outcome. In section 3 we provide some examples of this comparison with notable distributions. We conclude in Section 4.

2 Mixed spatial duopoly with non-uniform distribution

We consider a mixed spatial duopoly of the Hotelling type: a private profit-maximizing firm (firm 1) competes in prices and locations along a linear city of unit length with a public welfare-maximizing firm (firm 2). In order to focus on the role of the consumers' distribution, we preserve the following standard hypotheses: (a) firms share the same technology and produce at constant unit costs, normalized to zero; (b) consumers' transportation costs are quadratic in distance; (c) the gross consumer surplus is always greater than the price gross of the transportation cost, so that each consumer buys one unit of the good. We depart from previous analyses by relaxing the hypothesis of uniform distribution. Rather, we assume the following:

Assumption 1. For any location $x \in [0,1]$, a log-concave density $f(x)$ of consumers is defined with the following properties: (i) $f(x) \geq 0$ for all $x \in [0,1]$, and $f(x) > 0$ for all $x \in (0,1)$; (ii) if $f(0) = 0$ then $\lim_{x \to 0^+} f'(x) > 0$.

Given this set-up, in the next subsection we discuss the solution of the two-stage game in prices and locations between the private firm and the public firm. In subsection 2.2 we solve for the efficient solution and verify under which conditions the two solutions coincide.

2.1 The non-cooperative equilibrium

Denote with x_1 and x_2 the distances of firms 1 and 2 from the the left end point 0, and assume, without any loss of generality, that $x_1 < x_2$.[1] Given quadratic transportation costs, and the prices p_1 ad p_2 set by the firms, the location z of the consumer who is indifferent between patronizing either firm satisfies:

$$p_1 + (z - x_1)^2 = p_2 + (x_2 - z)^2$$

so that

$$z = z(p_1, p_2; x_1, x_2) = \frac{1}{2}\left(\frac{p_2 - p_1}{x_2 - x_1} + x_2 + x_1\right) \tag{1}$$

Accordingly, the demand functions faced by the firms are respectively:

$$D_1 = \int_0^z f(x)\, dx = F(z), \qquad D_2 = \int_z^1 f(x)\, dx = 1 - F(z)$$

where $F : [0,1] \to [0,1]$ is the cumulative consumers' distribution. Therefore, the objective function of firm 1 is:

$$\pi_1 = p_1 F(z) \tag{2}$$

Since firm 2 maximizes welfare - the sum of both firms' profits and the consumers' net surplus - its objective collapses to minimizing total transportation costs T:

$$T = \int_0^z (x - x_1)^2 f(x)\, dx + \int_z^1 (x - x_2)^2 f(x)\, dx \tag{3}$$

[1] Notice that we are not imposing any *a priori* boundary on the location of firms. Following Cremer et al (1991), we assume that firms have different locations. This is confirmed at equilibrium.

The price stage At the price stage, minimization of (3) by the public firm yields the following FOC:

$$\frac{\partial T}{\partial p_2} = f(z)\frac{dz}{dp_2}\left[(z-x_1)^2 - (z-x_2)^2\right] = 0$$

Since the SOC is verified, this implies that independently of the shape of $f(\cdot)$ the reaction function of firm 2 is:

$$p_2 = p_1 \tag{4}$$

As to the private firm 1, profit maximization requires:

$$\frac{\partial \pi_1}{\partial p_1} = F(z) + p_1 f(z)\frac{\partial z}{\partial p_1} = 0$$

which, using (1), boils down to:[2]

$$p_1 = 2\frac{F(z)}{f(z)}(x_2 - x_1) \tag{5}$$

Given (4) and (5), we can now establish the following Proposition.

Proposition 1. For all $f(\cdot)$ satisfying Assumption 1 there exists a unique Nash equilibrium in prices for any pair of locations (x_1, x_2).

Proof. By total differentiation of (5), the slope of the reaction function of firm 1, $p_1(p_2)$, is

$$\frac{dp_1}{dp_2} = \frac{\left(1 - \frac{F(z)}{f(z)}\frac{f'(z)}{f(z)}\right)}{1 + \left(1 - \frac{F(z)}{f(z)}\frac{f'(z)}{f(z)}\right)} < 1$$

due to the log-concavity of $f(\cdot)$. Assume now that $p_2 = 0$. Since $x_1 < x_2$, there always exists a positive price p_1 which ensures positive profits to firm 1 by attracting customers located near the left end of the linear city. Therefore, the best reaction to $p_2 = 0$ is some $p_1(0) > 0$. Given that along (5) $dp_1/dp_2 < 1$, there exists a unique price $\widehat{p}_2$ such that $p_1(\widehat{p}_2) = \widehat{p}_2$ and both (4) and (5) are verified.■

Therefore, the Nash equilibrium in prices is:

$$\widehat{p}_1 = \widehat{p}_2 = \frac{2F\left(\frac{x_1+x_2}{2}\right)}{f\left(\frac{x_1+x_2}{2}\right)}(x_2 - x_1) \tag{6}$$

[2] As to the SOC for firm 1,

$$\frac{\partial^2 \Pi_1}{\partial p_1^2} = \frac{1}{(x_2 - x_1)}\left(-f(z) + F(z)\frac{f'(z)}{f(z)}\frac{1}{2}\right) < 0$$

since log-concavity implies $Ff' < f^2$.

The location stage At the location stage, the public firm minimizes (3) with respect to x_2, and the private firm maximizes (2) with respect to x_1, by taking into account the solution of the price stage – which implies $z = \widehat{z}(x_1, x_2) = (x_1 + x_2)/2$. Therefore, the public firm's FOC at the location stage is:

$$\frac{\partial T}{\partial x_2} = -2\int_{\widehat{z}}^{1} (x - x_2) f(x)\, dx = \mu - \int_0^{\widehat{z}} x f(x) dx - x_2 [1 - F(\widehat{z})] = 0$$

where $\mu = \int_0^1 x f(x) dx$ is the average consumers' location. Integrating by parts, we obtain:

$$\mu - \widehat{z}F(\widehat{z}) + S(\widehat{z}) - x_2 (1 - F(\widehat{z})) = 0 \tag{7}$$

where $S(z) = \int_0^z F(x)dx$. Equation (7) can be rewritten as:

$$x_2 = \frac{\mu - \widehat{z}F(\widehat{z}) + S(\widehat{z})}{(1 - F(\widehat{z}))} \tag{7'}$$

As to the private firm, its optimal location satisfies:

$$\left(2 - \frac{F(\widehat{z})}{f(\widehat{z})}\frac{f'(\widehat{z})}{f(\widehat{z})}\right)(x_2 - x_1) - \frac{2F(\widehat{z})}{f(\widehat{z})} = 0 \tag{8}$$

Equations (7') and (8) allow us to establish the following Proposition.

Proposition 2. For all $f(\cdot)$ satisfying Assumption 1, there exists a sub-game perfect Nash equilibrium in prices and locations at which $x_2 > x_1$.

Proof. In order to prove Proposition 2 we show that there is a pair (x_1^*, x_2^*) with $x_2^* > x_1^*$, which satisfies (7') and (8), and the SOCs are verified at that solution. We proceed by steps.
Step 1. By substituting $x_1 = 2\widehat{z} - x_2$, equation (8) can be rewritten as:

$$(2 - \theta(\widehat{z}))(x_2 - \widehat{z}) - \frac{F(\widehat{z})}{f(\widehat{z})} = 0 \tag{8'}$$

where $\theta(\widehat{z}) = F(\widehat{z}) f'(\widehat{z}) / f(\widehat{z})^2$. By substituting (7') in (8'), we can establish that an equilibrium exists if there is a location $\widehat{z}^* \in (0, 1)$ at which

$$(2 - \theta(\widehat{z}))\left(\frac{\mu - \widehat{z}F(\widehat{z}) + S(\widehat{z})}{(1 - F(\widehat{z}))} - \widehat{z}\right) - \frac{F(\widehat{z})}{f(\widehat{z})} = 0$$

and the SOCs are satisfied. Define now the continuous function:

$$\varphi(\widehat{z}) = (2 - \theta(\widehat{z}))\left(\frac{\mu - \widehat{z}F(\widehat{z}) + S(\widehat{z})}{(1 - F(\widehat{z}))} - \widehat{z}\right) - \frac{F(\widehat{z})}{f(\widehat{z})}$$

Notice that:

(i) $\lim_{\widehat{z}\to 0^+} \varphi(\widehat{z}) > 0$, since $\lim_{\widehat{z}\to 0^+} F(\widehat{z})/f(\widehat{z}) = 0$,[3] and $\lim_{\widehat{z}\to 0^+} \theta(\widehat{z}) < 1$ by log-concavity;
(ii) $\lim_{\widehat{z}\to 1^-} \varphi(1) < 0$, since $\lim_{\widehat{z}\to 1^-} (2-\theta(\widehat{z})) > 0$ by log-concavity, $\lim_{\widehat{z}\to 1^-} (((\mu - \widehat{z}F(\widehat{z}) + S(\widehat{z}))/(1-F(\widehat{z}))) - \widehat{z}) = 0$,[4] and $\lim_{\widehat{z}\to 1^-} F(\widehat{z})/f(\widehat{z}) < 0$.

Therefore, continuity of φ implies that there exists an internal location $\widehat{z}^*$ such that $\varphi(\widehat{z}^*) = 0$, at which $\varphi'(\widehat{z}^*) < 0$. Given $\widehat{z}^*$, equation (7') delivers x_2^*, while $x_1^* = 2\widehat{z}^* - x_2^*$. Moreover, x_2^* is strictly greater than x_1^* since (7') and (8') cannot be both satisfied for $x_2^* = x_1^*$

Step 2. The solution (x_1^*, x_2^*) is an equilibrium if the SOCs of both firms are satisfied at that solution. This is proved in Appendix A.■

2.2 The efficient solution

We consider now the efficient solution of the above price-location problem. Minimization of transportation costs with respect to prices implies that condition (4) must hold, so that for any pair of locations efficiency requires $z = \widehat{z}(x_1, x_2) = (x_1 + x_2)/2$.

As to locations, we first notice that the efficient locations must lie within the $[0,1]$ interval. The socially optimal choice requires $\partial T/\partial x_1 = 0$ and $\partial T/\partial x_2 = 0$. The latter condition amounts to equation (7'), while the former, using the definition of $\widehat{z}$, can be written as:

$$(x_2 - \widehat{z})\, F(\widehat{z}) - S(\widehat{z}) = 0 \tag{9}$$

Indeed, the Weierstrass Theorem ensures that the total transportation costs function has a minimum for $x_i \in [0,1]$, $i = 1,2$, $x_1 \leq x_2$; since we can rule out boundary minima, the system given by (7') and (9) has an internal solution, which identifies the efficient location pair $\left(x_1^{SW}, x_2^{SW}\right)$.[5]

We now compare the non-cooperative solution with the efficient solution. The former is given by equations (7') and (8'), or

$$\begin{aligned} x_2 &= \frac{\mu - \widehat{z}F(\widehat{z}) + S(\widehat{z})}{(1-F(\widehat{z}))} \\ \varphi(\widehat{z}) &= (2-\theta(\widehat{z}))\left(\frac{\mu - \widehat{z}F(\widehat{z}) + S(\widehat{z})}{(1-F(\widehat{z}))} - \widehat{z}\right) - \frac{F(\widehat{z})}{f(\widehat{z})} = 0 \end{aligned}$$

while the latter solves equations (7') and (9), or

$$\begin{aligned} x_2 &= \frac{\mu - \widehat{z}F(\widehat{z}) + S(\widehat{z})}{(1-F(\widehat{z}))} \\ \lambda(\widehat{z}) &= \left(\frac{\mu - \widehat{z}F(\widehat{z}) + S(\widehat{z})}{(1-F(\widehat{z}))} - \widehat{z}\right) F(\widehat{z}) - S(\widehat{z}) = 0 \end{aligned}$$

[3] This follows trivially if $f(0) > 0$; if $f(0) = 0$ it can be easily obtained by applying the Hôpital rule under our hypothesis that in this case $f'(0) > 0$.

[4] This relies on the fact that $\lim_{\widehat{z}\to 1^-} S(\widehat{z}) = 1 - \mu$. Indeed, integration by parts of $\mu = \int_0^1 x f(x)dx$ yields $\mu = 1 - \int_0^1 F(x)dx$.

[5] Boundary minima are excluded by checking that (a) $\partial T/\partial x_1$ is negative at $x_1 = 0$ and positive at $x_1 = x_2$, and (b) $\partial T/\partial x_2$ is positive at $x_2 = 1$ and negative at $x_1 = x_2$.

where the function $\lambda(\widehat{z})$ has been obtained by substituting (7') into (9).

Given that (7') must hold in both systems, simple inspection clarifies that the two solutions coincide if:

$$\frac{\frac{F(\widehat{z})}{f(\widehat{z})}}{(2-\theta(\widehat{z}))} = \frac{S(\widehat{z})}{F(\widehat{z})}$$

Indeed, under a uniform consumers' distribution − with $f(x) = 1$, $f'(x) = 0$, $F(x) = x$ and $S(x) = x^2/2$ − both the above ratios collapse to $\widehat{z}/2$, and this ensures that the non-cooperative solution be socially efficient.[6] In general, however, this coincidence does not occur and the efficiency property of the sub-game perfect equilibrium under the uniform distribution must be considered as an exception rather than the rule. We provide some examples in the next section.

3 Examples: the Normal and Beta distributions

Tables 1 and 2 show the discrepancy between the sub-game perfect Nash equilibrium and the efficient solution for two types of symmetric distributions, namely the Normal distribution (normalized over the unit interval) and the symmetric Beta distribution.[7] In the tables, each column synthesizes the market and efficient solutions under different values of a concentration parameter - the variance in the case of the Normal distribution, the shape parameter γ in the case of the symmetric Beta. It is worth recalling that the Beta distribution collapses to the uniform distribution for $\gamma = 1$, so that the first column of Table 2 displays the benchmark results of the standard formulation of the model.

Table 1: Values of relevant variables under different concentration parameters: the Normal distribution

	$\sigma = 0.5$	$\sigma = 0.2$	$\sigma = 0.05$
x_1^{SW}	0.27007	0.34552	0.46011
x_2^{SW}	0.72993	0.65448	0.53989
x_1^*	0.30040	0.44157	0.48913
x_2^*	0.74112	0.70147	0.55478
$F(x_1^*)$	0.52426	0.64114	0.67003
$\frac{T^{NE}-T^{SW}}{T^{SW}}$	1.5116×10^{-2}	0.24315	0.31190

[6] The same applies for the linear distribution $f(x) = 2x$, where both the above ratios collapse to $\widehat{z}/3$.

[7] We use a Normal density defined over $[0,1]$, $f(x,\sigma) = \exp\left(-(z-1/2)^2/(2\sigma^2)\right)/\sigma\sqrt{2\pi}E\left(\sqrt{2}/4\sigma\right)$, where $E(t) = \frac{2}{\sqrt{\pi}}\int_0^t e^{-k^2}dk$ is the canonical error function. The symmetric Beta density is $g(x,\gamma) = x^{\gamma-1}(1-x)^{\gamma-1}/B(\gamma)$, where $B(\gamma) = \int_0^1 u^{\gamma-1}(1-u)^{\gamma-1}du$.

Table 2: Values of relevant variables under different concentration parameters: the symmetric Beta distribution

	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
x_1^{SW}	0.25	0.3125	0.34375
x_2^{SW}	0.75	0.6875	0.65625
x_1^*	0.25	0.35526	0.39749
x_2^*	0.75	0.70446	0.67917
$F(x_1^*)$	0.5	0.54474	0.57159
$\frac{T^{NE}-T^{SW}}{T^{SW}}$	0	3.829×10^{-2}	0.53809

The two tables exhibit some common patterns. The efficient solution is obviously symmetric, with both firms moving symmetrically towards the centre as the distribution becomes more concentrated. The market solution is asymmetric. Notwithstanding the symmetry of the distribution and the two firms setting identical prices, the marginal consumer is located to the right of the modal (and median) location, so that the market share of the private firm exceeds that of the public firm.

The intuition behind this result is the following. Assume that the public firm locates at its socially optimal location. The private firm sets its location by balancing the marginal gain of moving inwards (the demand effect of reaching a larger share of consumers) and the marginal gain of moving outwards (the strategic effect of relaxing competition). We know that under the uniform distribution this balance occurs at the socially optimal location. But when the distribution of consumers is not uniform and central locations are more populated than external ones, at its socially optimal location the private firm perceives that the demand effect outweighs the strategic effect. With respect to the efficient location, the private firm moves to the right, choosing a more central location. In order to minimize the transportation costs, also the public firm moves to the right. This implies that the marginal consumer lies at the right of the modal location and that the market share of the private firm is larger than that of its public competitor.

The incentive of the private firm to capture the consumers located in the central area becomes stronger, the more concentrated is the consumers' distribution. Indeed, in our examples the share of the market served by the private firm increases as concentration increases, with the interesting implication that a greater homogeneity in consumers' tastes amplifies the percentage welfare loss (the excess of total transportation costs with the respect to the social optimum) due to strategic interaction.

4 Conclusions

The main finding of this paper is that under a generic log-concave consumers' distribution the solution of a mixed spatial duopoly model is inefficient. In our examples this inefficiency increases as the distribution becomes more concentrated. When compared with the existing literature, our results can be looked at as a warning: in models where heterogeneity is crucial, the robustness of the results with respect to distributional assumptions is indeed a sensitive issue. In our case the inefficiency of the market solution has important policy implications, as it lets us envisage a role for those partial privatization policies which are ruled out as irrelevant under the uniform distribution assumption.

Appendix A. The Second Order Conditions at the Location Stage

Notice that $\frac{d}{d\widehat{z}}\frac{F(\widehat{z})}{f(\widehat{z})} = 1 - \theta(\widehat{z}) > 0$ by log-concavity. Straightforward calculations show that the SOC of the private firm requires that:

$$-\theta'(\widehat{z}) < (3 - 2\theta(\widehat{z}))\frac{2}{(x_2 - x_1)} \tag{A.1}$$

This is true at equilibrium. Indeed, recalling that $(x_2 - \widehat{z}) = (x_2 - x_1)/2$, the derivative of the $\varphi(\bullet)$ function can be written as:

$$\varphi'(\widehat{z}) = -\left(\tfrac{x_2 - x_1}{2}\right)\theta'(\widehat{z}) + \tfrac{f(\widehat{z})}{1 - F(\widehat{z})}\left[2 - \theta(\widehat{z})\right]\left(\tfrac{x_2 - x_1}{2}\right) - \left[3 - 2\theta(\widehat{z})\right]$$

so that $\varphi'(\widehat{z}^*) < 0$ at equilibrium implies

$$-\theta'(\widehat{z}^*) < \left[3 - 2\theta(\widehat{z}^*)\right]\frac{2}{\left(x_2^* - x_1^*\right)} - \frac{f(\widehat{z}^*)}{1 - F(\widehat{z}^*)}\left[2 - \theta(\widehat{z}^*)\right] \tag{A.2}$$

Since $1 - \theta(\widehat{z}) > 0$, inequality (A2) holding at equilibrium implies that also inequality (A1) is satisfied at equilibrium.
Consider now the public firm, the SOC for which requires

$$2\left(1 - F(\widehat{z})\right) > \tfrac{x_2 - x_1}{2} f(\widehat{z})$$

At $\varphi(\widehat{z}) = 0$, $(x_2^* - x_1^*) f(\widehat{z}^*)/2 = F(\widehat{z}^*)/(2 - \theta(\widehat{z}^*))$, so that the above inequality is equivalent to

$$2\left(1 - F(\widehat{z}^*)\right) > \frac{F(\widehat{z}^*)}{2 - \theta(\widehat{z}^*)}$$

i.e.,

$$F(\widehat{z}^*) < 2\frac{2 - \theta(\widehat{z}^*)}{5 - 2\theta(\widehat{z}^*)} \tag{A.3}$$

Notice that log-concavity implies $\theta'(\widehat{z}) < \left[1 - \theta(\widehat{z})\right] f(\widehat{z})/F(\widehat{z})$, so that (A2) implies

$$-\left[1 - \theta(\widehat{z}^*)\right]\frac{f(\widehat{z}^*)}{F(\widehat{z}^*)} < \left[3 - 2\theta(\widehat{z}^*)\right]\frac{2}{\left(x_2^* - x_1^*\right)} - \frac{f(\widehat{z}^*)}{1 - F(\widehat{z}^*)}\left[2 - \theta(\widehat{z}^*)\right]$$

that is

$$\left(\frac{2-\theta(\widehat{z}^*)}{1-F(\widehat{z}^*)}-\frac{1-\theta(\widehat{z}^*)}{F(\widehat{z}^*)}\right)\frac{x_2^*-x_1^*}{2}f\left(\widehat{z}^*\right)<3-2\theta\left(\widehat{z}^*\right)$$

At $\varphi=0$, this expression becomes

$$\left(\frac{2-\theta(\widehat{z}^*)}{1-F(\widehat{z}^*)}-\frac{1-\theta(\widehat{z}^*)}{F(\widehat{z}^*)}\right)\frac{F(\widehat{z}^*)}{2-\theta(\widehat{z}^*)}<3-2\theta\left(\widehat{z}^*\right)$$

which implies

$$F\left(\widehat{z}^*\right)<\frac{7-8\theta(\widehat{z}^*)+2\theta(\widehat{z}^*)^2}{9-9\theta(\widehat{z}^*)+2\theta(\widehat{z}^*)^2} \tag{A.4}$$

so that (A.3) is verified since $\frac{7-8\theta(\widehat{z}^*)+2\theta(\widehat{z}^*)^2}{9-9\theta(\widehat{z}^*)+2\theta(\widehat{z}^*)^2}<2\frac{2-\theta(\widehat{z}^*)}{5-2\theta(\widehat{z}^*)}$ as $\theta\left(\widehat{z}\right)<1$.

References

[1] Cremer, H., Marchand, M., Thisse, J.F., 1991, Mixed oligopoly with differentiated products, Int. J. Ind. Organ., 9, 43-53.

[2] Kitahara, M., Matsumura, T., 2013, Mixed duopoly, product differentiation and competition, Manch. Sch., 81, 730-744.

[3] Lu, Y., Poddar, S., 2007, Firm ownership, product differentiation and welfare, Manch. Sch., 75, 210-217.

[4] Matsumura, T., Matsushima, N., 2003, Mixed duopoly with product differentiation: sequential choice of location, Aust. Econ. Pap., 42, 18-34.

[5] Matsumura, T., Matsushima, N., 2004, Endogenous cost differentials between public and private enterprises: a mixed duopoly approach, Economica, 71, 671-688.