Drives power reduction procedure to fill in the multiple-input multiple-output random control reference matrix

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Abstract

In Multiple-Input Multiple-Output (MIMO) random control testing, one of the challenges is related to the MIMO definition of the target in case where a full set of field measurements are unavailable and the specifications are therefore retrieved from standards. For multi-axis control tests in fact, although the target is a fully Spectral Density Matrix (SDM) in the frequency band of interest, the test specifications are often provided as multiple PSDs, coming from the single-axis practice. Therefore, additional information about the cross-correlation between pairs of control channels needs to be included, defining the Cross Spectral Densities (CSDs) as off-diagonal terms of the reference SDM.
This paper proposes an innovative fully automatic target definition procedure to fill in the MIMO random reference matrix in cases where no information about the CSD terms is available. The developed method yields a set of coherences and phases of the CSD terms that is able to minimize the drives power needed to reach the test specifications. Additionally, the retrieved target complies with the required property of the spectral density matrix being positive semi-definite and with the constraint imposed on the PSDs, considered as test specifications. A series of tests with a three-axial electrodynamic shaker is carried out comparing the proposed target definition procedure with respect to other state-of-the-art solutions.

1 Introduction

Vibration control tests are performed to verify that a system and all its sub-components can withstand the vibration environment during the operational life. These tests aim to accurately replicate via controlled shaker excitation the in-service structural response of a unit under test in the main axis of vibration and in all the possible axes where the levels exceed the acceptance thresholds [1].
Due to the multidirectional nature of the field environments, multi-axis random control tests are the nowadays best solution to represent real-life structural responses of test specimen [2], [3], [4]. Although several works [5], [6], [7] underline the advantages of using Multiple-Input Multiple-Output (MIMO) vibration control strategies, their practice still needs to grow. The high degree of expertise needed to perform these tests and decades of single axis controlled excitation built meanwhile a legacy of Single-Input Single-Output (SISO) standards that currently represent the main reference for the environmental test engineers. For these reasons nowadays MIMO vibration control tests are still considered as a *pioneering* testing methodology.

There are different types of MIMO tests (random, sine, time waveform replication), depending on the environment a test article needs to be exposed. For automotive and aerospace systems and subsystems, a random vibration test is required for all the main mechanical and electrical components. This type of test is performed to simulate the response of the unit under test to a broadband random Gaussian vibration environment. For the SISO case the test specification is a Power Spectral Density (PSD, usually in g^2/Hz) profile that needs to be replicated for a user-defined control channel by exciting the unit under test with a single-axis shaker. In the MIMO case, it is possible to define required test levels for multiple control channels that will be controlled simultaneously. Additional information about the cross-correlation between the control channels is also included. This information must be provided in terms of Cross Spectral Densities (CSDs) between pairs of control channels defining desired phase and coherence profiles [4], [8], [9]. The definition of these terms is essential to also replicate the cross-correlation that naturally exists between difference responses. These terms are also controlled by modern vibration controllers. For these systems the control target is thus a full reference Spectral Density Matrix (SDM). The target definition process plays already a key role for MIMO random control tests as documented in recent studies [10], [11].

Theoretically, a successful MIMO random control test can be performed in case the operational environment is fully replicated in the laboratory. As pointed out in [2], unfortunately operational measurements are not always available and often the test specifications are provided just in terms of PSDs at the control locations. This is due to several reasons. First of all, the gradual transition from sequential SISO testing to simultaneous multi-axial testing needs to face the aforementioned legacy of SISO standards and specifications, provided in terms of PSDs. Second, the standardization of the CSD terms is impractical to implement in a specification due to a lack of knowledge that makes challenging (and even impossibile) to average, smooth or envelope coherence and phase information from different operational conditions. In this case, the choice of setting appropriate values to fill in the full reference matrix *must reflect the desires of a knowledgeable environmental test engineer* [2]. Defining the reference matrix with no a-priori knowledge of the cross-correlation between control channels is very challenging. Filling in the off-diagonal terms, in fact, must guarantee that the reference matrix will have in the end a physical meaning (realizable). This is translated in the algebraic constraint that this matrix needs to be positive semi-definite and at the same time the test needs to guarantee the required PSDs at the control locations. The solution of finding a full reference matrix with fixed PSD terms is not unique. These considerations reflect the need of having a method rather than standardized values to define the specification of a multi-axial test.

The objective of this work is to provide a fully automated method to define the MIMO random reference matrix in case of missing operational measurements and the test specifications are provided in terms of (SISO) PSDs only. The full reference matrix returned by the method needs to be positive semi-definite in order to be used for actual testing and ensure good narrowband control performances on the provided PSDs. In [12] a solution to the aforementioned problems has been shown to be directly linked to the solution of the so-called problem of *meeting the minimum drives criteria* [2]. In [12] the author proposes the so-called Extreme Input/Output method. The idea behind the method is to find the CSDs that minimize the input power required to reach the (specified) PSDs. Unfortunately, this target definition procedure is not able to guarantee that the final target will be positive semi-definite. This makes the method not suitable for practical testing, as will be shown in the section 2. This issue is further tackled by the same author that proposes, in the later work [13], a constrained optimization algorithm to superimpose the required property.

Extending the work of Smallwood [12], rather than solving an optimization problem as proposed in [13], the fully automated procedure developed in this paper, named *Minimum (Maximum) Drives Method*, makes use of a direct phase selection as introduced in [14] and [15], to overcome the limitation of obtaining positive semi-definite target matrices.

Following this introduction, Section 2 explains the mathematical formulation of the Minimum (Maximum) Drives Method. In Section 3, a series of *normal-end* tests will be used to validate the developed procedure using an electrodynamic three-axial shaker. An industrial test case is considered to show how the method can be applied for testing automotive components.

Since in this work most of the derivations are in the frequency domain, all the arrays are functions of the frequency f (in Hz), if not specified otherwise. Vectors are denoted by lower case bold letters, e.g. **a**, and matrices by upper case bold letters, e.g. $\mathbf{A}$. An over-bar $\overline{\square}$ is used to indicate the complex conjugate operation and the Hermitian superscript $\square^H$ to indicate the complex conjugate transpose of a matrix, e.g. $\bar{\mathbf{a}}$ and $\mathbf{A}^H$. The dagger symbol $\square^\dagger$ is used to indicate the Moore-Penrose pseudo-inverse of a matrix, whereas the hat $\hat{\square}$ is used to emphasize the estimation of a quantity, e.g. $\hat{\mathbf{A}}$ is an estimate of the matrix $\mathbf{A}$.

2 Drives power reduction procedure

In a MIMO random control test where m inputs (the so-called *drives*) and ℓ outputs (the so-called *controls* or *pilots*) are considered, if the full dynamic system is linear and time invariant, it is possible to write the Input-Output relation in terms of SDMs as [16]

$$\mathbf{S_{yy}} = \mathbf{H}\mathbf{S_{uu}}\mathbf{H}^H \tag{1}$$

where $\mathbf{S_{yy}} \in \mathbb{C}^{\ell\times\ell}$ and $\mathbf{S_{uu}} \in \mathbb{C}^{m\times m}$ are the output SDM and the input SDM respectively, and $\mathbf{H} \in \mathbb{C}^{\ell\times m}$ is the Frequency Response Functions (FRFs) matrix.
The objective of the MIMO random vibration control test is to replicate a full referecnce SDM $\mathbf{S_{yy}^{ref}}$. Theoretically, the test target can be directly achieved by sending the input drives that have the specified input spectral density matrix [17].

$$\mathbf{S_{uu}} = \mathbf{Z}\mathbf{S_{yy}^{ref}}\mathbf{Z}^H \tag{2}$$

where the $\mathbf{Z} = \mathbf{H}^\dagger \in \mathbb{C}^{m\times\ell}$ is often referred as the *System's Mechanical Impedance Matrix*.
In case a full set of measurements is not available and test specifications provided in terms of PSD-only breakpoints, the choice of setting the cross-correlation information between pairs of control channels is given to the test engineer. Most of the MIMO vibration controllers give the possibility of defining element-wise the CSDs in terms of (ordinary) coherence and phase profiles [2] (typical values of low coherence and high coherence are 0.01 - 0.05 and 0.95 - 0.98, respectively). All the CSDs are then easily computed as

$$S_{jk} = |S_{jk}|\exp(\mathrm{i}\phi_{jk}) = \sqrt{\gamma_{jk}^2 S_{jj}S_{kk}}\exp(\mathrm{i}\phi_{jk}) \qquad \forall j,k \quad j\neq k \tag{3}$$

where i is the imaginary unit and j and k are the j-th and the k-th control channels, respectively.

2.1 The state-of-the-art solution: Extreme Inputs/Outputs Method

The idea of the method proposed in [12] is to find, with fixed PSD levels, the set of coherences and phases between the control channels that minimizes the trace of the drives SDM. The choice of the drive trace as quantity to be minimized is undoubtedly advantageous because a closed form expression can be derived in terms of the specified PSDs and the unknown coherences and phases between pairs of control channels. Considering the basic equation (2) for Linear Ttime Invariant (LTI) systems, the diagonal terms of the input SDM can be expressed as

$$S_{uu,ii} = \sum_{j=1}^{\ell}\sum_{k=1}^{\ell} Z_{ik}S_{yy,jk}^{ref}\bar{Z}_{ij} \qquad \forall i = 1:m \tag{4}$$

The trace of the drives SDM is the sum of the diagonal terms [18], where for the sake of brevity, the superscript $\square^{ref}$ will be dropped in the following derivation

$$P = \mathrm{Tr}(S_{uu}) = \sum_{i=1}^{m}\left(\sum_{j=1}^{\ell}\sum_{k=1}^{\ell} Z_{ik}S_{yy,jk}\bar{Z}_{ij}\right) = \sum_{j=1}^{\ell}\sum_{k=1}^{\ell} S_{yy,jk}\sum_{i=1}^{m}\bar{Z}_{ij}Z_{ik} \tag{5}$$

where the single sum on the right-hand-side of equation (5) can be interpreted as the kj-th entry of an hermitian matrix $\mathbf{F} \triangleq \mathbf{Z}^H\mathbf{Z}$.

By noticing that $\mathbf{S_{yy}^{ref}}$ needs to be hermitian too, the trace P must be a real number [18] and equation (5) can be rewritten as

$$P = \sum_{j=1}^{\ell}\sum_{k=1}^{\ell} S_{yy,jk}F_{kj} = \sum_{j=1}^{\ell} S_{yy,jj}F_{jj} + 2\sum_{j=1}^{\ell-1}\sum_{k=j+1}^{\ell} |S_{yy,jk}||F_{jk}|\cos(\phi_{jk} - \theta_{jk}) \tag{6}$$

where $S_{jk} = |S_{jk}|\exp(\mathrm{i}\phi_{jk})$ and $F_{jk} = |F_{jk}|\exp(\mathrm{i}\theta_{jk})$.
Equation (6) is explicit in the unknown reference matrix CSD terms. By using the relation (3), equation (6) can be finally expressed in terms of coherences and phases between the pairs of control channels (the information that needs to be provided to the vibration controller)

$$P = \sum_{j=1}^{\ell} S_{yy,jj}F_{jj} + 2\sum_{j=1}^{\ell-1}\sum_{k=j+1}^{\ell} \sqrt{\gamma_{jk}^2}\sqrt{S_{yy,j}S_{yy,k}}\,|F_{jk}|\cos(\phi_{jk} - \theta_{jk}) \tag{7}$$

The matrix $\mathbf{F}$ can be easily computed from the identified system $\hat{\mathbf{H}}$, and therefore the terms $|F_{jk}|$ and θ_{jk} can be considered as known quantities. Also the PSD terms are known and considered as test specifications. The first term on the right hand side of equation (7) is always positive and fixed for the given test specifications and test setup. The second term contains the quantities ϕ_{jk} and γ_{jk}^2, unknowns of the target definition procedure. This term can be negative because of the cosine contained in the double sum and can therefore be a negative contribution, reducing the drive trace.
The expression in equation (7) has a minimum (maximum) when the coherences are all unitary and the cosines all equal -1 (1). This observation leads to the following conditions (addressed here as *Extreme Drives Conditions*) that lead to the theoretical minimum (maximum) drive traces

$$P \text{ is minimum} \iff \begin{cases} \gamma_{jk}^2 = 1 \\ \phi_{jk} = \theta_{jk} + \pi \end{cases} \qquad \forall j,k = 1:\ell, j \neq k \tag{8a}$$

$$P \text{ is maximum} \iff \begin{cases} \gamma_{jk}^2 = 1 \\ \phi_{jk} = \theta_{jk} \end{cases} \qquad \forall j,k = 1:\ell, j \neq k \tag{8b}$$

In the following the conditions (8a) and (8b) will be referred as the *Minimum Drives Condition* and the *Maximum Drives Condition*, respectively. All the other possible combinations of coherences and phases return drive traces that fall in the range between the minimum and the maximum value.

2.2 Proposed solution: Minimum (Maximum) Drives Method

Condition (8a) requires to fill in the MIMO reference matrix *element by element* with $\big(\ell(\ell+1)/2 - \ell\big)$ unitary coherence and phase profiles equal to the $(\theta + \pi)$ phase angle of the $\mathbf{F}$ matrix corresponding entry (the same considerations hold for (8b)). This could result in the reference matrix being negative definite. A mathematical proof can be found by analyzing the simple case of three control channels. Stated that the only physical values that the coherence can assume are between 0 and 1, for the reference matrix to be positive semi-definite it is necessary and sufficient that the Sylvester's Criterion holds [18] and therefore that

$$det(\mathbf{S_{yy}^{ref}}) = 1 - \gamma_{12}^2 - \gamma_{13}^2 - \gamma_{23}^2 + 2\cos(\phi_{12} - \phi_{13} + \phi_{23})\sqrt{\gamma_{12}^2\,\gamma_{13}^2\,\gamma_{23}^2} \geq 0 \tag{9}$$

Unitary coherence implies also the cosine to be unitary, in order for the determinant to be greater or equal than zero and thus the cosine's argument must nullify

$$\begin{cases} det(\mathbf{S_{yy}^{ref}}) \geq 0 \\ \gamma_{12}^2 = \gamma_{13}^2 = \gamma_{23}^2 = 1 \end{cases} \Rightarrow \quad \phi_{12} - \phi_{13} + \phi_{23} = 0 \tag{10}$$

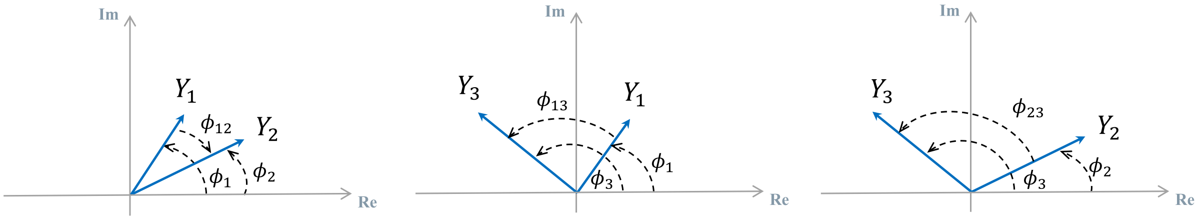


Figure 1: Example of phase relation for three fully coherent control channels. Given the phases ϕ_{12} and ϕ_{13}, the phase ϕ_{23} is unequivocally defined as the difference between the other two.

A strong deterministic relation needs to exist between the phases to be selected. This can be physically explained by associating these phases to the ones of the respective recorded spectra, as shown in Figure 1. Setting the phases ϕ_{12} and ϕ_{13} means to set a *relative constraint* in the phase information carried by the two pairs of recorded signals, i.e. that between controls 1 and 2 and controls 2 and 3 there are phase angles (in radians) of ϕ_{12} and ϕ_{13}, respectively. Therefore, the phase ϕ_{23} between the control channels 2 and 3 is unequivocally defined as the difference between ϕ_{13} and ϕ_{12}. Keeping unitary coherence, equation (10) highlights that, unless the $(\theta + \pi)$ angles does not already respect the aforementioned relation, a physical realizable target to be used for testing purposes cannot be set with the conditions (8).

Nevertheless, equation (7) still provides a solution to reduce the drives power. A positive semi-definite matrix can be obtained, according with the condition (8a), selecting just $(\ell - 1)$ phases equal to the respective $(\theta + \pi)$ phase angles whereas all the others needs to be obtained in order to fulfill the condition (10), i.e. as the difference between the selected ones. This means that an automatic target definition procedure can return positive semi-definite reference matrices and drastically reduce the drives power (without any modification on the PSDs) if these $(\ell - 1)$ phases are opportunely chosen and the remaining ones calculated in order to agree with this simple principle. Depending on the value of the resulting phase angle, they can contribute or not to the drive trace's reduction. It is worth to notice that, increasing the drive traces with respect to the one obtained from the standard method, is a direct consequence of the resulting reference matrix being positive semi-definite. A negative definite matrix can have a reduced traces due to the effect of negative eigenvalues (potentially, in the standard method, nothing prevents the drive traces to be also negative). In order to guarantee the biggest trace reduction, the $(\ell - 1)$ chosen phases can be defined as the one corresponding with the vectors with biggest amplitude $|F_{jk}|\sqrt{S_{yy,jj}\, S_{yy,kk}}$. More details of the proposed procedure can be found in [19].

The same procedure can be applied choosing the phases according to the condition (8b) in place of the condition (8a), hence maximizing the drive traces. However, although the advantages of having reduced drives are clear, the applicability (rather than purely comparative) of a method that requires maximum drives is still under investigation.

3 Test Cases

In order to show the general applicability of the algorithm, the Minimum (Maximum) Drives Method is tested with the electrodynamic three-axial shaker Dongling 3ES-10-HF-500 at the University of Ferrara, shown in Figure 2. This advanced actuation system is an assembly of three independent electrodynamic shakers, connected via a patented coupling hydrostatic bearing. The head expander is a squared 0.5 m plate with blunt corners and a total mass of 14 Kg. As data acquisition system and vibration controller, a Siemens LMS SCADAS Mobile SCM205V is used, driven by Simcenter LMS Test.Lab MIMO Random Control.
More details related to the applicability of the algorithm can be found in [19], where the developed procedure is tested considering different actuation mechanisms, number of drives and control channels.

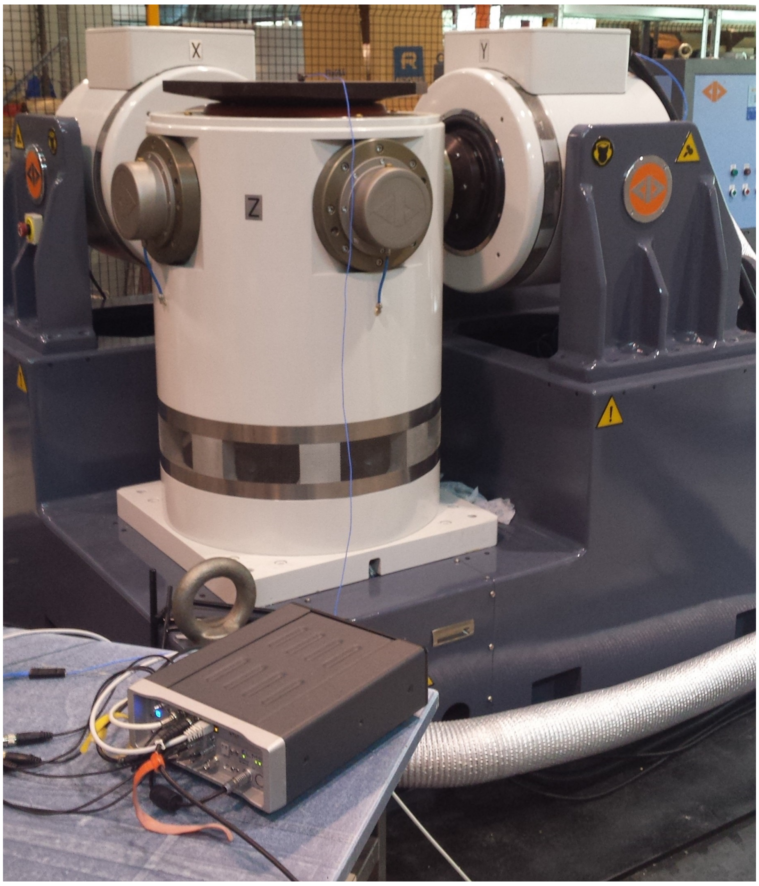


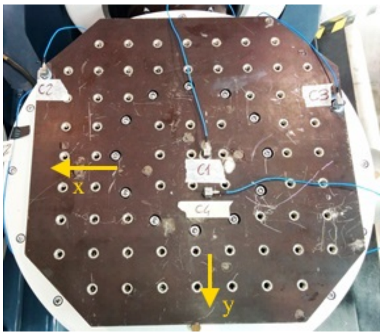
Figure 2: Dongling 3ES-10-HF-500 three-axial shaker at the University of Ferrara.

Two different series of tests are performed. First, the bare head expander (HE) is tested for the maximum allowed bandwidth. Subsequently, an automotive component is tested in a single simultaneous three-axial test to match the single axis specifications (longitudinal, lateral and vertical) in a simultaneous three-axial test. In order to avoid possible energy sinks associated with the pseudo-inversion, just *square* well-conditioned configurations are considered with realizable PSDs [20].

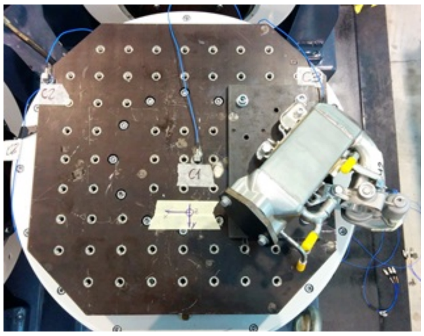
All the results are shown for *normal end* tests, meaning that the full level (0 dB) has been successfully run for 1 min (within the abort limits). The results in terms of control SDM are just shown for the tests run with the Minimum Drives Target. These results are illustrated in a matrix subplot fashion. The subplots on the diagonal show the reference profiles and the PSDs achieved at the normal end, together with the abort and alarm thresholds; in the upper and lower triangular parts of the plot the amplitude (in g^2/Hz) and phases (in degrees) of the upper triangular CSDs are reported, respectively. Since the matrices are positive semi-definite and therefore hermitian, the subplots are also representative for the lower triangular CSDs (complex conjugates). It is fundamental to notice that for the same test setup, different tests are run with the same PSD profiles, considered as test specifications, and different CSDs, coming from the application of the Minimum Drives Method and also with common choices of coherence and phases between control channels.

To show the advantage of the developed procedure, the comparison with other choices for the CSDs is shown in terms of:

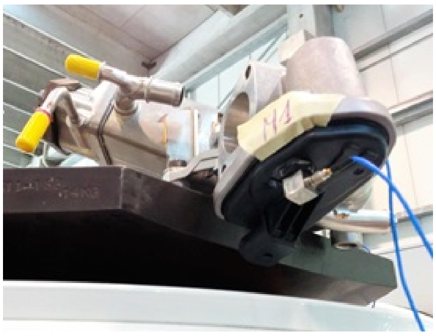
- predicted drives RMS from a system verification run prior to the test (to check the tests levels). These values are simply the RMS (over the selected bandwidth) calculated from the drive PSDs returned by equation (2);
- drive traces, to show the narrow band drive traces reduction;



(a) bare head expander test configuration.



(b) test configuration with the EGR valve mounted on the head expander, top view.



(c) test configuration with the EGR valve mounted on the head expander, side view.

(d) test configuration with the EGR valve mounted on the head expander, front view.

Figure 3: test configuration for the tests performed with the three-axial shaker at the University of Ferrara.

- drive traces RMS, as global indicator for the drives' power reduction. During the test narrowband unpredictable differences could arise due to the several reason, such as the randomness of the process, the on-line control action that tunes the drives to match the target and/or system's non-linearities.

3.1 Bare head expander

The HE is tested in the full frequency range allowed by the shaker's manufacturer ($[10 - 2000]$ Hz) with the test configuration shown in Figure 3a and a frequency resolution of 1.5625 Hz. The control channels are the X, Y and Z axis of the accelerometer C1. The bandwidth pushes the limits of the shaker system. In order to get meaningful PSDs to be set as diagonal elements for the MIMO random control test, an open-loop pre-test is run with uncorrelated drives (pseudo random signals with a random phase randomization). The X, Y and Z response RMS levels are 1.29, 1.25 and 1.45 g_{RMS}, respectively.

Figure 4 shows the predicted drive levels V_{RMS} and their sum from the pre-test System Verification. The different bars correspond to different choices in defining the CSDs. In the figure, the threshold for the overload of the data acquisition DAC is also reported. The figure allows to make some considerations. Low coherence between responses does not mean low power, as shown via the equation (8a). The phase information set between the control channels is playing a major role. Choosing the phases with the Minimum Drives Method returns minimum predicted drives power, compared to the other choices.

It is relevant to notice that the method shows a reduction of the single drives V_{RMS}. In case the test levels need to be increased, the method provides the biggest scaling factor. In case the same test levels are kept, choosing the CSDs with the Minimum Drives Method preserves the shakers and the amplifiers, subjected to lower voltages.

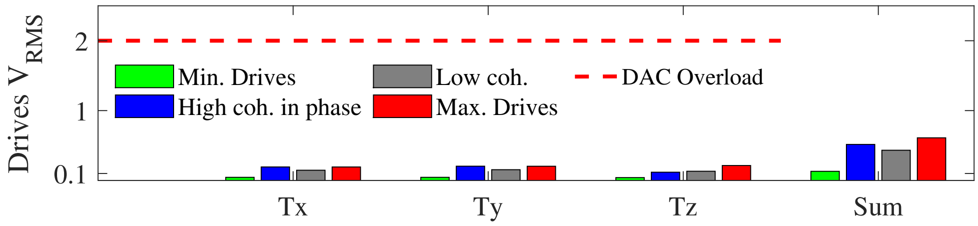


Figure 4: predicted drives and their sum, for the tests performed on the bare head expander with reference PSDs from an open loop pre-test.

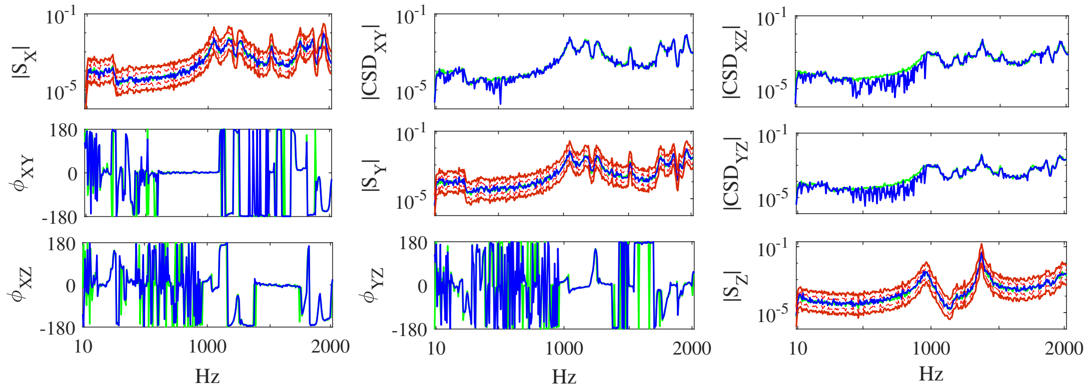


Figure 5: MIMO random control results (solid blue curve) for the test performed on the bare head expander of the three-axial shaker by setting the target (solid green curve) with the Minimum Drives Method. PSDs (diagonal subplots) and CSDs amplitudes (upper triangular subplots) in g^2/Hz, phase angles (lower triangular subplots) in degrees. The solid red and dashed orange lines are the abort and alarm thresholds, respectively.

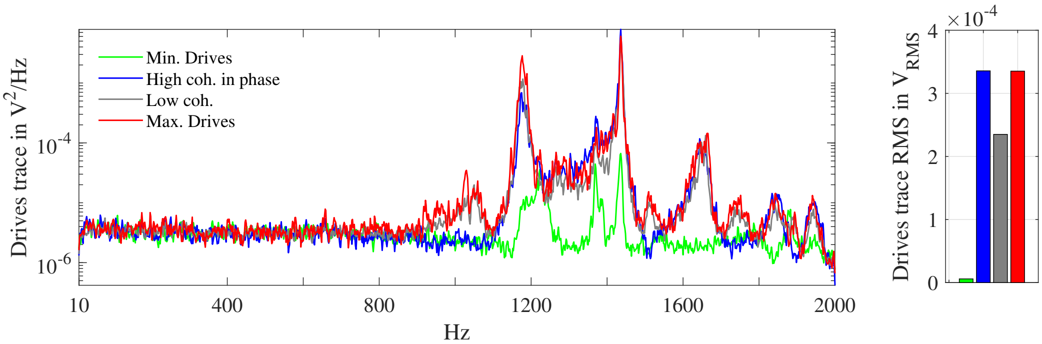


Figure 6: drive traces for the tests performed on the bare head expander with reference PSDs from an open loop test.

The control results for the normal end test with the target set with the Minimum Drives Method are shown in Figure 5 and the resulting drive traces are reported in Figure 6. The drive traces of the tests run by setting the CSDs with the Minimum and the Maximum Drives Method bound the drive traces returned by other phase and coherence selections.

3.2 Automotive test article: Exhaust Gas Recirculation (EGR) valve

The successful tests motivated to use the Minimum Drives Method on an automotive OEM (Original Equipment Manufacturer) component to be tested to random vibration in the three different directions. The test specification are defined in terms of PSDs only, inherited from single axis test standard practices. The

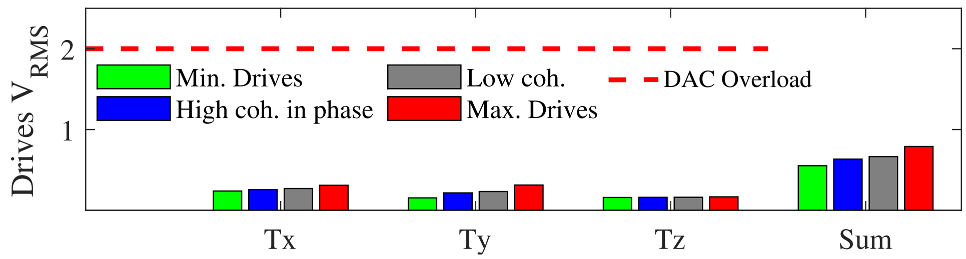


Figure 7: predicted drives and their sum, for the tests with the EGR valve mounted on the head expander.

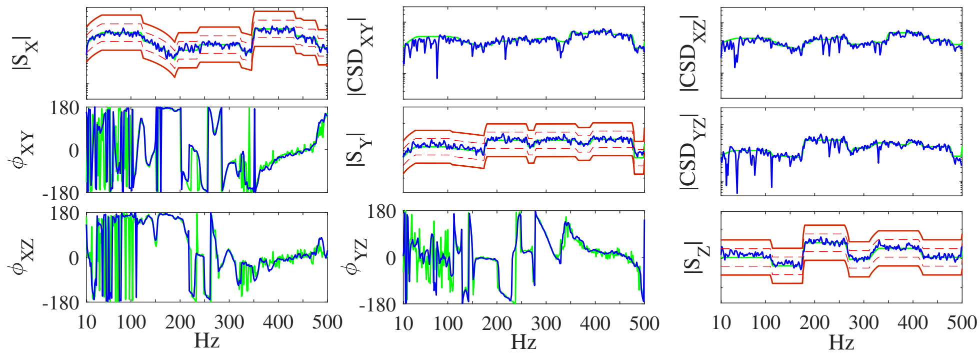


Figure 8: MIMO random control results (solid blue curves) for the test performed with the EGR valve mounted on the bare head expander of the three-axial shaker by setting the target (solid green curve) with the Minimum Drives Method. PSDs (diagonal subplots) and CSDs amplitudes (upper triangular subplots) in g^2/Hz, phase angles (lower triangular subplots) in degrees. The solid red and dashed orange lines are the abort and alarm thresholds, respectively.

automotive component tested is an Exhaust Gas Recirculation (EGR) valve, used to reduce the internal combustion engines emissions. The test configuration is shown in the Figures 3b, 3c and 3d. For these tests, the same control channels adopted for the bare head expander test case are used. The PSD shapes come from single axis test specifications. They are defined in the frequency range $[10-500]$ Hz, with a frequency resolution of 1.5625 Hz.

The predicted drive voltages are illustrated in Figure 7 and show again the drives reduction obtained by the method. In this application the advantage in terms of voltage's reduction is evident but less significant compared to the previous test case. This is due to the reduced bandwidth, limited to a region where the cross-coupling between orthogonal axes is small and therefore the reference matrix's CSDs influence on the drives power reduces consequently.

Figure 8 shows the narrowband results of the MIMO random control process, with the response SDM controlled to achieve the Minimum Drives Target. The difference in the narrowband drive traces at the normal end can be seen in Figure 9, plotted in linear scale. The Minimum Drives Method shows a non negligible drives trace reduction. In the figure the theoretical Minimum (Maximum) drives trace are also reported. These traces are calculated substituting in equation (2) the response SDM with the target obtained applying the developed method and the system's FRF matrix with the one obtained from the system identification. The only region with an unexpected behavior is a narrow band around 300 Hz, possibly due to the control action performed to try to control the system's non-linearities. The adaptive feature of the Siemens LMS MIMO Random Control can cope with system non-linearities, as shown in the good control results of Figure 8. On the contrary, the assumptions made to derive the Minimum Drives Target will be no longer valid and the effects of the drives correction could be unpredictable.

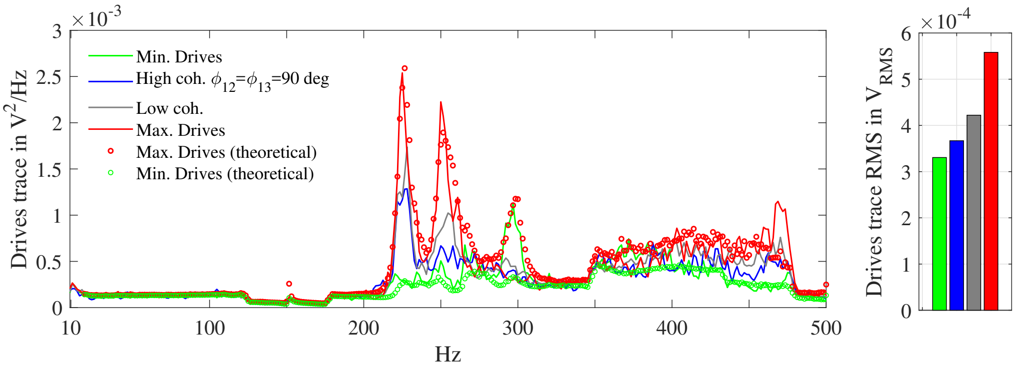


Figure 9: drive traces for the tests performed with the EGR valve mounted on the head expander.

4 Conclusions

This paper proposes the Minimum (Maximum) Drives Method as innovative target definition procedure for MIMO random control tests. The only information needed for this target definition method is the System Identification, anyhow required for the vibration control algorithm, and the PSD profiles, representing the test specifications. The target definition process can therefore be fully automated.

With the developed procedure it is possible to generate the missing CSDs in such a way that (i) the reference matrix is positive semi-definite in the whole test bandwidth and (ii) the drives power needed to reach the test specifications (the PSDs) is systematically reduced with respect to other possible state-of-the-art solutions. These features open the possibility to candidate the proposed methodology as attractive solution to the problem of meeting *the minimum drives criteria* and therefore to be included in the current standard practice for multi-axial testing.

Compared to standard choices or methods currently used to fill in the CSDs, the added value of reducing the total drives power is undoubtedly advantageous: for fixed test response levels, it guarantees that the delicate and expensive excitation hardware (shakers and amplifiers) is driven with reduced voltages. In case the test levels need to be increased, the proposed solution would allows a maximum scale factor.

Furthermore, the generated targets incorporate information coming from the system identification, that could result in exciting the system in agreement with its dynamic behaviour.

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