Simplified numerical approach for the thermo-mechanical analysis of steelmaking components under cyclic loading: an anode for electric arc furnace

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In the last decades the need of a strong improvement in term of productivity and reliability led the adoption of advanced modelling techniques in the design of steelmaking plants components. However, due to the lack of experimental data and to the necessity to achieve results in a time that has to be compatible with industrial requirements, a customized industry-oriented approach should be implemented. In this work a procedure based on a finite element simulation is proposed in order to perform a durability analysis. This component undergoes cyclic thermal loads, which produces a partial melting of one part, meanwhile the other is maintained at almost constant temperature by means of a cooling system. A simplified, but effective, procedure is developed to take into account steel melting during the heating phase. Due to the cyclic loading conditions, an investigation on material cyclic plasticity models, and their effect on the thermal fatigue behaviour, is performed. The proposed approach permits the component fatigue life to be achieved with a simple and fast uncoupled thermo-mechanical simulation in steady-state conditions.

Keywords: thermo-mechanical analysis; phase change; finite elements; constitutive models; steelmaking; industry- oriented design

Subject classification codes: Blast furnace practice; numerical modelling; finite element method; thermomechanical fatigue; failure analysis; life prediction

Introduction

In the last decades the design of steelmaking plants made use of well established, although quite simplified methodologies [1]. In fact, the uncertainties related to a strongly approximated simulation of the technological process, as well as the lack of accurate mathematical models of the structural behaviour, could easily be overcome by simply over sizing the most critical mechanical elements. Recently, the demand of a strong improvement in term of productivity and reliability, accompanied by cost reduction requirements, has completely been changing the design approach of such components. Therefore, the design should take into account, often with numerical models, quite complex phenomena such as plasticity at different temperatures, low-cycle thermal fatigue or phase transition. Although these phenomena have already been addressed in other technological areas (e.g. aerospace and automotive industry) [2], their use for designing steelmaking plants is not straightforward and still requires further study. In fact, the modelling of the abovementioned complex issues requires a calibration with experimental data, often at operative conditions. For this purpose, even simplified, but reliable procedures, have to be developed, whenever measurements cannot be carried out at affordable costs. This is an industrially oriented approach that permits the best balance between accuracy and model complexity to be achieved.

The work aims to describe an example of such an industrial oriented approach applied to the thermo-mechanical analysis of a steelmaking component undergoing cyclic thermal loads. The presented procedure could be thus adopted for all the components which work in close contact with molten steel or metal at high temperatures, such as furnace components, moulds and working rolls. In particular, in this work the case of a water cooled anode for electric arc furnace is considered. In fact, this component experiences thermal cycling, plasticity at elevated temperatures and partial melting. Despite the fact that electric arc furnaces (and their electrodes) are a key part of steelmaking plants, technical literature lacks detailed treatises dealing with thermo-mechanical design of anodes; in fact, in [3, 4] the general layout of electric arc furnaces is described. In [5], the main requirement concerning a bottom electrode is discussed and different possible designs are proposed. In [6], a detailed description of the whole electric arc furnace is presented, with only a qualitative scheme of the bottom anode. Other works [7, 8] are mainly focused on the development of electric arc and heat flux.

In this treatise, a procedure based on a Finite Element simulation is proposed in order to perform a durability analysis of the component. Due to the cyclic loading conditions, particular attention will be devoted to investigate how the material hardening model influences the thermal fatigue behaviour. An original procedure to take into account partial melting will be also described. The proposed approach permits the component fatigue life to be achieved with a simple and fast uncoupled thermo-mechanical simulation in steady-state conditions.

Component description and modelling

Usually an anode of an electrical furnace is an axisymmetric component, which is constituted by an upper steel billet joined to a cooled copper base, in order to have the part in contact with the scrap charge of a compatible material and to exploit the better thermal conductivity of the copper.

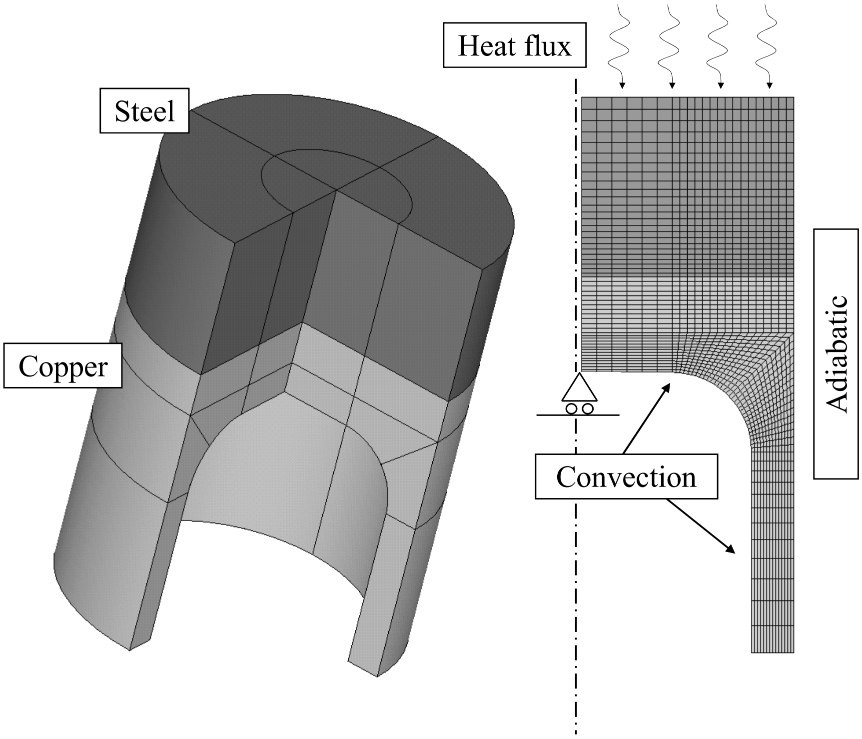


Figure 1. Component geometry and Finite Element model details.

Different configurations are adopted: the so called Kvaerner-Clecim billet electrode [6] presents the cooling fluid surrounding the copper base; in a more recent design solution, a cavity or a pattern of pipes is obtained in the inner part of the copper base [5]. In both cases the service life assessment requires the evaluation of the stress and strain behaviour of the component, characterized by cyclic thermal load producing a partial melting of the steel billet and a high thermal gradient in the copper part. The anode considered in this work can thus be modelled (Figure 1) by an axisymmetric plane model constituted by a steel billet in the upper part and a water cooled copper jacket in the lower region. The thermal flux due to the furnace operation heats the upper part and it is almost completely dissipated only through the cooling system, as the external surface is surrounded by refractory material that gives adiabatic conditions. The typical duty cycle that the component undergoes is constituted by a sequence of heating and cooling phases. As an example, in this work a component with an external diameter of 0.3 m and subjected to a maximum thermal flux of 210000 W/m^2 is considered. The thermo-mechanical behaviour of the component can be considered uncoupled and therefore it can be studied by using a thermal and a mechanical Finite Element model characterized by a quite simple geometry. Taking advantage of axial symmetry, plane elements can be used with different level of mesh refinements, depending on the kind of analysis considered. The thermal model shows the following imposed boundary conditions: heat flux in the top edge, convection in the inner filleted portion and null flux through the outer contour. The mechanical model is obviously self-balanced, since only thermal loads are present. To avoid rigid body motion, a single node is constrained.

Material properties and models

Within each duty cycle the upper steel billet partially melts and might undergo crack formation, which however does not affect furnace effectiveness. On the contrary local cracking in the copper jacket have to be absolutely avoided, as it can produce water leakage and eventual catastrophic failure [9, 10]. Cracks are initiated by cyclic stresses and strains in the plastic regime produced by cyclic thermal loads. Therefore, the mechanical cyclic response of copper has to be accurately modelled, whereas for steel, thermal material behaviour is of greater interest. An accurate numerical modelling requires the knowledge of material properties also at different temperatures.

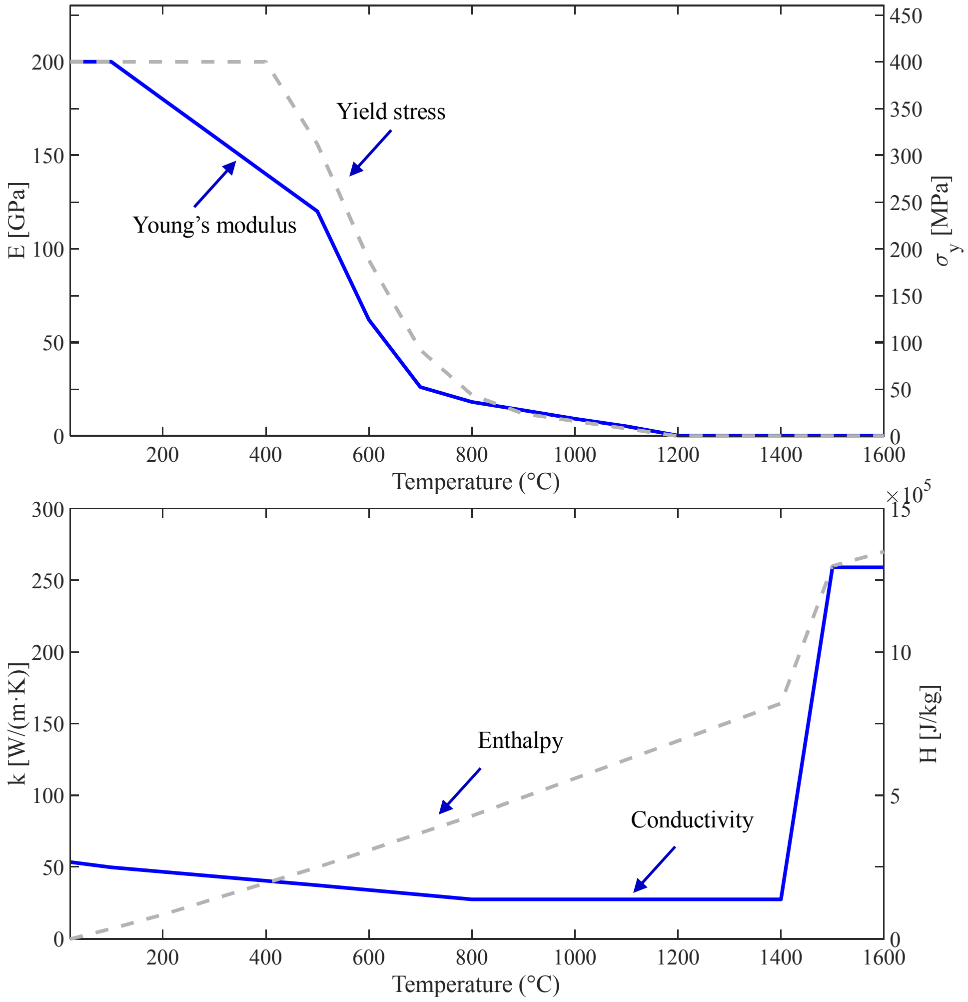


Figure 2. Steel properties (Data from [11, 12, 13]).

Table 1. Copper properties: kinematic or isotropic model.

	Temperature, (°C)			
	20	200	400	550
E (GPa)	118	118	98	68
σ_y (MPa)	80	69.1	51.6	29.8

Table 2. Copper properties: combined model (Data from [14]).

	Temperature, (°C)			
	20	200	400	550
σ_0 (MPa)	3	3	3	3
Q (MPa)	76	60	36	16
b	8	15	25	40
C (MPa)	64257	44324	31461	17188
γ	888	923	952	979

A survey of steel (0.27%C) properties at different temperatures is available in [11, 13, 15], where values of thermal conductivity (k), as well as Young's modulus (E) and yield strength (σ_y) are proposed. The mechanical properties collapse for temperature above 600°C. In the anode the steel part undergoes phase change during heating. In [13] the *solidus* and *liquidus* temperature were found to be around 1400°C and 1500°C respectively, and both enthalpy (H) and conductivity were shown to have a sharp variation at melting interval. Figure 2 represent the considered material properties versus temperature. In the case of copper, mechanical and thermal properties at room temperature are considered according to the stabilized curve from [16], while a suitable variation with temperature is considered as suggested in [17]. In order to correctly model the mechanism that give rise to hardening or softening behaviour, the evolution of yield surface and the so-called flow rules need to be considered.

Several models [14, 18, 19, 20] are available in literature to properly describe material elasto-plastic behaviour in monotonic and cyclic loading. As it is well known, a change of the yield surface size is connected to isotropic hardening and a change of location of the centre to kinematic hardening [21]. The von Mises yield criterion can formally be written as:

$$F = \left(\frac{3}{2} \mathbf{s} : \mathbf{s} \right)^{\frac{1}{2}} - \sigma_0 = 0 \qquad (1)$$

where $\boldsymbol{s}$ is the deviatoric stress tensor, σ_0 is the initial yield stress (prior to plastic deformation) and symbol "**:**" indicates the contracted tensorial product.
In the case of kinematic hardening Eq. (1) has to be modified as:

$$F = \left[\frac{3}{2} (\boldsymbol{s} - \boldsymbol{\alpha}) : (\boldsymbol{s} - \boldsymbol{\alpha}) \right]^{\frac{1}{2}} - \sigma_0 = 0 \qquad (2)$$

where $\boldsymbol{\alpha}$ is the backstress tensor, which controls the position of the yield surface as plastic deformation increases. The increment of back stress $\boldsymbol{\alpha}$ is given by the following equation for a non-linear kinematic hardening:

$$\mathrm{d}\boldsymbol{\alpha} = \frac{2}{3} C \,\mathrm{d}\boldsymbol{\varepsilon}_p - \gamma \boldsymbol{\alpha} \mathrm{d}\varepsilon_{\mathrm{p,acc}} \qquad (3)$$

where C is the initial hardening modulus, $\boldsymbol{\varepsilon}_{\mathrm{p}}$ the plastic strain tensor, $\varepsilon_{\mathrm{p,acc}}$ the accumulated plastic strain and γ is a material parameter that governs the decrease rate of the hardening modulus. A linear kinematic model can be obtained by putting γ=0 in Eq. (3), therefore $\mathrm{d}\boldsymbol{\alpha} = (2/3)C\,\mathrm{d}\boldsymbol{\varepsilon}_p$. In the case of isotropic hardening the increase of the yield stress R due to accumulated plastic strain is introduced in Eq. (1):

$$F = \left[\frac{3}{2}(\boldsymbol{s}):(\boldsymbol{s})\right]^{\frac{1}{2}} - R = 0 \qquad (4)$$

In the non-linear isotropic model developed by [18] the following expression of R is proposed:

$$R = \sigma_0 + Q_\infty\left(1 - e^{-b\varepsilon_{\mathrm{p,acc}}}\right) \qquad (5)$$

where Q is the asymptotic value of yield stress corresponding to stabilized regime and b a material parameter that governs the rate of change of the yield surface. Combined kinematic and isotropic models are used to take into account the increase of yield stress and back stress tensor variation:

$$F = \left[\frac{3}{2}(\boldsymbol{s}-\boldsymbol{\alpha}):(\boldsymbol{s}-\boldsymbol{\alpha})\right]^{\frac{1}{2}} - R = 0 \qquad (6)$$

which permits softening and hardening cyclic behaviour to be described. The main aspects of the abovementioned methods are now shortly discussed, with particular emphasis on their practical use in steelmaking component design.

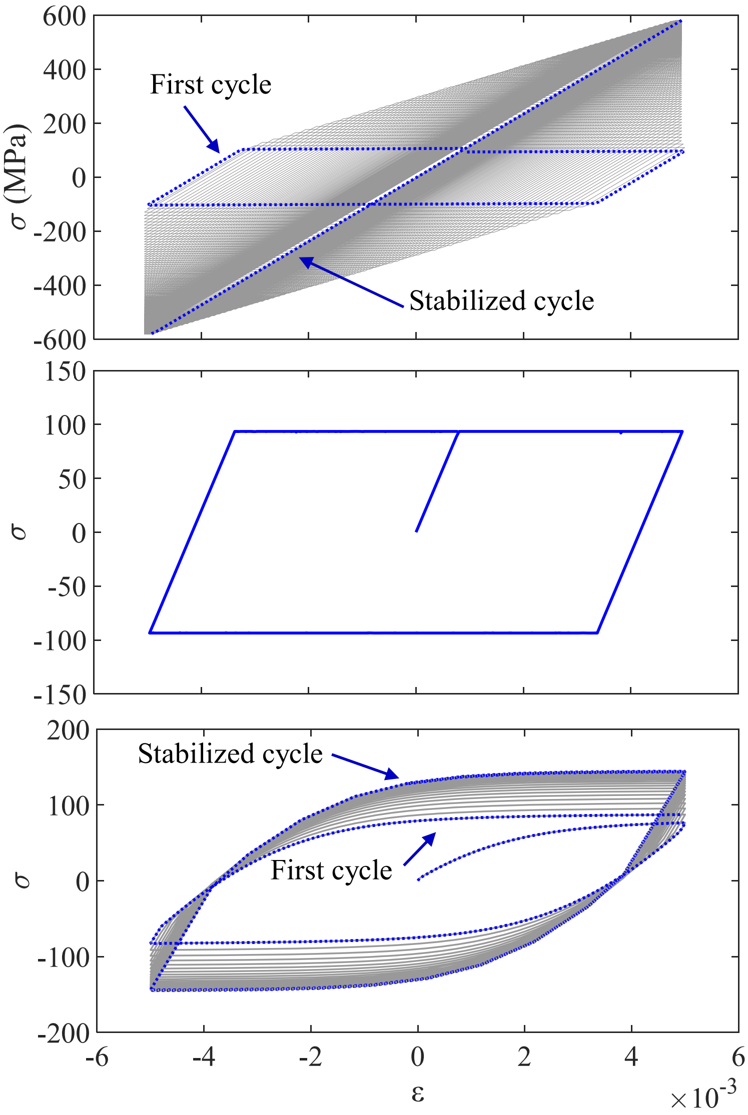


Figure 3. Cyclic response under strain control of pure copper: kinematic, isotropic and combined models.

Linear isotropic and kinematic models provide the easiest modelling approach, as they simply require 3 parameters: elastic modulus E, initial yield stress σ_0 (named σ_y in Table 1) and initial hardening coefficient C (also named tangent modulus). Isotropic models are used in [22] and in [23] to study the thermal and mechanical behaviour of copper mould during thin slab casting. Nevertheless, in the case of imposed cyclic strain, the isotropic model shows a shakedown into a purely elastic loop after few cycles. On the contrary, a kinematic hardening model captures Bauschinger's effect and, in the considered uniaxial case, it stabilizes into an elastic-plastic hysteresis loop after the first cycle. In [24] and [25], the thermomechanical behaviour of hot rolling mills is analyzed by using a kinematic model. In [26], a combined model is used to evaluate permanent deformation during the roll levelling of thin strip steel. With respect to pure isotropic or kinematic models, a combined model (kinematic plus isotropic) permits the cyclic hardening or softening material behaviour to be captured, despite a more expensive experimental effort to estimate the material parameters needed to calibrate the model [14, 19].

The differences between the three modelling strategies are better clarified by the comparison of the cyclic response under fully-reversed strain cycles at room temperature shown in Figure 3. Similar trends can be obtained at different temperatures. Material parameters for pure kinematic and isotropic hardening model are collected from [16] and summarized in Table 1, while for the combined model the data from [14], resumed in Table 2, are adopted. It can be noticed that, as previously pointed out, if large plastic deformation occurs, the physical behaviour of the material is grasped more efficiently according to the kinematic approach; isotropic model leads to elastic shakedown within few cycles (i.e. purely elastic response) and it cannot thus provide any information about the amount of plastic range achieved, thus it will not be considered in the following. The combined model, instead, seems to be the most appropriate, as it is able to capture both the shape of stress-strain cycles and the cyclic hardening behaviour. Figure 4 shows the cyclic behaviour obtained with the combined model at three strain amplitudes. The stabilized stress-strain curve experimentally obtained by [16] is also presented for comparison, confirming the validity of the used model parameters. From the same figure it is also clearly arguable that the material stabilizes just after 15-20 cycles. This result is confirmed in [14]. In [27] a similar behavior has been reported for polycrystalline copper. In [28], strengthened Cu-Ag reaches stabilization after less than 20 cycles.

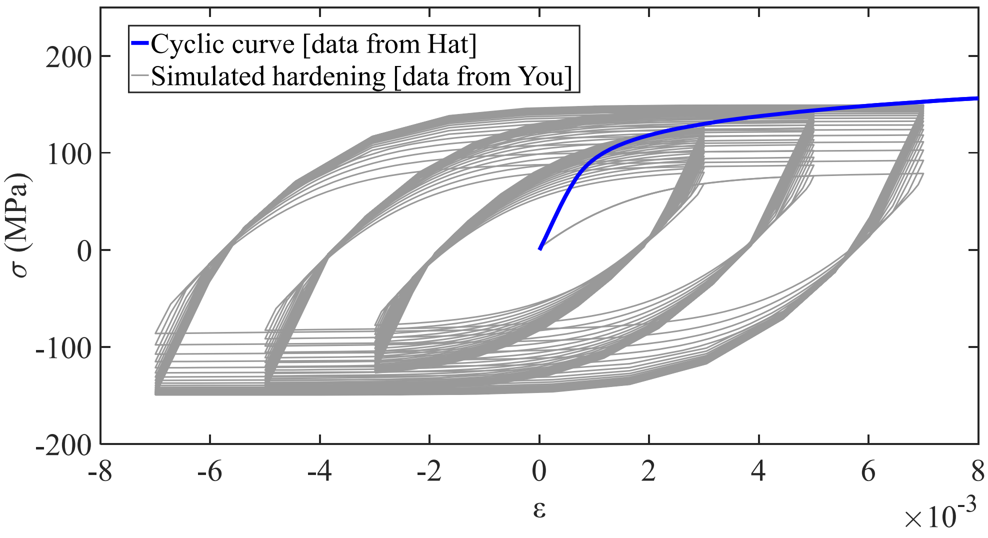


Figure 4. Numerically evaluated cyclic stress-strain curve for copper.

As it will be shown in the next section, once the cyclic plasticity parameters have been introduced in the Finite Element model, a sequence of cycles has to be simulated, in order to compute the stress and strain evolution until stabilization. Stabilization generally occurs at half fatigue life [29], it follows that often the computational time could be unfeasible for industrial needs when large numerical models have to be used.

Thermomechanical analysis

To study the behaviour of the anode, it should be necessary to simulate a sequence of heating and cooling, which produces a partial melting and subsequent solidification. In order to take into account phase transition, a numerical model involving transient analysis is required. Moreover, as thermal loads induce stresses exceeding yielding, cyclic plasticity has also to be considered with a suitable material model, thus further increasing the complexity of the analysis, which could require a huge computational time. It has to be remarked that the steelmaking process can tolerate localized collapses in the upper hot part of the component as periodic maintenance is generally planned when the plant is switched off [5]. On the contrary, the formation of cracks in the inner surface of the copper jacket has to be absolutely avoided, as it could produce a leakage of water that could reach the molten metal with catastrophic consequences. It follows that the accuracy of the model has to be verified with respect to its capability to give good results in term of stress and strain only in the copper jacket close to the cooling water. Model simplifications are thus acceptable, if they do not affect significantly the mechanical response of this specific portion. The suitability of the modelling techniques presented in the following will be therefore analyzed according to this design strategy. Firstly, the analysis of a single heating phase involving phase change will be considered; then the cyclic behaviour in elasto-plastic conditions will be evaluated. The finite element mesh (**io metterei model) developed is proposed in Figure 1. It consists of 1492 axial symmetric elements and 1593 nodes.

Component heating at operative condition: phase change model

In Finite Element modelling, melting can be simulated by a transient thermal analysis with a change in enthalpy from solid to liquid phase. This analysis is strongly non-linear, since it has to account for the change in material properties against temperature already shown in Figure 2. In particular, the sharp variation of enthalpy and conductivity between the *liquidus* and *solidus* temperatures could lead to difficulties in achieving convergence of results. For this reason, an accurate and suitable time step level need to be implemented in order to correctly follow the whole transient phase.

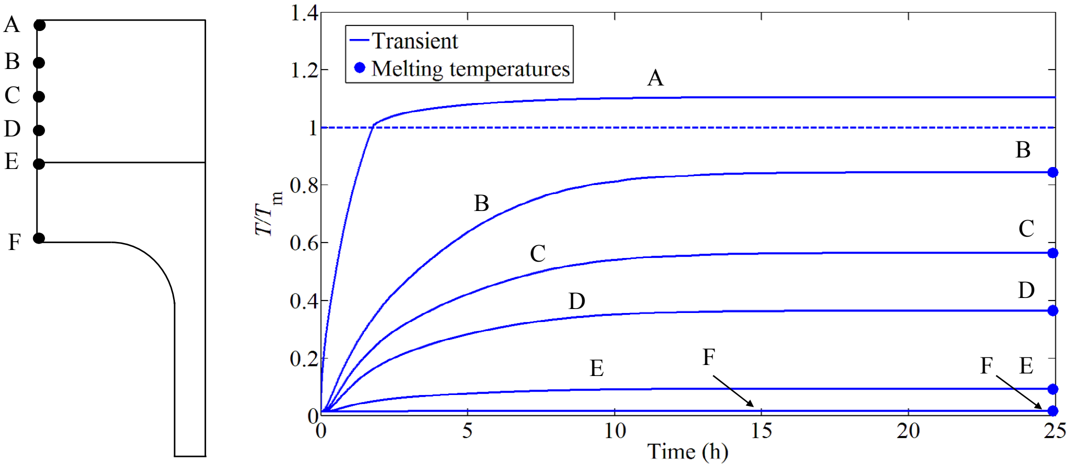


Figure 5. Temperature distribution during transient heating and according to the so called “melting temperature” method.

Figure 5 shows the temperature evolution at different points of the anode heated by a constant thermal flux applied on the uppermost surface and cooled by convection in the lower part, over a time interval of 25 hours. As it can be seen, at the beginning of the simulated heating sequence the temperature of the upper portion of the steel billet (points A) show a steep increase and stabilizes within the melting interval. Temperature on the lower part (points B, C and D) requires greater time to stabilize without any phase change. The copper jacket (points E and F) remains at rather constant low temperature. A further increase in computational effort is due to the fact that a subsequent transient mechanical analysis has to be performed, if the cyclic thermo-mechanical response has to be computed. The computational complexity could be unfeasible for industrial needs and alternative simplified model strategy should be investigated. An obvious option could be that of substituting the transient analysis with a steady state simulation. As shown in Figure 6, the obtained temperature distribution is clearly unrealistic from a physical point of view, because in the upper part of the component the melting point is largely exceeded. On the other hand, this error in the temperature evaluation can be tolerated as far as it does not influence the mechanical behaviour of the cold part (upper part of the anode). It is a matter of fact that the melted steel of the billet, characterized by poor mechanical strength, slightly affects the mechanical behaviour of the cold part, independently from an accurate evaluation of its temperature. It could be of interest to investigate to what extent the mechanical response in the copper part is sensitive to an imposed correction of either the steady state temperatures or the mechanical strength in the upper part. For this purpose, different modelling techniques can be implemented and compared (see Figure 6).

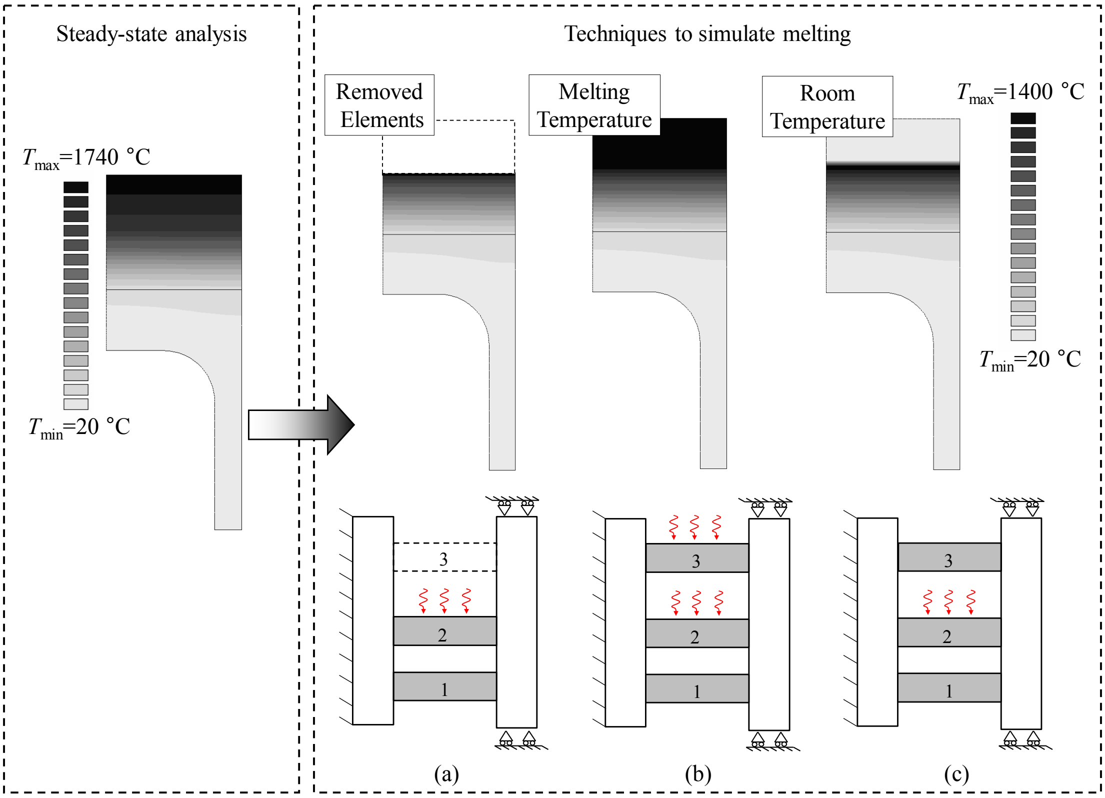


Figure 6. Techniques to simulate melting: (a) removed elements, (b) room temperature imposed and (c) melting temperature imposed.

One option could be that the finite elements corresponding to the molten part are simply removed or substituted with “soft” elements, characterized by a very low thermal expansion and modulus of elasticity. In fact, such “soft” elements do not affect the mechanical behaviour of the underlying structure (still solid). The described strategy, named “removed elements” (Figure 6a), would require, however, that “soft” elements be removed incrementally, when their temperature exceeds the melting point during the thermal transient. On the other hand, the analysis could be strongly simplified by removing all the melted elements not incrementally, but just after the steady state thermal analysis has been performed. If a suitable commercial code that makes the “removed element” feasible is not available, a slightly different method could be adopted, that will be named “melting temperature”. As shown in Figure 6b, the temperature above melting resulting from a steady state analysis is simply levelled out to the melting point. In Figure 5, the temperatures obtained with this approach are shown for comparison (see solid circles at t= 25 h); they clearly represent the asymptotic solution of the transient analysis.

It is now of interest to understand, at least qualitatively, how the two different modelling strategies influence the mechanical response of the anode. It has to be pointed out that the molten portion of the steel billet represents only a weak mechanical constraint at the interface with the underlying solid material. From this standpoint, the “removed element” strategy represents a case in which all nodes at the interface are completely free. In the case of the “melting temperature” approach interface nodes are slightly constrained in order to preserve compatibility. The latter strategy permits convergence to be improved and it also accounts for the levelling of temperatures that occurs during phase change. These two approaches are clearly expected to give comparable results of stress and strain in the filleted part of the copper jacket, as they are characterized by similar constraint conditions at the interface nodes. To complete the sensitivity analysis, it could be of interest to consider also the case in which nodes at the interface are over-constrained (see Figure 6c) by imposing a fixed fictitious room temperature to all the melted steel billet. Owing to the room temperature imposed, the upper steel billet now shows a high mechanical strength and a low thermal expansion, thus constraining the lower solid part. This technique, named “room temperature”, is expected to give the lower bound case and therefore it gives probably the less accurate results.

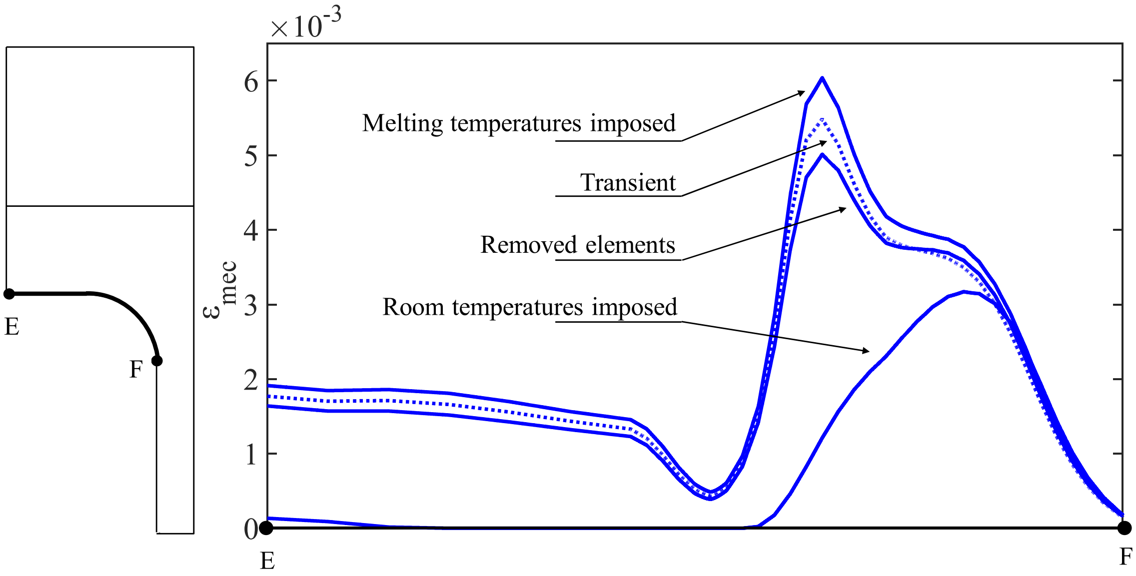


Figure 7. Total mechanical strain in the filleted region after one single heating, calculated by different simulation approaches.

The qualitative comparison of the three modelling strategies, sketched in Figure 6, can be further investigated by comparing the results of the mechanical analysis. Figure 7 shows the total mechanical strain ε_{mec} in the filleted part of the water cooled copper jacket after one heating, obtained with a transient analysis and also with the three approaches previously described. The solution of transient analysis, of course, has to be interpreted as the reference in comparison. It can be noticed that the “removed element” and the “melting temperature” methods give results very close to those obtained with the transient case, while the “room temperature” method brings to strain values that are more than halved, and thus not reliable.

The obtained results, and thus the mechanical behaviour of the copper jacket, can be interpreted by means of the simple one dimensional three bars model proposed in Figure 6, where bar 1 represents the cooled material (copper jacket), bar 2 the hot solid material and bar 3 the upper melted material (steel billet). One end is fixed, the opposite is free to translate to take into account the radial expansion of the component. The cold material (bar 1) is tensioned due to its tendency not to follow the thermal expansion of the hot part (bar 2 and 3). In the "removed element" technique bar 3 is obviously not present and it follows that the tensile load in bar 1 is exactly counterbalanced by the compressive force in bar 2. In the "melting temperature" technique, the melted material is not removed and it behaves like a heated third bar with low stiffness, which tends to expand. As a consequence, a slightly higher tension load is produced in bar 1; this explains why the "melting temperature" method gives a total strain slightly higher than the strain resulting from the "removed element" simulation, see Figure 7. If the melted material is forced to assume the room temperature, it exhibits a high stiffness not allowing the expansion of bar 2. It follows that bar 1 undergoes a lower load and the strain is considerably reduced. This means that the "room temperature" method underestimates the values of strain in the filleted part. The above considerations give physical evidence that the "melting temperature" method is the most conservative approach and it will then be adopted in the following analysis, as it is also easier to be implemented and it permits a faster convergence to be achieved.

Thermomechanical behaviour of the component under cyclic heating and cooling

Thermo-mechanical analysis is performed as shown in Figure 8 where the thermal flux is cyclically imposed and removed to account for plant switch on and switch off. This thermal loading constitutes the most critical condition, which corresponds to a prolonged maintenance switch off: the anode cools down, with a complete solidification of the steel billet. Even if such load cycle occurs only few times in the anode lifespan, the most conservative approach requires to design the component under such a condition. In the simulation, the anode is thus heated up to the operative conditions (maximum thermal flux) and subsequently brought back to room temperature.

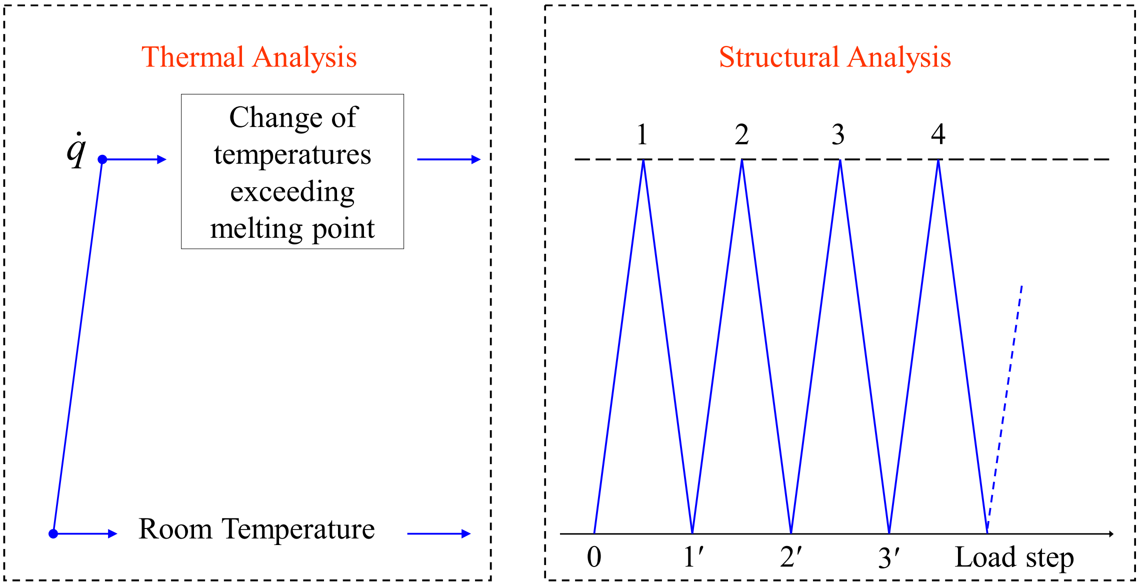
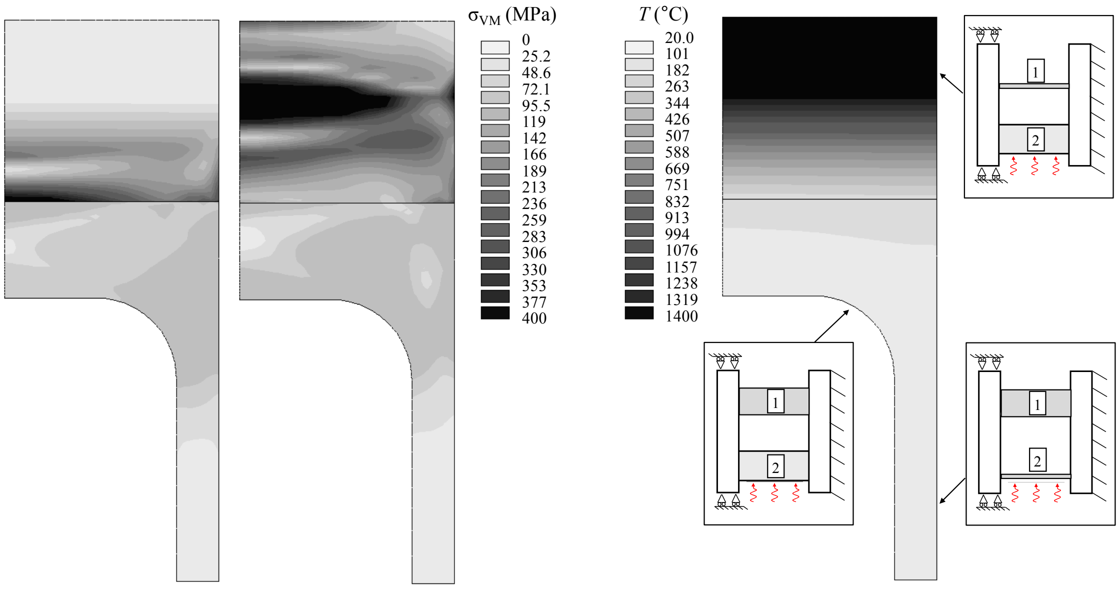


Figure 8. Load cycles.

According to the “melting temperature” approach, the nodal temperatures exceeding the phase transition point are changed to the melting temperature. Figure 9a and 9c show respectively Von Mises stresses and temperature distribution after the first heating (point 1), while in Figure 9b the Von Mises stresses at room temperature (point 1’) are depicted.



(a) (b) (c)

Figure 9. Von Mises stress contours (a) after heating, (b) after cooling to room temperature and interpretative models referring to temperature distribution (c).

Owing to the high temperature gradients, relevant stresses occur during heating and yielding is reached in the steel part and in the filleted portion of the copper jacket. When the flux is removed and the component cools down, the non-uniform plastic strain distribution produces residual stresses, which locally exceed the yield stress. A simplified interpretive model of the component thermo mechanical behaviour can be proposed. As schematically represented in Figure 9c, the behaviour of the component can be compared to that of a two-bar assembly in which the ends of the bars are connected to two rigid plates (rotations are not allowed). The two bars have area and Young modulus A_1, E_1 and A_2, E_2, respectively, and the same thermal expansion coefficient $\alpha_1=\alpha_2=\alpha$. One end of the bars is fixed, the opposite can translate axially. Bar 2 is heated and tends to expand. Its expansion, however, is partially restrained by bar 1.

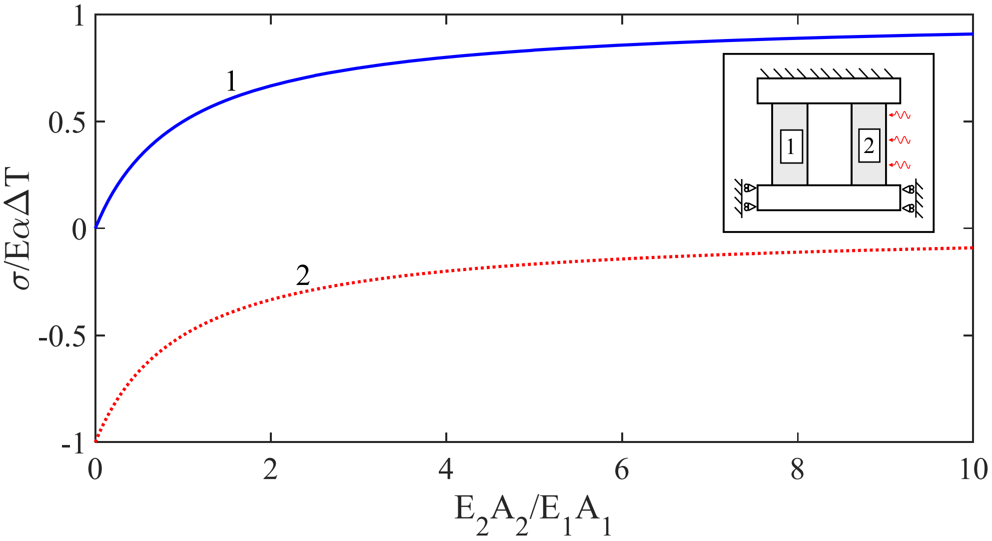


Figure 10. Stress (strain) in a two-bar assembly versus axial stiffness ratio.

In order to obtain an analytical description of this simple model, originally proposed by [29], it has to be reminded that, for the one-dimensional case, the total strain ($\varepsilon_{\mathrm{tot}}$) is due to mechanical ($\varepsilon_{\mathrm{mec}}$) and thermal ($\varepsilon_{\mathrm{th}}$) strain:

$$\varepsilon_{tot} = \varepsilon_{mec} + \varepsilon_{th} \tag{7}$$

where $\varepsilon_{mec} = \varepsilon_{el} + \varepsilon_{pl}$ and the subscripts *el* and *pl* correspond respectively to elastic and plastic strains. Considering that the bars have the same initial length, compatibility requires that displacements, and therefore total strains, must be the same. It is thus possible to write:

$$\varepsilon_{mec,1} = \varepsilon_{mec,2} + \varepsilon_{th,2} \tag{8}$$

since ($\varepsilon_{\mathrm{th},1}$) is null (bar 1 is not heated); it is $\varepsilon_{th,2} = \alpha\Delta T$. Equilibrium requires that:

$$\sigma_1 A_1 = -\sigma_2 A_2 \tag{9}$$

Considering a linear elastic behaviour of the material, stress is related to mechanical strain with the Hooke's law $\sigma_1 = E_1\varepsilon_{mec,1}$ and $\sigma_2 = E_2\varepsilon_{mec,2}$. Combining Eq. (8) and (9), the axial stress in the bar 2 and 1 can be finally related to the axial stiffness ratio E_2A_2/E_1A_1:

$$\sigma_2 = \frac{-\alpha\Delta TE_2}{1+\left(\frac{E_2A_2}{E_1A_1}\right)} \quad \sigma_1 = \frac{\alpha\Delta TE_1}{1+\left(\frac{E_1A_1}{E_2A_2}\right)} \tag{10}$$

As an example, if the axial stiffness ratio increases, σ_2 reaches higher compressive values (while bar 1 is characterized by lower values of tensile stress). Figure 10 shows stress variation against axial stiffness in dimensionless form, in such a way that the ordinate values can also be interpreted as the ratio between the mechanical and the thermal strain of the bar. The first asymptotic condition is reached when E_2A_2/E_1A_1 tends to ∞ and bar 2 is free to expand (hence σ_2 and $\varepsilon_{mec,2}$ tends to 0). This case occurs in the upper portion of the component, where there is a quite uniform temperature distributions and thus no significant stresses are exhibited. The second asymptotic condition is reached when E_2A_2/E_1A_1 tends to 0. This case represents the condition in which the thermal strain is completely converted into mechanical strain and thus σ_2 and $\varepsilon_{mec,2}$ reaches its maximum value and σ1 (and $\varepsilon_{mec,1}$) tends to 0. This occurs in the lower portion of the copper jacket. In this case only the cold bar has to be considered.

The part closer to the cooling fluid is maintained at constant, low temperature even if it is surrounded by a significant amount of copper material characterized by a relevant thermal gradient. In this case, as it can be easily shown, the hot portion, with respect to the cold one, has a more relevant extension (A_2) with lower mechanical properties (E_2). This corresponds to the case where the ratio E_2A_2/E_1A_1 >>1 and a significant positive in-plane stress close to the fillet profile occurs.

As it was previously emphasized, the main design requirement is to avoid the risk of crack formation on the lower part of the component. The most critical area is thus subjected to cyclic strains produced by cyclic temperature gradients, occurring far from this region, which is maintained at constant temperature by the cooling fluid. It is thus possible to refer to this particular loading condition as thermal fatigue under isothermal condition. In particular, there is no necessity of fatigue data at high temperatures, because the considered zone does not undergo temperature variation.

In literature relations between cyclic load and life at room temperature are available, see for instance [16], [20] and [30]. Usually, in the case of similar components, an approach based on the plastic strain range is adopted [23, 31]; nevertheless, recently in [32] an approach based on the total mechanical strain range is suggested. In fact, firstly it has been noticed that in the case of the same material undergoing different types of of technological treatments, even if its cycling behaviour is quite different (hardening or softening) in term of stress, its durability is only affected by the total strain level. Moreover, total strain range can be measured more easily and in general it is less sensitive, in a numerical analysis, to the adopted material model.

In this work the total strain-life relation obtained in [17] is adopted,

$$\Delta\varepsilon_{tot} = 77.37 \cdot N_f^{-0.63} + 0.94 \cdot N_f^{-0.12} \qquad (11)$$

where $\Delta\varepsilon_{\text{tot}}$ is the total strain range and N_{f} is the number of cycles to failure. For the determination of the strain range, a sequence of cycles with a thermal load oscillating between room temperature and operative condition has to be simulated, taking into account the elasto-plastic behavior of the material. As already pointed out, several models of cyclic plasticity are available in literature, with different accuracy level.

Apparently a more elaborate model seems to be the best choice, otherwise, very often an industry oriented approach, which makes use of simplified models, could be more advisable. In fact, firstly the experimental determination of the material parameters is not easy, then, it must be pointed out that in general a combined isotropic kinematic model requires simulating of a huge number of cycles, in order to reach complete stabilization.

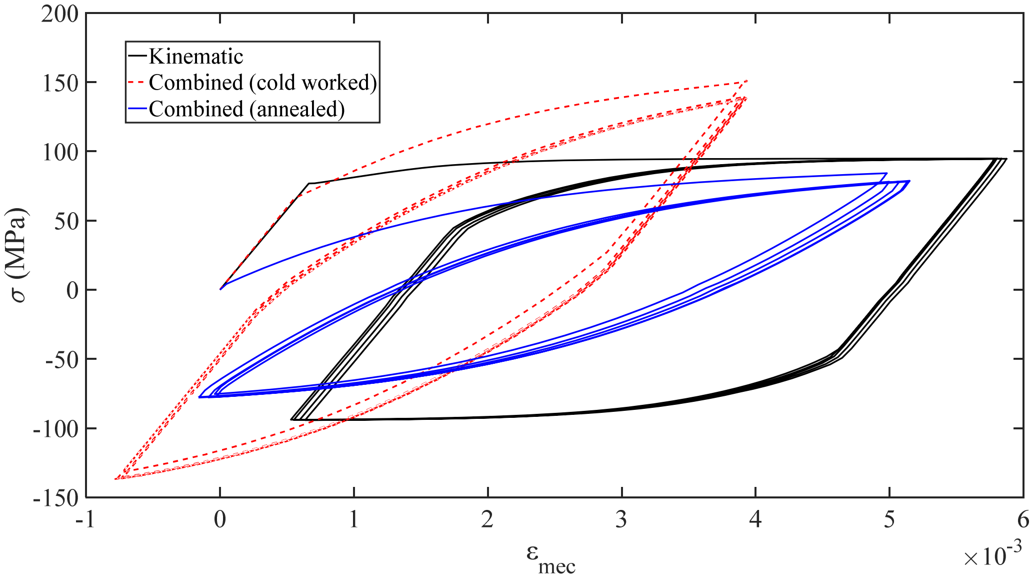


Figure 11. Stress-strain hysteresis loops.

As stated before, from the fatigue life assessment, the most critical point, occurs on the surface of the filleted area, where the highest values of plastic strain are reached. A comparison among the cyclic plasticity models previously proposed is here performed. Figure 11 shows the stress-strain hysteresis loop evaluated adopting a kinematic model. By comparison, a simulation with a combined isotropic kinematic model, calibrated according to the experimental data reported in [14], is performed. In the authors' experience, very often the copper jacket is made of cold worked copper; as the data from [14] refers to an annealed material, a higher yield stress is also considered. The stress-strain curve was obtained after 10 cycles; further cycles simulation shows in this case negligible variation in term of strain range.

Table 3. Strain life evaluation.

	$\Delta\varepsilon_{el}$	$\Delta\varepsilon_{pl}$	$\Delta\varepsilon_{tot}$	N_f
Kinematic model	0.0012	0.0039	0.0051	12370
Combined model (cold worked)	0.0019	0.0026	0.0045	18850
Combined model (annealed)	0.0011	0.0040	0.0051	12370

All the results confirm that, during heating, the cooled filleted portion exhibits a positive strain, due to the action of the surrounding heated material, as sketched in Figure 9c. Yielding occurs, with a relevant plastic strain. When the thermal flux is removed, a compressive stress arises, which produces copper yielding again. The material thus undergoes a sequence of yielding in tension and in compression, in phase with the temperature variation. It can be finally observed that the three elasto-plastic models give different values of elastic and plastic strains; on the other hand, the total strain range is quite similar for all the adopted models. In principle, a total conversion of the thermal strain in mechanical strain occurs only if $E_2A_2>>E_1A_1$ (see Figure 10). In this case, even if the elastic and plastic strain depend on the material model adopted, the total mechanical strain remains quite constant. As previously pointed out, limited amount of “cold” material is counterbalanced by its higher elastic modulus. It can therefore be concluded that this condition corresponds to a point located in the right side of the graph of Figure 10, thus justifying a relatively small difference among the total strain range obtained with the different models of plasticity. The mixed model for the cold worked copper gives a slightly higher number of cycles with respect to the other two approaches. The kinematic model is therefore the simpler and the more conservative and could represent the most suitable design choice.

Conclusions

In the last decades, the strong demand for improvement in terms of productivity and reliability, accompanied by cost reduction requirements, has been leading to make use of advanced modelling approaches in the design of steelmaking plants. Nevertheless, the lack of experimental data and the need to speed up the time to market, suggest a customized industrial oriented approach to be adopted.

In this work the thermo-mechanical analysis of an anode used in electric arc furnaces is presented. This component undergoes cyclic thermal loads, which produces a partial melting of one part, meanwhile the other is maintained at almost constant temperature by means of a cooling system. Firstly, the analysis of the single heating involving phase change was considered, then the cyclic behaviour in elasto-plastic conditions was evaluated.

A first concluding remark concerns modelling of stress and strain behaviour due to a heating which produces a partial melting of the steel billet. In this case two modelling strategies corresponding to impose zero displacements or to keep free the nodes of the *solidus-liquidus* interface can be implemented. When, as usual, the material mechanical properties show a huge decay at high temperature, the actual behaviour of the steel is closer to the free-nodes model and therefore the “melting temperature” approach can also be adopted, thus reducing the computational effort and improving convergence.

A final observation concerns the thermo-mechanical cycling analysis, which has to be focused on the outer surface of the copper jacket. It can be said that often the total mechanical strain range could be a more suitable parameter to evaluate the component durability, as it is less sensitive to material hardening rule: When thermal strain is almost completely converted in mechanical, accurate results can be achieved with a simple kinematic model.

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