Journal of Hydrology 527 (2015) 20–29

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Estimation of bathymetry (and discharge) in natural river cross-sections by using an entropy approach

G. Farina [a], S. Alvisi [a,*], M. Franchini [a], G. Corato [b], T. Moramarco [c]

[a] Engineering Department, University of Ferrara, via Saragat, 1, 44121 Ferrara, Italy
[b] Département Environnement et Agro-biotechnologies, Centre de Recherche Public Gabriel Lippmann, 41 Rue du Brill, 4422 Belvaux, Luxembourg
[c] Research Institute for Geo-Hydrological Protection, National Research Council, Via Madonna Alta,126, 06128 Perugia, Italy

ARTICLE INFO

Article history:
Received 17 November 2014
Received in revised form 13 April 2015
Accepted 18 April 2015
Available online 27 April 2015
This manuscript was handled by Konstantine P. Georgakakos, Editor-in-Chief

Keywords:
Bathymetry
Entropy
Streamflow measurements
Remote sensing

SUMMARY

This paper presents a new method for reconstructing the bathymetric profile of a cross section based on the application of the principle of maximum entropy and proposes a procedure for its parameterization. The method can be used to characterize the bathymetry of a cross-section based on a reduced amount of data exclusively of a geometric type, namely, the elevation of the lowest point of the channel cross-section, the observed, georeferenced flow widths and the corresponding water levels measured during the events.

The procedure was parameterized and applied on two actual river cross-sections characterized by different shapes and sizes. In both cases the procedure enabled us to describe the real bathymetry of the cross-sections with reasonable precision and to obtain an accurate estimate of the flow areas. With reference to the same two cases, we show, finally, that combining the bathymetry reconstruction method proposed here and an entropy-based approach for estimating the cross-sectional mean velocity previously proposed (Farina et al., 2014) enables a good estimate of discharge.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

The discharge of a river, quantifiable by multiplying the mean stream velocity by the flow area of the cross-section, represents a fundamental parameter for water resource management, flood control and land protection.

In common practice, discharge is generally estimated using the velocity-area technique, in which the cross-section area is discretized into an adequate number of segments, each delimited by two verticals. The discharge associated with each segment is determined as the product of the segment surface area and the corresponding mean velocity, the latter being obtained by means of direct current-meter measurements in points within (on an axis) or on the perimeter of the segment itself; finally, the total discharge is calculated as the sum of the discharges of the individual segments.

This procedure for estimating discharge thus entails sampling the stream velocity in points located at different depths along a sufficient number of verticals distributed within the flow area; these verticals are generally spaced in such a way as to provide an adequate representation of the variations in velocity across the cross-section.

At the same time, the cross-section profile is schematically represented by connecting the bottom points of the different verticals so that the flow area is the area between the cross-section profile and the free surface of the stream.

Although this technique proves to be particularly accurate, it is not easy to implement, as it relies on measurements of an episodic type that are difficult to automate, take a considerable amount of time and are not very accurate in proximity to the river bed due to the presence of vegetation; moreover, the strong currents that typically occur during exceptional flood events may expose operators to hazards or even make it impossible to take proper flow velocity measurements.

Acoustic Doppler Current Profiler (ADCP) can represent an alternative, since it provides the spatial distribution of velocity, but it is costly because of all the operations tied to post-processing and data filtering.

Another valid alternative for estimating discharge is provided by the entropy method, which, based on the principle of entropy maximization (Jaynes, 1957), was applied by Chiu (1987, 1988) to reconstruct the (probability) distribution of velocity in a channel cross-section. Chiu was able to identify a linear relationship, which is the function of a parameter – M – between the mean velocity

* Corresponding author. Tel.: +39 0532974849; fax: +39 0532974870.
E-mail address: stefano.alvisi@unife.it (S. Alvisi).

http://dx.doi.org/10.1016/j.jhydrol.2015.04.037
0022-1694/© 2015 Elsevier B.V. All rights reserved.

o verlineU and the maximum velocity $u_{\max}$ of a river cross-section (Chiu, 1988, 1991; Xia, 1997), i.e., $\overline{U} = f(u_{\max}, M)$.

In practical terms, starting from a single measurement of the cross-sectional maximum velocity $u_{\max}$ – easily determinable as it generally manifests itself in the upper-middle portion of the flow area (Chiu, 1991; Chin and Murray, 1992; Chiu and Said, 1995; Moramarco et al., 2004), which remains easily accessible for sampling even during substantial flooding – it is possible to arrive at an estimate of the mean velocity *o verlineU* and then, by multiplying the latter by the flow area of the cross-section, at an estimate of the discharge.

However, the parameter M must first be estimated in order to convert the maximum observed velocity $u_{\max}$ into the cross-sectional mean velocity *o verlineU*; this dimensionless parameter does not vary with the velocity (or discharge) and represents a typical constant of a generic cross-section of a channel/river (Xia, 1997; Moramarco et al., 2004). The parameter M is generally estimated by linear regression performed on a substantial set of pairs of values $u_{\max}$–*o verlineU*, which are obtained by means of the velocity-area method; hence, numerous current-meter measurements taken during multiple flood events are required. Indeed, these latter are the same measurements typically required for building a stage-discharge relationship.

However, some recently proposed procedures (Farina et al., 2014) enable the parameter M to be estimated relying on a more limited set of velocity measurements; these range from current-meter measurements across the entire flow area to one measurement of surface maximum velocity alone. In short, once the parameter M has been estimated by means of these procedures, it will be possible, as already stated, to estimate the mean velocity, and then multiplying the latter by the flow area of the cross-section, to have an estimate of discharge.

However, it should be observed that in order to be able to estimate the discharge using the entropy-based approach just outlined, we need to know the geometry of the river cross-section for the purpose first of estimating the parameter M and then of estimating discharge (quantification of the flow area). The problem to be confronted, therefore, is to reconstruct the cross-section geometry.

In order to determine the cross-section geometry we can presently rely on various bathymetric survey techniques, depending on the size of the cross-section we are investigating. The active bed of non-navigable rivers (water less than 1 m deep) can be surveyed simply by wading and taking direct measurements of points at the gage site or using GPS measurements; if, during exceptional flood events, the depth increases to such an extent that it is no longer possible to stand in the water, it will be necessary to rely on an ultrasonic bathymeter or an ADCP connected to a GPS survey system which can provide a precise, real-time survey of the course of the float it is mounted on (Costa et al., 2000; Yorke and Oberg, 2002). In addition to providing a spatial distribution of velocity, this technology makes it possible to have a high-definition map of the bed investigated, but as previously observed it is both time-consuming and costly because of all the operations tied to post-processing, data filtering, graphic rendering and inputting data to the databases.

Several authors have therefore sought to determine the bathymetric profile indirectly by analytic means based on the measurement of surface velocity, as this parameter can be easily determined by using latest-generation non-contact radar sensors (Costa et al., 2006; Fulton and Ostrowski, 2008). The method developed by Lee et al. (2002) is based on the assumption of a logarithmic velocity profile, but it requires knowledge of such hydraulic variables as the energy slope and Manning's roughness, which are very often not known. In a manner analogous to what Chiu (1987, 1988) did for velocity, Moramarco et al. (2013) applied the principle of maximum entropy to estimate the probability density function of water depth and the flow depth distribution along the cross-section, assuming a priori that the cumulative probability distribution function increases monotonically with the surface flow velocity.

In this paper we describe the theoretical development and practical application of a new analytical method that is likewise derived from the principle of maximum entropy, but is able to dissociate the bathymetry estimate from the surface velocity measurement. Using the proposed method, in fact, we can describe the bathymetric profile of the cross-section investigated on the basis of a smaller amount of information, exclusively of a geometric character, namely, the elevation of the lowest point of the channel cross-section and the georeferenced flow width (i.e. a flow width whose end coordinates are known) associated with a precise water level recorded during a sufficient number of flood events.

Below we shall start off by presenting a summary overview of what is already known from the literature concerning the entropy concept and the principle of maximum entropy, as they represent the theoretical assumptions underlying this paper. We shall then illustrate the method for reconstructing the bathymetry of a river cross-section and show how this method can be parameterized for operational purposes. The proposed procedure is applied in two case studies, one regarding the Ponte Nuovo gage site along the Tiber River in central Italy and the other the Mersch gage site along the Alzette River in Luxembourg. Making reference to the same two sites, we then show how the cross-sections thus reconstructed can be effectively used to estimate discharge by combining (multiplying) the estimate of the flow area with the cross-sectional mean velocity as determined using the entropy approach proposed by Chiu (1987, 1988). We conclude the paper by presenting some final considerations.

2. Entropy and the principle of maximum entropy (POME)

The entropy of a system was first defined by Boltzmann (1872) as "a measure of our degree of ignorance as to its true state". In his information theory, Shannon (1948) introduced what is today called "*entropy of information*" or also Shannon entropy, defining it quantitatively in probabilistic terms for a discrete system as:

$$H(X) = -\sum_j p(X_j) \ln p(X_j) \tag{1}$$

where $p(X_j)$ represents the (a priori) probability mass function of a system being in the state X_j, belonging to the set $\{X_j,\ j = 1, 2, \ldots\}$.

If the variable X is continuous, the entropy is expressed as:

$$H(X) = -\int p(X) \ln p(X) \tag{2}$$

where $p(X)$ now represents the probability density function.

The Principle of Maximum Entropy (Jaynes, 1957) affirms that, in the presence of data and/or experimental evidence regarding a given physical phenomenon, for the purpose of estimating the associated probability distribution it will be sufficient to choose a model that is consistent with the available data and at the same time has the *maximum* entropy.

From a strictly mathematical viewpoint, the form of the probability density function $p(X)$ which maximizes the entropy $H(X)$ defined by Eq. (2) and subject to a number m of assigned constraints in the form:

$$G_i = \int_a^b \psi_i(X, p)\, dX \quad i = 1, 2, \ldots m \tag{3}$$

can be obtained by solving the following equation:

$$\frac{\partial[-p\ln p]}{\partial p} + \sum_{i=1}^{m}\lambda_i \frac{\partial\psi_i(X,p)}{\partial p} = 0 \quad (4)$$

where λ_i is the i-th Lagrange multiplier (Vapnyarskii, 2002).

The principle of maximum entropy was applied by Chiu (1987, 1989) to describe the two-dimensional flow velocity distribution in a channel cross-section based on the cross-sectional maximum velocity u_{max} and the dimensionless parameter M, and by Moramarco et al. (2013) to estimate the probability density function of water depth and the flow depth distribution along the cross-section, assuming a priori that the cumulative probability distribution function increases monotonically *with the surface flow velocity*. In a similar manner, below we outline a method for determining the geometry of a natural channel that uses the entropy maximization principle, *but is independent* of the surface velocity measurement.

3. Reconstruction of the bathymetry of a river cross-section

Let us consider a generic cross-section of a river with a free surface flow and let D be the maximum depth in the cross-section; assuming a Cartesian reference system whose origin is fixed on the surface at the top of the vertical where the depth is greatest, the coordinate y in Fig. 1 represents the depth and x the horizontal distance from the vertical where we have the maximum depth D; moreover, let h be the water depth (relative to the surface), corresponding to a vertical at a horizontal distance x from the reference vertical of the system. Finally, let us assume that the depth h decreases monotonically along the transverse direction, going from the maximum value D at the reference vertical ($x = 0$) to 0 on the river bank $x = L$ (see Fig. 1).

Assuming that the depth h represents a random variable, let $F(h)$ be the corresponding cumulative probability distribution function and $p(h)$ the probability density function given by:

$$p(h) = \frac{dF(h)}{dh} \quad (5)$$

In particular, the probability density function $p(h)$ to be identified must satisfy the unity constraint:

$$\int_0^D p(h)dh = 1 \quad (6)$$

where D is the maximum depth, i.e., the maximum value of h at the point in which $x = 0$.

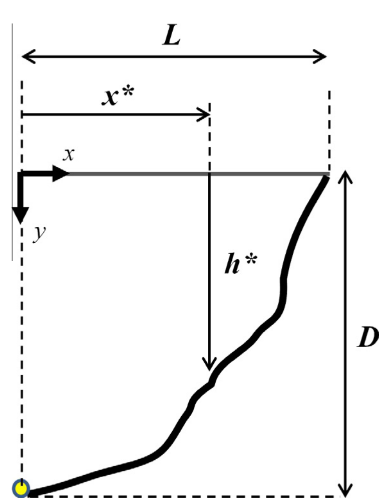

Fig. 1. Example of a generic half cross-section and associated reference system.

An additional constraint on the flow depth distribution is represented by the mean value H_m of the depth h, which can be expressed as:

$$\int_0^D hp(h)dh = H_m \quad (7)$$

If the cross-section geometry is not known, nor will the corresponding probability density function $p(h)$ be known; however, it can be estimated by applying the principle of maximum entropy through the constrained maximization of the entropy (see Eq. (4) with $\psi_1 = p(h)$ and $\psi_2 = hp(h)$):

$$\frac{\partial[-p(h)\ln p(h)]}{\partial p} + \lambda_1\frac{\partial p(h)}{\partial p} + \lambda_2\frac{\partial[hp(h)]}{\partial p} = 0 \quad (8)$$

The solution of Eq. (8) provides the expression of the probability density function:

$$p(h) = e^{\lambda_1 - 1}e^{\lambda_2 h} \quad (9)$$

On the basis of Eqs. (9), (5) becomes:

$$\frac{\partial F(h)}{\partial h} = e^{\lambda_1 - 1}e^{\lambda_2 h} \quad (10)$$

which relates the depth h to the corresponding cumulative probability function $F(h)$.

By substituting Eq. (9) in the first constraint Eq. (6) and integrating and substituting the result in Eq. (10) and integrating, the following expression is obtained:

$$h(x) = \frac{1}{\lambda_2}\ln\left[1 + (e^{\lambda_2 D} - 1)F(h(x))\right] \quad (11)$$

which gives us the depth h of a cross-section at a point corresponding to a vertical at a distance x from the vertical where we have the maximum depth. This expression is clearly a function of the cumulative probability distribution $F(h)$ and, therefore, in order to be able to estimate the shape of the cross-section it is necessary to formulate an expression with which to quantify $F(h)$.

To this end, Moramarco et al. (2013) assume for the cumulative probability distribution function $F(h)$ an expression given by the ratio between the surface velocity $u_s(x)$ and the maximum surface velocity u_{smax}. However, as also assumed in Moramarco et al. (2013), since the link existing between the two variables x and h is monotonic, the probability that the depth $h(x)$ will remain less than or equal to a given value h^* coincides with the probability that the x coordinate will be greater than or equal to the corresponding x^*:

$$F(h^*(x^*)) = P(h \leqslant h^*) = P(x \geqslant x^*) = 1 - P(x < x^*) = 1 - x^*/L \quad (12)$$

Therefore, Eq. (11) can be rewritten as:

$$h(x) = \frac{D}{W}\ln\left[e^W - (e^W - 1)\frac{x}{L}\right] \quad (13)$$

where $W = \lambda_2 D$ is a dimensionless parameter characteristic of the river cross-section.

Eq. (13) thus enables us to describe the bathymetry pattern of a cross-section once the parameter W is known.

Since at this point $F(h)$, and thus $p(h)$, are formally known, the solution of integral in Eq. (7) produces the following result:

$$H_m = \left(\frac{e^W}{e^W - 1} - \frac{1}{W}\right)D = \Phi(W)D \quad (14)$$

It is worth observing, incidentally, that the value of the parameter W varies from very small values (close to zero) for triangular cross-sections (that is, where $H_m/D \cong 0.5$) up to very high values for approximately rectangular cross-sections (that is, where $H_m/D \cong 1$). Theoretically, the parameter W could thus be estimated by linear regression performed on a substantial set of pairs of values H_m–D (Moramarco et al., 2013), but this approach would entail carrying out a bathymetric survey across the entire river cross-section in order to quantify H_m.

In the paragraph below we describe a new procedure for estimating the dimensionless parameter W that does not require any bathymetric survey to be conducted and is based only on a reduced amount of information of an exclusively geometric type.

3.1. Estimate of the parameter W

Let us suppose that n (with $n \geqslant 2$) flood events have occurred over time in the river cross-section whose geometry we want to reconstruct and that observed, georeferenced flow width (i.e. whose extremes have known coordinates) and water level data are available for every case. Let us consider the flood event during which the *maximum* water level was observed and set the origin of the Cartesian reference system on the free surface associated with that event, in the point of maximum depth, implicitly assuming that the elevation of the deepest point of the cross-section is known. For all practical purposes, the point of maximum depth ($x = 0$) can be positioned in correspondence of the vertical in which the maximum surface velocity is observed.

Based on the available geometric data, and once the aforesaid reference system has been defined, it will be possible to quantify (see Fig. 2):

- The maximum depth D, i.e., the largest distance between the river bed and the free surface of the event.
- The coordinates ($l_{l,i}$, δ_i) and ($l_{r,i}$, δ_i) of the extremes, respectively on the left and right banks, of the flow width corresponding to each event, with the exception of the largest ($i = 1, 2 \ldots n - 1$) (given that δ_i represents the depth associated with the i-th water surface/flow width relative to water surface/flow width associated with the largest/maximum flood event).
- The distances L_l and L_r of the extremes of the flow width of the largest flood relative to the vertical of the reference system, respectively on the left and right banks.

Eq. (13), which describes the variation in the depth h (calculated relative to the free surface of the largest flood event) along the horizontal coordinate x, can thus be rewritten to the left and right of the reference vertical by setting $L = L_l$ and $L = L_r$, respectively:

$$\begin{cases} h(x_l) = \frac{D}{W} \ln \left[e^W - (e^W - 1)\frac{x_l}{L_l} \right] \text{ with } 0 \leqslant x_l \leqslant L_l \\ h(x_r) = \frac{D}{W} \ln \left[e^W - (e^W - 1)\frac{x_r}{L_r} \right] \text{ with } 0 \leqslant x_r \leqslant L_r \end{cases} \quad (15)$$

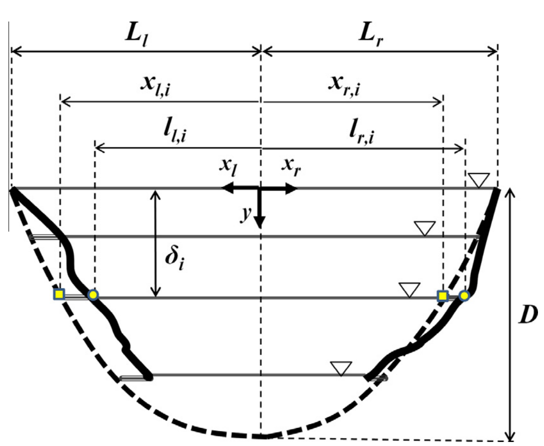

Fig. 2. Parameters used to estimate the parameter W.

In this manner we describe two functions, both constrained to passing through two fundamental points, namely, the lowest point of the channel cross-section ($x_l = x_r = 0$, $h = D$) and the extreme corresponding to the greatest flow width ($x_l = L_l$, $h = 0$ on the left and $x_r = L_r$, $h = 0$ right banks of the river, respectively). The combination of these two functions delineates the bathymetric profile of the entire cross-section, which, for given values of D, L_l and L_r, varies its shape with variations of the parameter W.

Let $x_{l,i}$ and $x_{r,i}$ be the coordinates, respectively on the left and right banks, obtained by rearranging Eq. (15) and imposing $h = \delta_i$ ($i = 1, 2 \ldots n - 1$); that is, $x_{l,i}$ and $x_{r,i}$ represent the coordinates of the bathymetric profile described by Eq. (15) at the depth associated with the flow widths of the $n - 1$ events (see Fig. 2) calculated starting from the flow width associated with the maximum event:

$$\begin{cases} x_{l,i} = \frac{L_l}{(e^W - 1)}(e^W - e^{W\delta_i/D}) \text{ with } i = 1, 2, \ldots n - 1 \\ x_{r,i} = \frac{L_r}{(e^W - 1)}(e^W - e^{W\delta_i/D}) \text{ with } i = 1, 2, \ldots n - 1 \end{cases} \quad (16)$$

It should be noted that a variation in the parameter W is reflected in the shape of the bathymetric profile and, consequently, in the coordinates $x_{l,i}$ and $x_{r,i}$. It is assumed, therefore, that the optimal estimate of W is the one whereby the profile defined by Eq. (15) best reproduces the entire cross-section, i.e., the value that minimizes the sum of horizontal deviations (in absolute value) between the profile itself and the extremes of the flow widths of the $n - 1$ events at an equal depth δ_i defined as follows:

$$\text{err}(W) = \sum_{i=1}^{n-1} |l_{l,i} - x_{l,i}| + |l_{r,i} - x_{r,i}| \quad (17)$$

4. Case studies

The proposed method for reconstructing bathymetry and estimating the parameter W was applied and verified using data regarding the Ponte Nuovo gage site located along the Tiber River (Central Italy) and the Mersch gage site located along the Alzette river (Luxembourg) (see Fig. 3).

The basin closed at Ponte Nuovo drains an area of around 4135 km^2 and is equipped/monitored with a cableway that enables current-meter velocity measurements to be made at different depths and depth measurements on different verticals.

The Mersch station subtends a more limited drainage area, about 707 km^2, and velocity measurements are performed with an Acoustic Doppler Current Profiler.

Fig. 3 shows a map of the two basins subtended by two river cross-sections concerned and their positions, while Fig. 4 shows the bathymetric survey data. In particular, for Ponte Nuovo section the bathymetry shown in Fig. 4 was obtained by a topographic survey done in 2005, whereas for Mersch was obtained by elaboration of the ADCP measurements done during the most severe flood event occurred in 2006.

As can be seen from the figure, the sites under examination are characterized by cross-sections of different size and shape; in particular, the Ponte Nuovo cross-section has a trapezoidal shape, while the shape at the Mersch site more closely resembles a triangle. Topographic surveys and flow depth measurements conducted at the sites over a number of years have shown no significant modifications in geometry, which can thus be considered as unchanged over time.

The data set for the Ponte Nuovo site consists of $n = 9$ flood events recorded from December 1999 to April 2004, whereas data for the Mersch site consists of $n = 14$ flood events recorded from December 2004 to July 2007. The main hydraulic characteristics of the events are provided in Tables 1 and 2 for the Ponte Nuovo and Mersch site respectively. For each event the discharge Q, the

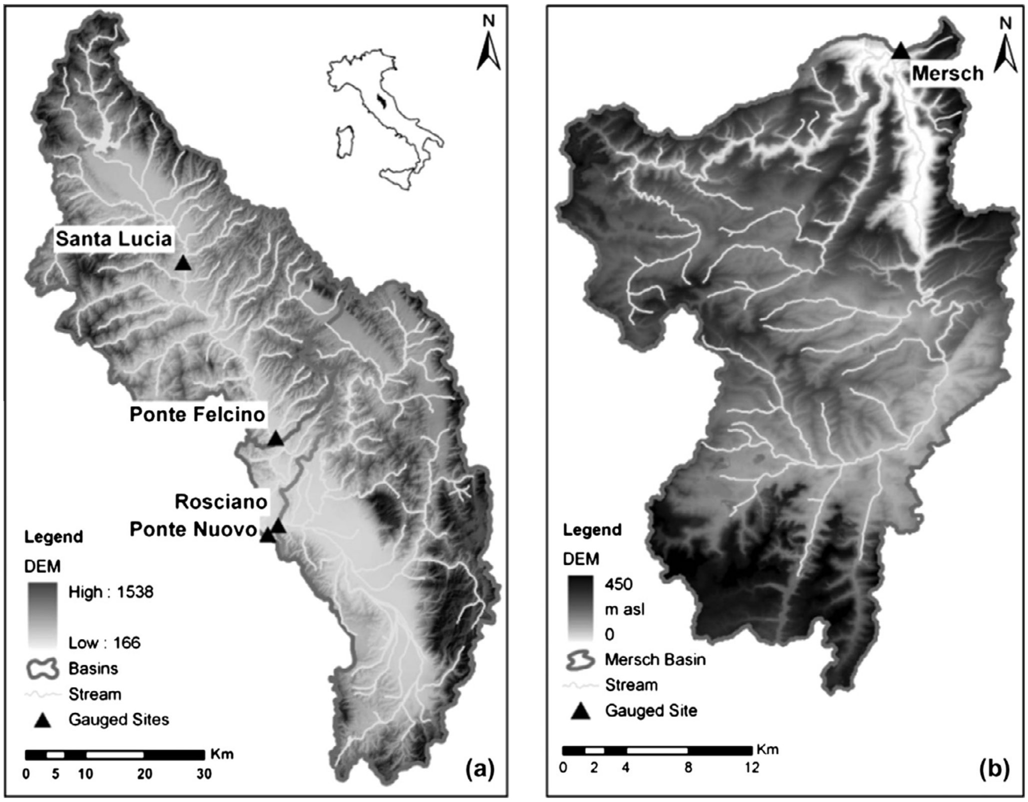

Fig. 3. Areas of study: (a) Upper Tiber basin and (b) Alzette basin.

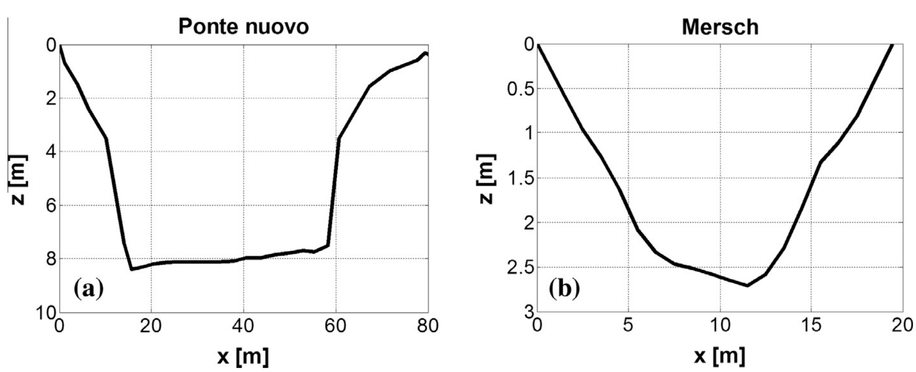

Fig. 4. Topographical survey of the analyzed river sites.

Table 1
Main hydraulic characteristics of the flood events observed at Ponte Nuovo cross-section.

ID	Date	Q (m^3/s)	D (m)	L (m)	A (m^2)	H_m/D
1	16/12/1999	427.46	5.88	58.44	274.95	0.80
2	20/04/2004	397.70	4.78	51.09	214.69	0.88
3	30/01/2001	316.67	3.98	49.44	174.60	0.89
4	30/03/2000	274.25	3.88	49.28	169.67	0.89
5	07/11/2000	227.72	3.50	48.66	151.06	0.89
6	27/11/2003	108.27	2.60	47.20	107.92	0.88
7	16/06/2000	29.55	1.51	45.42	57.45	0.84
8	14/01/2004	16.60	1.28	45.05	47.04	0.82
9	25/09/2000	6.70	1.00	44.59	34.49	0.78

Table 2
Main hydraulic characteristics of the flood events observed at Mersch cross-section.

ID	Date	Q (m^3/s)	D (m)	L (m)	A (m^2)	H_m/D
1	24/11/2006	37.26	2.50	18.50	28.72	0.62
2	23/03/2007	34.35	2.15	16.80	22.64	0.63
3	19/01/2005	21.48	1.93	15.70	19.10	0.63
4	05/12/2005	20.57	1.93	15.66	18.98	0.63
5	14/02/2005	25.62	1.87	15.37	18.11	0.63
6	30/05/2005	18.55	1.78	14.85	16.79	0.63
7	18/01/2006	17.72	1.76	14.71	16.45	0.64
8	12/02/2005	15.83	1.49	12.87	12.75	0.66
9	19/01/2006	9.95	1.33	11.69	10.71	0.69
10	27/12/2004	7.46	1.12	10.72	8.40	0.70
11	31/05/2005	5.17	1.04	10.36	7.58	0.70
12	10/07/2007	4.69	1.00	10.19	7.16	0.70
13	08/12/2004	2.94	0.96	10.03	6.78	0.70
14	17/08/2006	2.27	0.73	9.03	4.54	0.69

maximum water depth D, the flow width $L_{tot} = L_l + L_r$, the flow area A and the ratio of the mean and maximum water depth H_m/D are provided. In particular, the discharge Q was calculated on the basis of point velocity measurements using a variant of the Mean-Section Method (UNI EN ISO 748, 2008). As can be observed the events considered are characterized by a broad range of discharge values, between 6.70 and 427.46 m^3/s for Ponte Nuovo and between 2.27 and 37.25 m^3/s for Mersch. Furthermore, it is worth noting that Ponte Nuovo is characterized by higher values of the H_m/D ratio (0.8–0.9) than Mersch (0.62–0.7), in agreement with the trapezoidal and nearly triangular shapes of the two cross-sections respectively, given that the ratio H_m/D varies from 0.5 for triangular section up to 1 for rectangular section.

Based on numerous pairs of values of H_m–D, Moramarco et al. (2013) estimated by means of a least squares linear regression W values equal to 6.6 and 2.2 for Ponte Nuovo and Mersch respectively. These two values of W were taken as reference values with which to compare the corresponding values furnished by the method for estimating the parameter W proposed here.

Below we present and discuss the results we obtained in our estimation of the parameter W, as well as the reconstruction of the bathymetry for each of the two cross-sections considered. The results in terms of the discharge estimates obtained by combining the flow areas estimated using the method proposed here with the cross-sectional mean velocities estimated by applying the method proposed by Farina et al. (2014) are also presented for both sites.

5. Analysis and discussion of results

5.1. Analysis and discussion of results of bathymetry reconstruction

For both cross-sections under examination, the bathymetry was reconstructed using Eq. (13) after the parameter W had been estimated using the procedure described in Section 3.1. In particular, for the purpose of estimating the parameter W, Eq. (17) was minimized using the "*fmincon*" function from the optimization toolbox available in the Matlab™ environment based on Sequential Quadratic Programming (Powell, 1983, Schitlowski, 1985). Making reference to the entire set of data available for the two gage sites, the optimal value of W was computed to be 6.5 for the Ponte Nuovo cross-section and 1 for the Mersch cross-section; these values are in line with those typically representative of trapezoidal/rectangular and triangular cross-sections, respectively, and with those obtained by linear regression of the pairs of H_m–D values (Moramarco et al., 2013) and taken here as a reference, equal to 6.6 and 2.2, respectively.

Fig. 5 shows a comparison between the actual bathymetry and the bathymetry estimated by means of the proposed procedure for both cross-sections considered; Fig. 5 also shows, by way of example, the bathymetry obtained for the same cross-sections with the procedure proposed by Moramarco et al. (2013), that is, taking the reference values of W estimated through the linear regression previously mentioned and using the surface velocity profiles measured and modeled by means of a parabolic function.

As can be observed for both cross-sections, the proposed procedure provides a reasonable approximation of the actual bathymetry. In particular, in the case of Ponte Nuovo, the bathymetry reconstruction resulted in a percentage error of just over 6% in the estimation of the flow area for the most severe event, versus an error of between 9% and 11% when we considered the cross-section obtained with the procedure proposed by Moramarco et al. (2013), using the measured and modeled surface velocity profiles, respectively. Similarly, in the case of the Mersch site, the procedure proposed here resulted in a percentage error of about 11% in the estimation of the flow area for the most severe event, versus a percentage error of between 12% and 14% when we applied the procedure proposed by Moramarco et al. (2013) using the measured and modeled surface velocity profiles, respectively.

In both cases, the bathymetry reconstructed with the procedure proposed here enables an accurate estimate of the flow area that provides an improvement over previous efforts (e.g. Moramarco et al. (2013)), with no assumptions being made on the relationship between the flow depth distribution and the surface velocity.

It should be observed, however, that the estimate of the parameter W resulting from the procedure described in Section 3.1 depends on the number and characteristics of the events for which there are observed flow widths. In this regard, two sensitivity analyzes were performed to determine the sensitivity of the procedure for estimating the parameter W a) to the number n of events and b) to the characteristics of the events. First of all, for each of the two gage sites, the procedure was repeated $n - 1$ times, considering only the two largest events ($n = 2$) to begin with and eventually all the n available events, added one at a time. That is, once the flow width corresponding to the most severe event had been fixed, the immediately less severe event – or rather, the corresponding flow width – was added and so on until all n available events had been considered. For both cross-sections, Fig. 6 shows the trend in the value of W obtained with changes in the number n of events used to estimate it, whilst Fig. 7 shows a comparison between the n estimated flow areas and the corresponding observed flow areas with changes in the number n of events used to estimate the parameter W.

With reference to the case of Ponte Nuovo, it can be observed (see Fig. 6a) that the value of the parameter W varies, and

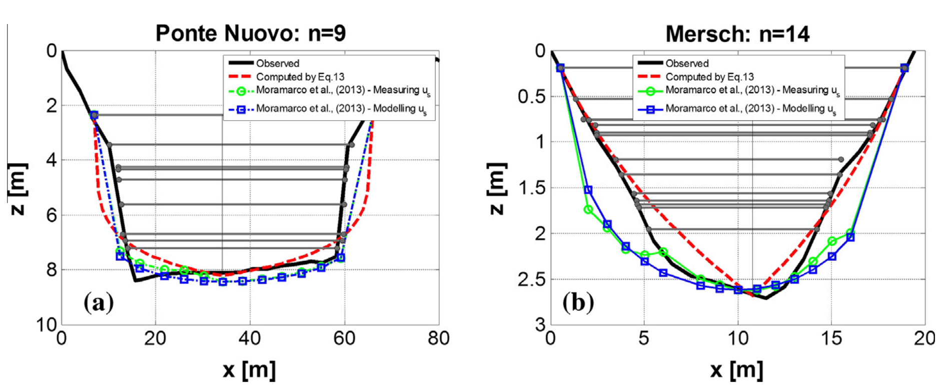

Fig. 5. Comparison between the observed bathymetry and the bathymetry reconstructed by means of the proposed procedure (Eq. (13)) and by means of the procedure proposed by Moramarco et al. (2013) (n: number of flood events for which observed, georeferenced flow width and water level data were used for parameterization of Eq. (13)).

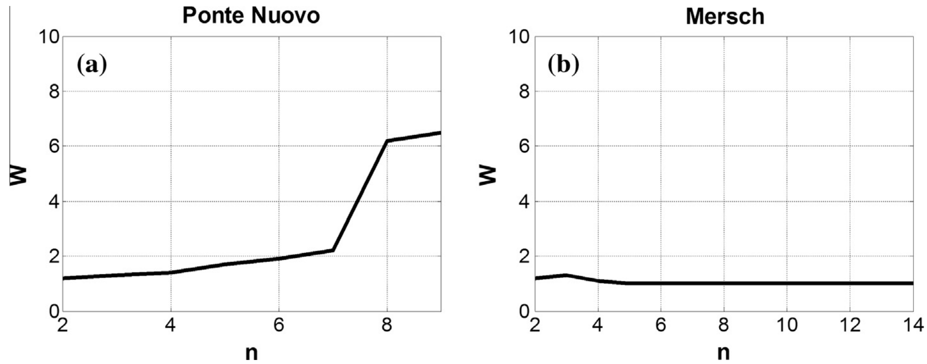

Fig. 6. Trend in the parameter W versus the number of events n used for its estimation.

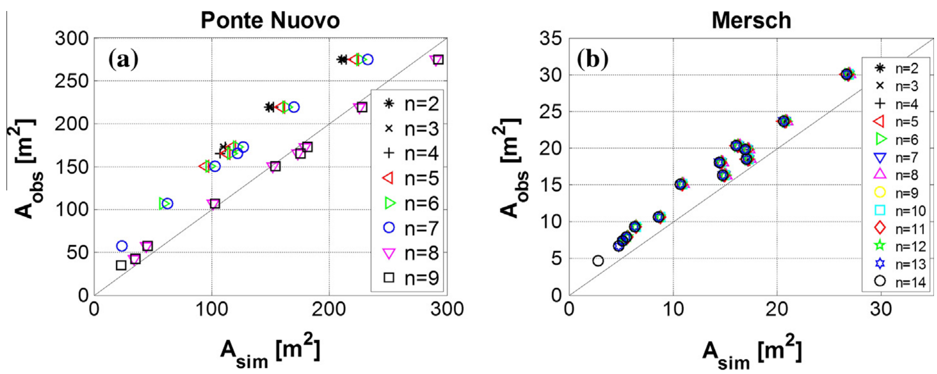

Fig. 7. Comparison between observed and estimated flow areas.

specifically it increases with an increasing number n of events used for its estimation, going from a minimum of $W = 1.2$ with $n = 2$ to $W = 6.5$ with $n = 9$. In practical terms, this means that for Ponte Nuovo if a smaller number of events ($n \leqslant 7$) is considered, the value of the parameter W will be underestimated and so will the corresponding flow areas (see Fig. 7a). Indeed, it is worth noting that, given the criteria used to add the events (from the most severe to the less severe event), small number n of events also imply that the corresponding observed flow widths are mainly located in the upper portion of the cross-section. The change of the value of the parameter W with n is thus understandable if we look at Fig. 8a, which shows, by way of example, the flow widths and reconstructed cross-section in the case of $n = 3$.

As may be observed, the Ponte Nuovo cross-section, though substantially trapezoidal in shape, shows a variation in the bank slope: the lower part of the banks slopes more steeply (nearly rectangular cross-section), whereas in the upper portion of the cross-section the bank slope is less steep. If we consider a reduced number of events characterized by high flow depths and flow widths prevalently determined by the geometry of the upper part of the cross-section, the estimation method tends to assume the "observed" portion of the cross-section with a gentler bank slope to be representative of the entire cross-section, thus clearly leading to an underestimation of W and hence of the flow area.

In the case of Mersch, on the other hand, the estimation of the value of the parameter W remains practically constant irrespective of the number n of flow widths (see Fig. 6b) and an analogous observation may thus be made for the flow area (see Fig. 7b). Moreover, at the latter gage site, the bank slope does not vary significantly with depth and hence even with a very limited number

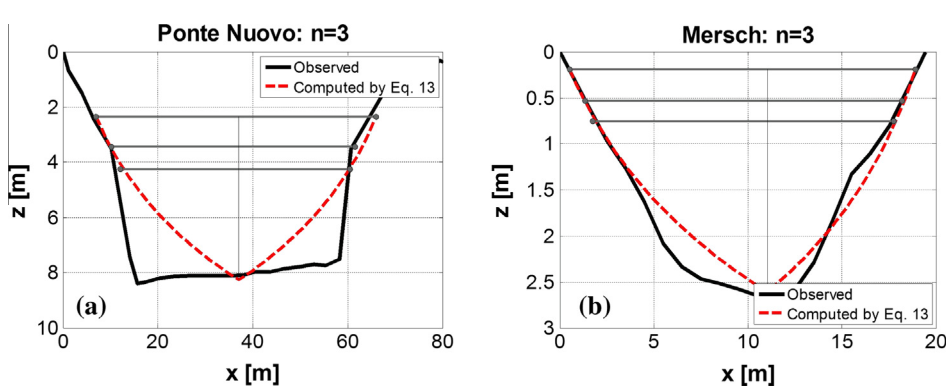

Fig. 8. Comparison between the observed bathymetry and the bathymetry reconstructed by means of the proposed procedure (Eq. (13)) (n: number of flood events for which observed, georeferenced flow width and water level data were used for parameterization of Eq. (13)).

of events and corresponding flow concentrated in the upper portion of the cross-section (see Fig. 8b) the procedure enables us to correctly estimate the shape of the entire cross-section and the value of the parameter W.

These considerations are confirmed also by the second sensitivity analysis performed. In this case n was kept fixed equal to 2, and different combinations of observed events were considered. More precisely, the flow width associated with the largest/maximum observed flood event was used as reference, whereas the second event (and its corresponding flow width) varied. Thus, the analysis was performed considering different values of δ, where δ represents the depth of the water surface/flow width of generic flood event with respect to the water surface/flow width associated with the largest/maximum observed flood event (see Fig. 2). The results obtained, shown in Fig. 9 substantially confirm the findings of the previous analysis. In fact, the analysis shows that for Ponte Nuovo section (Fig. 9a), given its variation in the bank slope, it is important to consider flow widths corresponding to rather different flow events in order to be representative of the entire cross-section. In fact, for δ values lower than 2–3 m the value of the parameter W is clearly underestimated. In the case of Mersch (Fig. 9b), given its cross-section shape characterized by a bank slope that does not vary significantly with depth, the estimation of the value of the parameter W remains much more constant.

Finally, it is worth observing that in any case, in order to successfully apply the proposed approach, the observed flow events should pertain to a time window during which the cross-section shape does not change significantly, as in the case study here considered. Indeed, the prosed approach is not aimed at modeling the temporal evolution of the cross-section shape due to sediment load and transport as done for example by more complex numerical flow models (see for example Lisle et al., 2000; Olsen, 2003; May et al., 2009).

5.2. Analysis and discussion of the results regarding discharge estimation

As previously observed, discharge, which represents a parameter of real practical interest in many hydraulic engineering and hydrological applications, can be estimated by multiplying the flow area by the cross-sectional mean velocity. Therefore, to conclude our analysis of the effectiveness of the proposed procedure for reconstructing bathymetry, we shall analyze the discharge estimate that can be obtained by combining the flow area estimated using the proposed bathymetry reconstruction procedure with the cross-sectional mean velocity $\overline{U} = f(u_{\max}, M)$ estimated using the entropy approach proposed by Chiu (1987,1988).

In order to apply the entropy-based approach to estimate the cross-sectional mean velocity, we first had to estimate the parameter M. For this purpose we relied on Method 3 proposed by Farina et al. (2014). The method requires solely a measurement of the maximum surface velocity u_{Di} of the i-th event with $i = 1, 2\ldots n$ and assumes the hydrometric geometry of the cross-section concerned to be known. We shall point out, therefore, that the estimated (not observed) cross-section geometry was used not only to quantify the flow area to be adopted for the purpose of estimating discharge, but also at a preliminary stage to estimate the parameter M.

In practical terms, the parameter M was determined using the same dataset as was employed to estimate W in the first sensitivity analysis: more specifically, we used the n maximum surface velocities recorded during the n events considered and the estimated cross-section, the latter being a function of the optimal value of W corresponding to the same number n of events. Therefore, as in the case of W, the calculation of M was performed $n - 1$ times, starting from the two most significant events ($n = 2$) and adding one by one the immediately less severe events until eventually considering all the n available events.

Once M was known, for each of the n events we converted the maximum observed surface velocity into the cross-sectional maximum velocity based on a velocity profile derived from the entropy model (Farina et al., 2014) and then estimated the corresponding cross-sectional mean velocity; finally, we calculated the discharge by multiplying the latter by the flow area of the reconstructed cross-section.

Fig. 10 shows a comparison, for both real-life cases, between the discharges estimated within the framework of the first sensitivity analysis previously described and those observed, given an increasing number n of events.

As can be observed for both cross-sections, the points fall around the diagonal representing a perfect correspondence between observed and simulated data, with values of the Nash–Sutcliffe (NS) index (Nash and Sutcliffe, 1970) of 0.92 and 0.96 and a mean percentage error in the discharge estimate of about 10.54% and 15.13%, respectively, for the Ponte Nuovo and Mersch gage sites. It is moreover worth pointing out that the estimate of the discharge values was obtained relying on relatively little information and measurements: (1) the elevation of the lowest point of the channel cross-section, (2) the observed, georeferenced flow widths occurring during different flood events and (3) the corresponding water levels measured while the event was in progress, used to estimate the bathymetry. The maximum surface velocity measured during the same flood events was the only data added to the other three parameters in order to estimate the cross-sectional mean velocity.

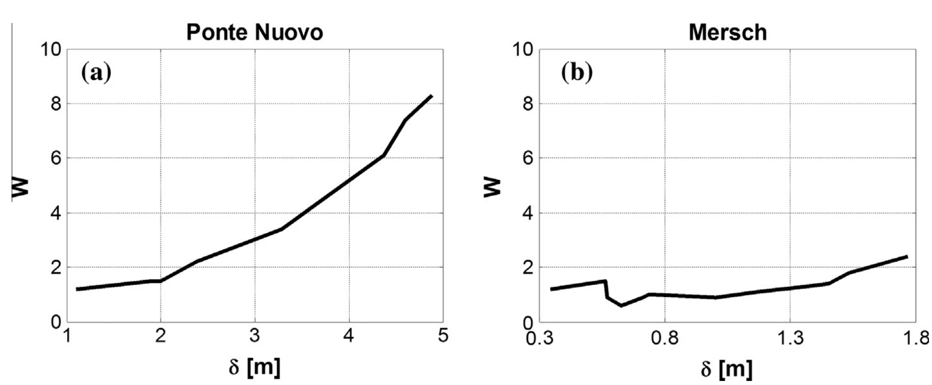

Fig. 9. Trend in the parameter W versus δ (depth of the water surface of generic flood event with respect to the water surface associated with the largest/maximum observed flood event).

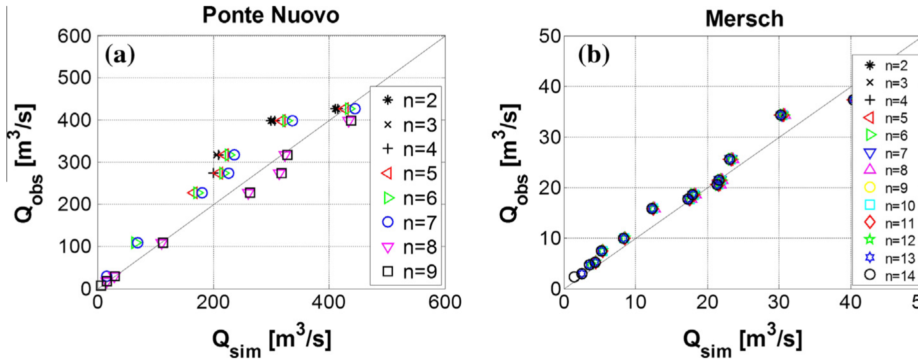

Fig. 10. Comparison between observed and estimated discharges.

From a practical viewpoint, therefore, combining the method proposed here for estimating *W* and reconstructing bathymetry with the entropy-based approach for estimating cross-sectional mean velocity – including the method proposed by Farina et al. (2014) for estimating *M* – represents a valid tool for determining discharge in a river cross-section where only the elevation of the lowest point of the cross-section and observed, georeferenced flow widths and corresponding water levels occurring during different flood events are available to characterize its geometry. Incidentally, among these data, the most difficult to obtain is represented by the elevation of the lowest point in the cross-section. In the case this elevation was not available, it could be estimated by using the regression approach recently proposed by Moramarco (2013) (see also Tarpanelli et al., 2014) which requires only measurement of water levels and corresponding maximum velocity observed for several events.

6. Conclusions

Relying on the principle of maximum entropy, we have developed a relationship for reconstructing the bathymetry of a river cross-section and proposed a method for estimating the parameter *W*. Unlike the method proposed by Moramarco et al. (2013) for reconstructing bathymetry, which is similarly based on the principle of entropy maximization, the approach we propose here does not require measurement of the surface velocity for bathymetry reconstruction. The parameter *W* can be estimated on the basis of a smaller amount of information, exclusively of a geometric type, i.e., the elevation of the lowest point of the channel cross-section, the observed, georeferenced flow widths occurring during different flood events and the corresponding water levels (from which we derive the estimate of *D*, or maximum depth in the cross-section).

The application of the method to two different natural river cross-sections showed it to be effective. By relying on a sufficient number of georeferenced flow widths and corresponding water levels, we can in fact accurately estimate the parameter *W* and arrive at a reasonable reconstruction of the bathymetry and estimate of the flow area. It was also observed, however, that the accuracy of the estimate of the parameter *W* diminishes as the amount of field information used to estimate it decreases, above all where such information refers to events of an analogous entity, that is, events characterized by similar water levels and flow widths. For this reason, the events for which georeferenced flow width and water level data are available should preferably be very different, especially in the case of a small number of events. The need to rely on multiple measurements taken during different flood events is all the greater when the cross-section considered is characterized by a change in bank slope. In such a case, in fact, in order to correctly estimate the parameter *W* it is necessary to have observed flow width data for various portions with a different slope. If, on the other hand, the slope of the river banks does not vary significantly, even only a few measurements will suffice to ensure good accuracy in the estimation of the parameter *W*.

Also, we observed that by combining the proposed method for estimating the flow area with the entropy-based method, parameterized according to the approach proposed by Farina et al. (2014) for estimation of the cross-sectional mean velocity, we can provide an accurate estimate of discharges, thus allowing the definition, on the basis of several events, of a stage-discharge curve relating the water surface elevation to discharge. This curve is certainly obtained with a smaller effort than that necessary when the section is directly detected and the discharge is estimated through point measurements as in the case of the mean-section method.

Finally, it worth noting that the methodology proposed has the potentiality of being easily coupled with remote sensing systems, considering that the main parameters it is based on, namely, maximum surface velocity and georeferenced flow widths, can be easily measured by the new non-contact radar sensors (see for example Moramarco et al., 2011; Fulton and Ostrowski, 2008) and/or satellites (see for example Smith, 1997; Barrett, 1998; Bjerklie et al., 2003). This aspect represents an interesting topic to be analyzed and the necessary investigations will be developed in the next future.

Acknowledgments

The authors are grateful to the reviewers for their precious remarks, which made it possible to improve the quality of the paper. This study was carried out under the framework of Terra&Acqua Tech Laboratory, Axis I activity 1.1 of the POR FESR 2007–2013 project funded by Emilia-Romagna Regional Council (Italy).

(http://fesr.regione.emilia-romagna.it/allegati/comunicazione/la-brochure-dei-tecnopoli).

References

Barrett, E., 1998. Satellite remote sensing in hydrometry. In: Herschey (Ed.), Hydrometry: Principles and Practices, second ed., Wiley, Chichester, pp. 199–224.

Bjerklie, D.M., Dingman, S.L., Vorosmarty, C.J., Bolster, C.H., Congalton, R.G., 2003. Evaluating the potential for measuring river discharge from space. J. Hydrol. 278, 17–38.

Boltzmann, L., 1872. Weitere Studien über das Wärmegleichgewicht unter Gasmolekülen. Sitzungsberichte Akademie der Wissenschaften 66, 275–370.

Chiu, C.L., 1987. Entropy and probability concepts in hydraulics. J. Hydraulic Eng. 113 (5), 583–600.

Chiu, C.L., 1988. Entropy and 2-D velocity in open channels. J. Hydraulic Eng. 114 (7), 738–756.

Chiu, C.L., 1989. Velocity distribution in open channels. J. Hydraulic Eng. 115 (5), 576–594.

Chiu, C.L., 1991. Application of entropy concept in open channel flow study. J. Hydraulic Eng. 117 (5), 615–628.
Chiu, C.L., Abidin Said, C.A., 1995. Maximum and mean velocities and entropy in open-channel flow. J. Hydraulic Eng. 121 (1), 26–35.
Chiu, C.L., Murray, D.W., 1992. Variation of velocity distribution along non-uniform open-channel flow. J. Hydraulic Eng. 118 (7), 989–1001.
Costa, J.E., Spicer, K.R., Cheng, R.T., Haeni, F.P., Melcher, N.B., Thurman, E.M., Plant, W.J., Keller, William C., 2000. Measuring stream discharge by non-contact methods: a proof-of-concept experiment. Geophys. Res. Lett. 27 (4), 553–556.
Costa, J.E., Cheng, R.T., Haeni, F.P., Melcher, N., Spicer, K.R., Hayes, E., Plant, W., Hayes, K., Teague, C., Barrick, D., 2006. Use of radars to monitor stream discharge by noncontact methods. Water Resour. Res., 42.
Farina, G., Alvisi, S., Franchini, M., Moramarco, T., 2014. Three methods for estimating the entropy parameter M based on a decreasing number of velocity measurements in a river cross-section. Entropy 16, 2512–2529.
Fulton, J., Ostrowski, J., 2008. Measuring real-time streamflow using emerging technologies: radar, hydro-acoustics, and the probability concept. J. Hydrol. 357, 1–10.
Jaynes, E.T., 1957. Information theory and statistical mechanics. I. Phys. Rev. 106, 620–630.
Lee, M.C., Lee, J.M., Lai, C.J., Plant, W.J., Keller, W.C., Hayes, K., 2002. Non-contact flood discharge measurements using an X-band pulse radar (II) improvements and applications. Flow Measure. Instrum. 13 (5), 271–276 (6).
Lisle, T.E., Nelson, J.M., Pitlick, J., Madej, M.A., Barkett, B.L., 2000. Variability of bed mobility in natural, gravel-bed channels and adjustments to sediment load at local and reach scales. Water Resour. Res. 36, 3743–3755.
May, C.L., Pryor, B., Lisle, T.E., Lang, M., 2009. Coupling hydrodynamic modeling and empirical measures of bed mobility to predict the risk of scour and fill of salmon redds in a large regulated river. Water Resour. Res. 45, W05402. http://dx.doi.org/10.1029/2007WR006498.
Moramarco, T., 2013. Monitoraggio della portata sulla base della velocità massima superficiale e in assenza di batimetria. Giornate dell'Idrologia della Società Idrologica Italiana 2013 (in Italian).
Moramarco, T., Saltalippi, C., Singh, V.P., 2004. Estimation of mean velocity in natural channels based on Chiu's velocity distribution equation. J. Hydraulic Eng. 9 (1), 42–50.
Moramarco, T., Saltalippi, C., Singh, V.P., 2011. Velocity profiles assessment in natural channels during high floods. Hydrol. Res. 42 (2–3), 162–170.
Moramarco, T., Corato, G., Melone, F., Singh, V.P., 2013. An entropy-based method for determining the flow depth distribution in natural channels. J. Hydrol. 497, 176–188.
Nash, J.E., Sutcliffe, J.V., 1970. River flow forecasting through conceptual models 1: a discussion of principles. Journal of Hydrology 10 (3), 282–290.
Olsen, N., 2003. Three-dimensional CFD modeling of self-forming meandering channel. J. Hydraul. Eng. 129 (5), 366–372.
Powell, M.J.D., 1983. Variable Metric Methods for Constrained Optimization, Mathematical Programming: The State of the Art. In: Bechem, A., Grostschel, M., Korte, B. (Eds.), Treatise on Geochemistry. Springer Verlag, Germany, pp. 288–311.
Schitlowski, K., 1985. NLQPL: A Fortran-subroutine solving constrained nonlinear programming problems. Operations Res 5, 485–500.
Shannon, C.E., 1948. The mathematical theory of communications, I and II. Bell Syst. Tech. J. 27, 379–423.
Smith, L.C., 1997. Satellite remote sensing of river inundation area, stage, and discharge: a review. Hydrol. Process. 11, 1427–1439.
Tarpanelli, A., Brocca, L., Barbetta, S., Faruolo, M., Lacava, T., Moramarco, T., 2014. Coupling MODIS and radar altimetry data for discharge estimation in poorly gauged river basin. IEEE J. Sel. Top. Appl. Earth Observations Remote Sensing. http://dx.doi.org/10.1109/JSTARS.2014.2320582.
UNI EN ISO 748, 2008. Hydrometry. Measurement of liquid flow in open channels using current-meters or floats.
Vapnyarskii, I.B., 2002. Lagrange Multipliers in Encyclopaedia of Mathematics. Springer and European Mathematical Society.
Xia, R., 1997. Relation between mean and maximum velocities in a natural river. J. Hydraulic Eng. 123 (8), 720–723.
Yorke, T.H., Oberg, K.A., 2002. Measuring river velocity and discharge with acoustic Doppler profilers. Flow Meas. Instrum. 13 (5–6), 191–195.