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Procedia Engineering 70 (2014) 379 – 388

Procedia Engineering

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12th International Conference on Computing and Control for the Water Industry, CCWI2013

Comparison between entropy and resilience as indirect measures of reliability in the framework of water distribution network design

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Abstract

The aim of this paper is to investigate which between the entropy and resilience indices represents a better indirect measure of reliability in the framework of water distribution network design. The methodology adopted consisted of (a) multi-objective optimizations performed in order to minimize costs and maximize reliability, expressed by means of one of the indirect indices at time; (b) retrospective performance assessment of the solutions of Pareto fronts obtained. Two case studies of different topological complexity were considered. Results showed that indices based on energetic concepts (resilience and modified resilience) represent a better compact estimate of reliability than the entropy.

Selection and peer-review under responsibility of the CCWI2013 Committee

Keywords: entropy; resilience; reliability; water distribution network; optimal design.

1. Introduction

The reliability of a water distribution system is classically defined as its capacity to fully satisfy users' demand in a given period of time (Hashimoto 1982). For a *direct* estimation of service reliability several specific performance indicators can be adopted (Gargano and Pianese 2000, Tanymboh et al. 2001, Ciaponi 2009, Creaco and Franchini 2012), for instance expressing the average (or weighted average) of the ratios of water discharge supplied to users to the corresponding water demand under various operation scenarios, including normal peak

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Selection and peer-review under responsibility of the CCWI2013 Committee
doi:10.1016/j.proeng.2014.02.043

operational conditions and critical operational scenarios such as segment isolation and hydrant service. The evaluation of these performance indicators may unfortunately turn out to be a computationally heavy task, especially in the case of complex real networks, since it generally implies the execution of numerous pressure-driven hydraulic simulations.

Therefore, in an attempt to limit the computation time (like in the context of network design phase, when reliability assessment has to be done several times), reliability is often expressed through *indirect* indices such as resilience (Todini 2000), modified resilience (Prasad et al. 2003) and entropy (Tanyimboh and Templeman 2000); these indices can in fact be evaluated by means of a single (even demand-driven) hydraulic simulation, in a bid to express the redundancy of the network under benchmark operation conditions.

It is thus of great interest to understand which of the above indirect measures of reliability is the most appropriate to better characterize the *full* reliability of the network within the framework of the design phase. In this context, Tanymboh et al. (2011) and Greco et al. (2012) investigated which indirect index is more correlated to network reliability. In their works the Authors generated various pipe diameter configurations in the case studies analyzed in order to obtain network featuring higher and higher values of the indirect reliability indices. Subsequently, they retrospectively assessed performance indicators for each of the configuration and analyzed the correlation between each indirect index and the performance index. In the end, Tanymboh et al. (2011) and Greco et al. (2012) arrived at contrasting results. As a matter of fact, by considering the retrospective assessment only of performance indicators related to segment isolation, the analysis of Tanymboh et al. (2011) indicates the entropy as the best indirect measure. On the other hand, Greco et al. (2012) indicate the resilience as the best indirect reliability measure.

This paper is aimed at analyzing in depth the issue of indirect reliability indices. Unlike the works of Tanymboh et al. (2011) and Greco et al. (2012), it considers the analysis in the framework of network multi-objective design (Gessler and Walski 1985, Todini 2000, Prasad et al. 2003), where the indices are related to the costs; furthermore, the comparison of the indices is made by retrospectively assessing performance indicators related to segment isolation and hydrant service, rather than only segment isolation.

In the following sections, the methodology is first proposed (section 2); then, the applications to two case studies of different complexity (a synthetic network and a real network) are presented (section 3) and conclusions are finally drawn (section 4).

2. Methodology

The methodology used in this paper consists of two steps. In Step 1, multi-objective design optimizations, aimed at simultaneously minimizing network total cost and maximizing a reliability indirect index, are performed on a water distribution network considering, as decisional variables, the network pipe diameters. The results of the optimizations are Pareto fronts of optimal solutions featuring increasing values of network cost and indirect reliability index.

Three different optimizations are performed using the NSGA-II multi-objective algorithm (Deb et al. 2002); the optimizations differ in the index adopted as indirect measure of reliability within the optimization process: optimization I – resilience index by Todini (2000); optimization II – modified resilience by Prasad et al. (2003); optimization III – entropy index by Tanyimboh and Templeman (2000).

In Step 2, in order to understand which of the optimizations (I, II or III) yields the best representation of the network reliability as the cost grows, for each of the optimization performed, all the optimal solutions of the Pareto front are *a posteriori* assessed in terms of direct performance indicators relative to the critical operation scenarios of network segments isolation and hydrant service, as proposed in Creaco and Franchini (2012). This retrospective assessment makes it possible to obtain relationships between the latter direct performance indicators and the costs produced by the different optimizations. Results are then compared and the best optimization approach, which leads to the highest reliability levels for given cost, is detected. The best optimization approach will give indication of which is the best indirect measure of reliability in the framework of network design.

In the following sub-sections, the indirect reliability indices (section 2.1) and the performance indicators (section 2.2) adopted are described.

2.1. Indirect reliability indices

Resilience index, I_r (Todini 2000), is connected with the hydraulic head surplus at network nodes compared to the minimum required heads under normal operation condition; this head surplus represents the "energy storage" that can be dissipated under critical operational conditions such as segment isolations (which cause an increase in head losses), thus preventing water supply to users being affected (Fortunato et al. 2012).

It is defined starting from the overall hydraulic power entering the network, P_{tot}, given by:

$$P_{tot} = \gamma \sum_{k=1}^{n_r} Q_k H_k \tag{1}$$

where Q_k and H_k are the flow entering the network and the head at the k-th reservoir, or supply point, respectively, n_r is the number of reservoirs supplying the network, and γ is the specific gravity of water.

The hydraulic power dissipated by the water flowing through the network, P_{int}, is given by the difference of P_{tot} and the total hydraulic power provided to the users, P_{ext}, which in turn is expressed by the following equation:

$$P_{ext} = \gamma \sum_{i=1}^{n_n} q_i h_i \tag{2}$$

wherein h_i and q_i respectively are the actual head and supplied flow at the i-th node (which during the design phase and the ordinary operation conditions is equal to the nodal demand, d_i), and n_n is the number of nodes of the system.

The maximum hydraulic power that can be dissipated within the system while meeting minimum heads constraints at network nodes, $h_{i\,min}$, to supply the required demands, is given by:

$$P_{\text{int max}} = P_{tot} - \gamma \sum_{i=1}^{n_n} q_i h_{i\,\text{min}} \tag{3}$$

Resilience index, I_r, is eventually defined as:

$$I_r = \frac{P_{\text{intmax}} - P_{\text{int}}}{P_{\text{intmax}}} = 1 - \frac{P_{\text{int}}}{P_{\text{intmax}}} = \frac{\sum_{i=1}^{n_n} q_i \left(h_i - h_{i\,\text{min}}\right)}{\sum_{k=1}^{n_r} Q_k H_k - \sum_{i=1}^{n_n} q_i h_{i\,\text{min}}} \tag{4}$$

In normal operational condition, having imposed $h_i \geq h_{i\,min}$ $\forall\ i$, during the design phase, I_r can only take on positive values and range within the interval [0, 1): it can never be equal to 1 as that would imply the total absence of energy dissipation.

Modified resilience index, $I_{r\,mod}$ (Prasad et al. 2003), is an upgrade of the index provided by Todini (2000); besides considering the head surplus at each node of the network, it also takes account of the uniformity of the pipes connected to each network node, as a further ingredient of network reliability. This index can be calculated through the following formula:

$$I_{r\,\text{mod}} = \frac{\sum_{i=1}^{n_n} C_i q_i \left(h_i - h_{i\,\text{min}}\right)}{\sum_{k=1}^{n_r} Q_k H_k - \sum_{i=1}^{n_n} q_i h_{i\,\text{min}}} \tag{5}$$

where coefficient C_i, representative of pipe uniformity at the generic i-th node, appears in the numerator of the expression. This coefficient can be calculated as:

$$C_i = \frac{\sum_{j=1}^{np_i} D_j}{np_i \times \max(D_j)} \tag{6}$$

where np_i is the number of pipes j connected to node i.

Entropy function, E (Tanyimboh and Templeman 2000), is linked to the uncertainty characterizing the paths that bring water to each network node; a high entropy entails existence of many equally important feeding paths and would guarantee the generic node to be properly supplied even if one of those paths is temporarily out of service due to maintenance works.

It is expressed by the equation:

$$E = E_0 + \sum_{i=1}^{n_n} P_i E_i \tag{7}$$

where E is the overall entropy for the water distribution network, E_0 is the entropy of supply sources, E_i is the entropy of the i-th node, $P_i = T_i / T$ is the fraction of the total flow entering the network which reaches node i, T_i is the total flow that reaches node i, T is the sum of the nodal demands (equal to the total flow entering the network, when the demands are fully satisfied as in normal operational conditions), and n_n is the number of demand nodes.

E_0 is given by:

$$E_0 = -\sum_{i \in I} \frac{Q_{0i}}{T} \ln\left(\frac{Q_{0i}}{T}\right) \tag{8}$$

where Q_{0i} is the supply inflow at the i-th source node, and I is the set of source nodes.

E_i is defined as:

$$E_i = -\frac{q_i}{T_i} \ln\left(\frac{q_i}{T_i}\right) - \sum_{ij \in ND_i} \frac{Q_{ij}}{T_i} \ln\left(\frac{Q_{ij}}{T_i}\right) \tag{9}$$

wherein q_i=d_i is the nodal demand, Q_{ij} is the pipe flow from node i to node j, ND_i is the set of pipe flows from node i, and i ranges from 1 to n_n.

2.2. Performance indicators

As a performance indicator, direct measure of reliability, for a given operation scenario j, the demand satisfaction rate S_j, defined as follows (Creaco and Franchini 2012), can be adopted:

$$S_j = \frac{\sum_{i=1}^{n_n} q_i}{\sum_{i=1}^{n_n} d_i} \tag{10}$$

where q_i is the actual water flow supplied to users at node i, calculated on the basis of a pressure-driven simulation of the network under the j-th scenario, and d_i is the water demand. The relationship between q_i and d_i depends on the value of pressure head h at node i and can be expressed as follows (Wagner et al. 1988):

$$\begin{aligned} q_i &= 0 && \text{if } h < h_{0,i} \\ q_i &= d_i\left(\frac{h_i - h_{0,i}}{h_{min,i} - h_{0,i}}\right)^{\gamma} && \text{if } h_{0,i} \le h \le h_{min,i} \\ q_i &= d_i && \text{if } h > h_{min,i} \end{aligned} \tag{11}$$

where $h_{min,i}$ is the minimum piezometric head required to fully satisfy nodal demands and $h_{0,i}$ is the minimum pressure head required to enable nodal outflow; the exponent γ is commonly set to 0.5 (for further details on γ value see also Fujiwara and Li 1998 and Tucciarelli et al. 1999).

Among the various possible operational scenarios, those featuring service disruption in some parts of the network (segments), which can be isolated from the water sources by operating isolation valves, are particularly relevant. S_j can be assessed with reference to operational scenarios in which a single segment is isolated at a time. Assuming that the network can be subdivided into n_s independent segments, it is possible to evaluate the S_j index associated with the operational scenario corresponding to the isolation of the generic network segment j, by performing a pressure driven hydraulic simulation for the part of the network remaining connected to the water sources after the isolation of the segment itself. After assessing S_j for the generic segment, the performance indicator "average satisfaction" I_{aS} (aS: average-Satisfaction) of the whole network can be calculated by averaging the demand satisfaction rate relative to each segment isolation as follows:

$$I_{aS} = \frac{\sum_{j=1}^{n_n} S_j}{n_s} \tag{12}$$

Further possible critical scenarios are those featuring the activation of a hydrant at a generic node. Assuming the presence of a hydrant at n_{nh} of the network nodes, the performance indicator "average satisfaction during fire conditions" I_{afS} (afS: average-fire-Satisfaction) can be assessed as:

$$I_{afS} = \frac{\sum_{j=1}^{n_{nh}} S_j}{n_{nh}} = \frac{1}{n_{nh}} \sum_{j=1}^{n_{nh}} \frac{\sum_{i=1}^{n_n} q_{i,j}}{\sum_{i=1}^{n_n} d_i} \tag{13}$$

where $q_{i,j}$ is the actual discharge delivered at node i when the j-th hydrant is activated.

To describe network performance as regards the operation of a hydrant at the generic j-th node, the following $S_{h,j}$ index can be introduced:

$$S_{h,j} = \frac{q_{hydr\,j}}{d_{hydr}} \tag{14}$$

in which d_{hydr} and $q_{hydr\,j}$ are the required and actual hydrant discharges, respectively (it is implicitly assumed that d_{hydr} is the same for all the hydrants) . The relationship between $q_{hydr,j}$ and d_{hydr} takes on the following form:

$$q_{hydr,j} = d_{hydr}\sqrt{h_j / h_{min,j}} \tag{15}$$

which yields $q_{hydr\,j} = d_{hydr}$ when h_j=$h_{min,j}$.
A global performance indicator "average satisfaction of hydrants" I_{ah} (ah: average-hydrant) can also be evaluated with reference to the total number n_{nh} of hydrants installed:

$$I_{ah} = \frac{\sum_{j=1}^{n_{nh}} S_{h,j}}{n_{nh}} \tag{16}$$

3. Numerical application

3.1. Case studies

Two case studies were considered. The first one is the rather simple network of Tanyimboh et al. (2011), made up of n_n=11 nodes with outflow, all with ground elevation of 0 m, n_p=17 pipes, all 1,000 m long with Hazen-Williams roughness coefficient equal to 130, and n_l=6 square (minimum) loops. The network features only one source node with 100 m piezometric head and a global peak demand of 444.5 l/s. The minimum desired pressure for full demand satisfaction is h_{min}=30 m in all the nodes whereas the lowest pressure head value h_0 that ensures nodal outflow is 5 m in all the nodes.

The second case study is the distribution network serving the part of the city of Ferrara (Northern Italy) inside the medieval walls (Alvisi et al. 2011), which features n_n=536 nodes with outflow, n_p=825 pipes with a total length of about 90 km, and n_l=288 (minimum) loops (Fig. 1).

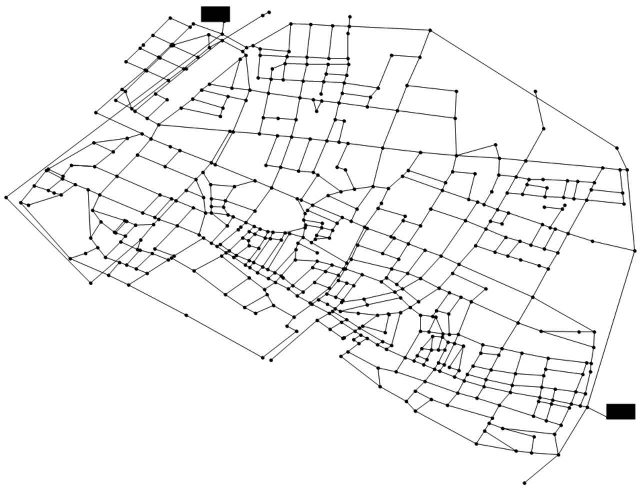

Fig. 1. Network serving the part of Ferrara city, Italy, lying inside the medieval walls (Creaco and Franchini 2012).

The whole network peak demand of 367 l/s is supplied by n_r=2 reservoirs. In the network layout adopted, all nodes have a ground elevation of 0 m, and the reservoirs have hydraulic heads of 30 m each. The roughness coefficients considered within the design phase are those relative to old cast iron pipes (Manning coefficient equal to 0.015 s/m$^{1/3}$). The minimum desired pressure for full demand satisfaction h_{min} was set at 25 m in all the nodes whereas the lowest pressure head value h_0 that ensures nodal outflow is 5 m in all the nodes.

Pipe diameters and unit costs considered for the design applications are reported in Table 1. As far as network segment isolation is concerned, it was assumed the presence of $2 \cdot n_p$ isolation valves, making it possible to consider each pipe as an independent segment. As regards the hydrants, for each hydrant the required operation outflow discharge d_{hydr} was assumed equal to 64 l/s in correspondence to h=h_{min} adopted for the specific network. For each network configuration found by the optimization algorithm, hydrants were assumed to be positioned in nodes which featured at least a connected pipe with diameter equal to or greater than 150 mm.

Table 1. Pipe diameters, *D*, and unit costs, *c*, adopted during the design phase for both case studies.

D[mm]	*c* [€/m]
45	185
60	203
80	227
100	231
150	272
200	299
250	328
300	360
350	399
400	439

3.2. Results

The multi-objective design optimization was applied to both case studies, using a population of 200 and 1,000 individuals in the first and second case-study respectively. The total number of generations was set at 200 and 5,000 in the first and second case-study respectively. In order to increase the computational efficiency in the optimization relative to the second case study, some solutions obtained by applying the methodology of Creaco and Franchini (2012) were inserted in the initial population. The Pareto fronts of optimal solution reported in Fig. 2 (a), (b), (c) and (d) were finally obtained.

Observing the Pareto fronts of Fig. 2, it can be noticed that, for both case studies, the three indirect reliability indices adopted grow as design configuration cost grows; I_r - C Pareto fronts presents the characteristic "knee", which is less pronounced in $I_{r\,mod}$ - C Pareto fronts and completely absent in E - C fronts. The analysis of the fronts show that, whereas the variation range of I_r and $I_{r\,mod}$ does not change significantly from case-study 1 to case-study 2, the variation range of E is strongly dependant on the kind of network. The higher values of E observed in the Ferrara network are due to the much higher number of loops in the network, that leads to a higher number of paths that may bring water to the generic node.

Once the three optimizations have been carried out for both case studies, the alternative optimal network configurations found were retrospectively evaluated in terms of direct performance indicators, I_{aS}, I_{afS} and I_{ah}, as described in the methodology section, thus obtaining the graphs of Fig. 3.

The first thing arising from the graphs of Fig. 3 is the absence of positive correlation between entropy E and the direct performance indicators, since network configurations with higher entropy and cost can be characterized by performance indicators lower than those corresponding to configurations with lower entropy and cost. That is enough to state that entropy index proved not to be, in itself, a good and consistent *indirect* measure of network reliability. Nevertheless, maximizing E can result in some well performing network configurations, which has however to be singled out by means of an *a posteriori* assessment of their performance. Furthermore, for both case studies, network configurations yielded by the optimization III had, in general, worse performance indicators than those yields by the optimizations I and II.

On the other hand, I_r and $I_{r\,mod}$, showed a much stronger positive correlation with the direct performance indicators, since higher resilience values generally entail better performance. Performance of the network configurations produced by I_r and $I_{r\,mod}$ maximization resulted rather similar, the latter being generally slightly higher; only for I_{aS} index, in the case of Tanyimboh's network (Fig. 3.e), a significant discrepancy was observed, with $I_{r\,mod}$ maximization producing much better performing network configurations. This occurrence is most probably due to the lowly redundant topological scheme of the network itself.

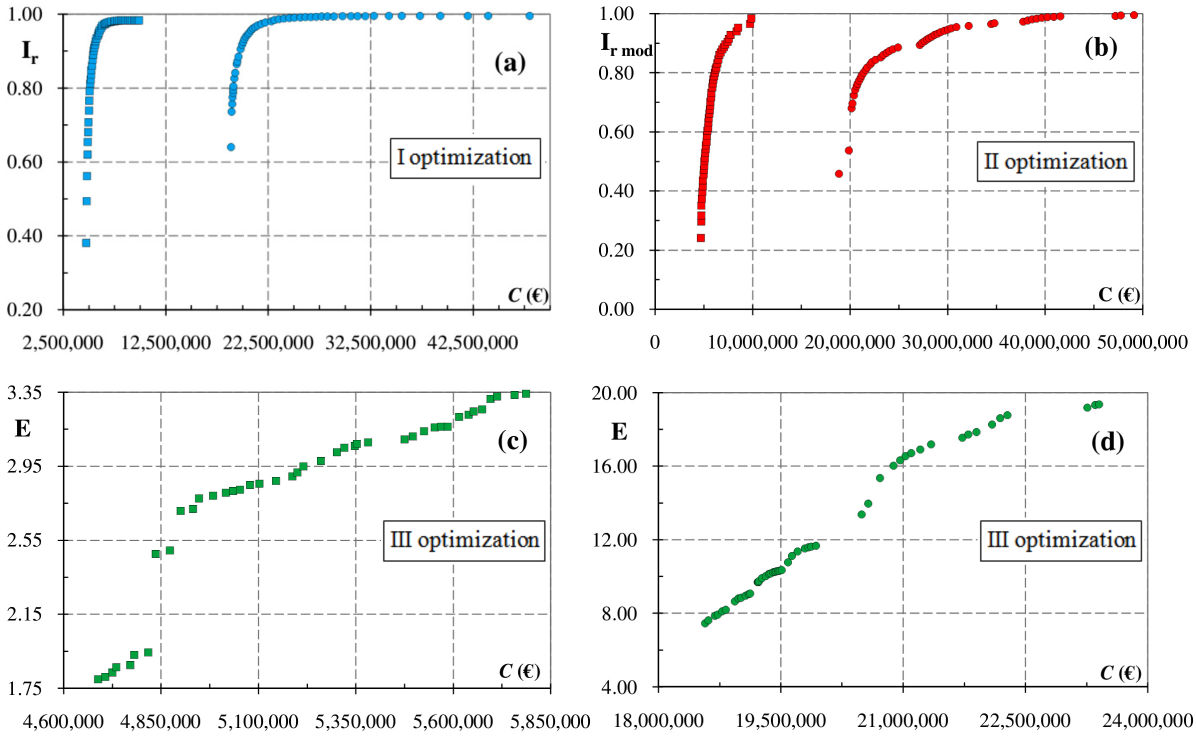

Fig. 2. Pareto fronts of optimal solutions. Optimal solutions corresponding to the I optimization (min cost - max resilience I_r) are marked in blue (a), those corresponding to the II optimization (min cost - max resilience $I_{r\,mod}$) are marked in red (b) and those corresponding to the III optimization (min cost - max entropy) are marked in green (c) and (d). Design configurations of Tanyimboh's network and of Ferrara city network are represented by square-dots and circle-dots, respectively.

4. Conclusions

The paper investigated which of the *indirect* measures of reliability (resilience I_r by Todini 2000; modified resilience $I_{r,mod}$ by Prasad et al. 2003; entropy E by Tanyimboh and Templeman 2000) is the most appropriate to better characterize the *full* reliability of the networks in the design phase.

To this end the *standard* multi-objective design approach, aimed at minimizing network costs while maximizing the *indirect* service reliability indices, was applied to two case studies: a rather simple benchmark network (Tanyimboh et al. 2011) and the real network of a city in Northern Italy. For each of the optimization performed, all the optimal solutions were *a posteriori* assessed in terms of direct performance indicators relative to the critical operation conditions of segment isolation and hydrant service.

Entropy index E, linked to the uncertainty characterizing the paths that bring water to each network node, proved not to be, in itself, a good and consistent *indirect* measure of network reliability in light of the absence of positive correlation with the direct performance indicators adopted.

On the other hand, resilience I_r and $I_{r\,mod}$, linked to the energy surplus in the network, showed a much stronger positive correlation with the direct performance indicators. Performance of the network configurations produced by I_r and $I_{r\,mod}$ maximization resulted rather similar, the latter being generally a bit higher. $I_{r\,mod}$ is therefore the more advisable *indirect* reliability index for both simple and complex networks.

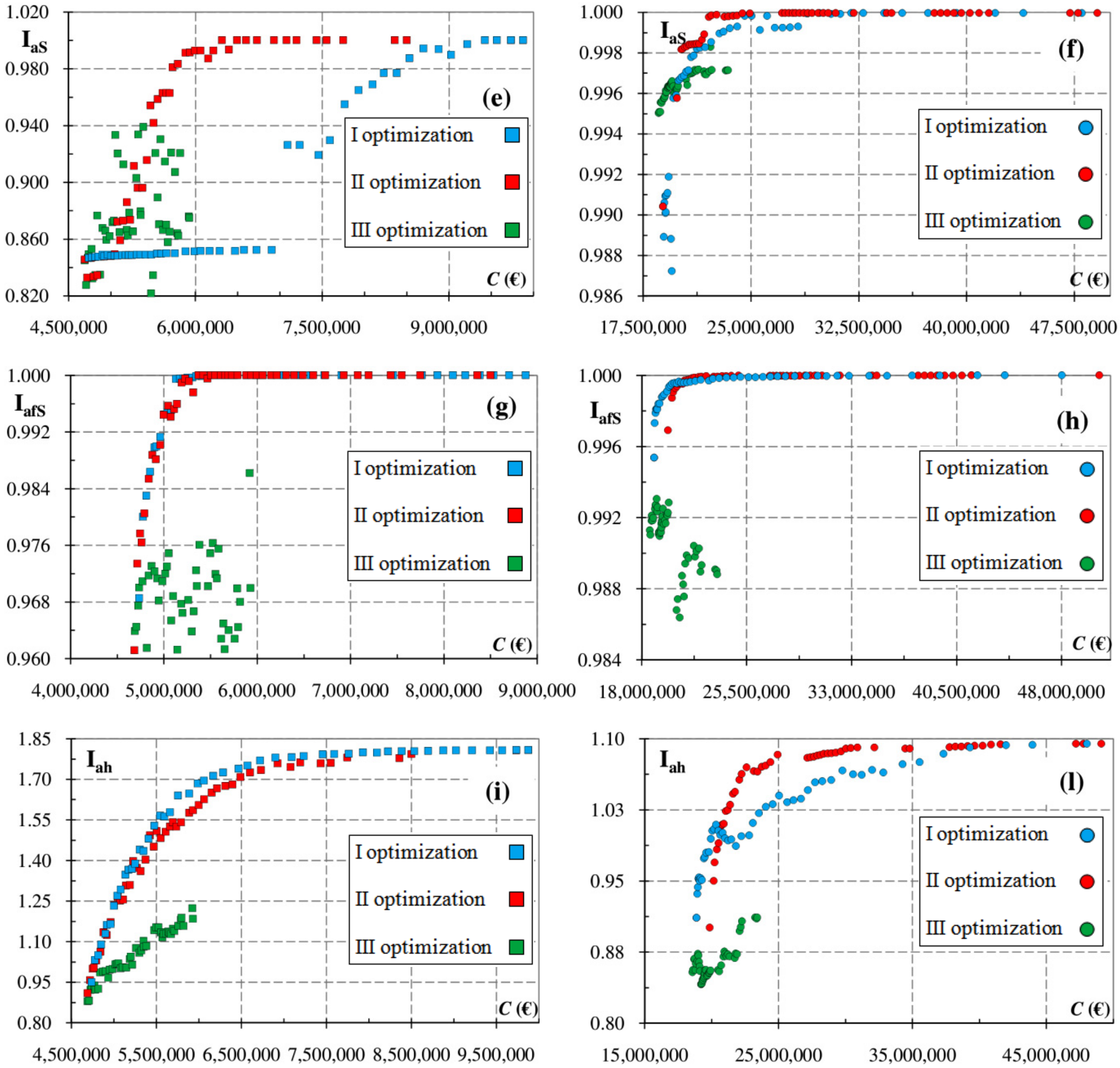

Fig. 3. Direct performance indicators, I_{aS}, I_{afS} and I_{ah}, of the alternative network configurations yield by the optimizations. Optimal solutions corresponding to the I optimization (min cost - max resilience I_r) are marked in blue, those corresponding to the II optimization (min cost - max resilience, $I_{r\,mod}$) are marked in red and those corresponding to the III optimization (min cost - max entropy) are marked in green. Design configurations of Tanyimboh's network and of Ferrara city network are represented by square-dots (plots on the left) and circle-dots (plots on the right), respectively.

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