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Evaluating size effects on fatigue life of 42CrMo4+QT steel using a statistical S-N model with highly-stressed volume and surface

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Abstract

A statistical S-N model based on the highly-stressed volume/surface concept is proposed for evaluating the size effects on fatigue strength of specimens from 42CrMo4+QT steel. The proposed model extends an existing physically valid model (Castillo et al. 2009) and allows the S-N curve to be extrapolated from small-scale to full-scale components. The model is validated against experimental fatigue data obtained by testing solid and hollow smooth cylindrical specimens under push-pull constant amplitude loadings. After checking that the proposed model agrees with experimental fatigue data, it is possible to apply the model to estimate the S-N curve of specimens of the highly-stressed volume up to 1802 mm^3. For this case, an S-N expression is provided that can directly be used in fatigue analysis without the necessity of running a non-linear optimization.

Keywords: size effects; fatigue; S-N curve; highly-stressed volume; highly-stressed surface

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1. Introduction

Full-scale fatigue tests of engineering components are generally time-consuming and costly, and often require special-purpose equipment. Therefore, fatigue tests are usually carried out by small-scale specimens and the relationship between the fatigue strengths of full-scale components and small-scale specimens is carefully established by a fatigue model able to account for size effects.

Full-scale and small-scale fatigue strengths are typically linked by the so-called weakest link principle. Based on this principle, Castillo et al. (1985) proposed a fatigue model to extend the S-N curves to any actual length of an evaluated component, e.g. to the length of a tendon of a cable-stayed bridge. However, this model is restricted to wires, strands and cables with varying length. Furthermore, it is not derived for any combination of minimum stress σ_m, maximum stress σ_M, or stress ratio $R = \sigma_m/\sigma_M$, as in the most general model also developed by Castillo et al. (2009).

By leading to a system of functional equations, Castillo et al. (2009) proposed a general fatigue model satisfying several physical, statistical and compatibility conditions. By applying the Castillo et al. (2009) model, the run-outs can be also considered in the fatigue analysis. This newer model does not include the size effect though it is more general compared to the mentioned Castillo et al. (1985) model, which considered the length effect. For this reason, a model used to extrapolate S-N curves from small-scale components to full-scale components would be welcome.

To deal with this extrapolation, stressed-based methods based on highly-stressed volume and highly-stressed surface are commonly used (Kuguel (1960) and Leitner et al. (2017)). The highly-stressed volume and highly-stressed surface concepts assume that fatigue strength decreases with increasing volume or surface area of material, according to an inverse linear function on log–log scale. Also, these concepts usually provide a basis for probabilistic fatigue assessment using the statistical theory (Leitner et al. (2017)). Aiming at devising a robust probabilistic fatigue model combining highly-stressed volume and surface, it is desirable to extend the Castillo et al. (2009) model to extrapolate the fatigue properties from small-scale specimens to full-scale components.

This paper proposes a statistical S-N model combining the Castillo et al. (2009) model and the highly-stressed volume or the highly-stressed surface to evaluate the size effects on fatigue life. After a short theoretical background on the Castillo et al. (2009) model and the highly-stressed volume and the highly-stressed surface concepts, the paper describes the proposed fatigue model used to account for the size effects. The proposed model is validated on experimental fatigue tests performed on specimens obtained from a single 42CrMo4+QT lot of steel bars. Solid and hollow unnotched cylindrical specimens with various diameters are tested under push-pull constant amplitude loadings. Before applying the proposed model to experimental data, finite element analysis is performed to determine the highly-stressed volumes and highly-stressed surfaces of each specimen configuration. After checking that the proposed model agrees with experimental fatigue data, the model enables the S-N curve of specimens of the highly-stressed volume up to 1802 mm^3 to be estimated. An S-N curve expression including the highly-stressed volume value is provided to be used in fatigue analysis without the necessity of a non-linear optimization program.

2. Theoretical background

2.1. The CFC model

This section introduces the robust fatigue model proposed by Castillo et al. (2009) to describe any S-N curve. The Castillo et al. (2009) model (also known as CFC model) makes use of the Buckingham theorem to provide a physically valid model. Using this theorem, the model selects dimensionless variables such as cycles to failure N^*, minimum σ_m^* and maximum stress ratios σ_M^* (asterisk refers to dimensionless variables). To satisfy several physical conditions, see Castillo et al. (2009), the only possible Gumbel model in terms of probability of failure p is:

$$p = 1 - \exp\left\{-\exp\left[\frac{(\ln N^* - B)(\sigma_M^* - \sigma_m^* - C) - E}{D}\right]\right\} \tag{1}$$

where $p = F^*(N^*; \sigma_M^*, \sigma_m^*)$ is also the cumulative distribution function of N^* for given σ_M^* and σ_m^*, B is the threshold value of log-lifetime, C is the endurance limit ratio, E defines the position of zero-percentile hyperbola and D is the scale factor. The dimensionless variables are defined as $N^* = N/N_0$, $\sigma_m^* = \sigma_m/\sigma_0$ and $\sigma_M^* = \sigma_M/\sigma_0$, where both denominators σ_0 and N_0 can be chosen arbitrarily (e.g. $\sigma_0 = 100$ MPa and $N_0 = 1$), and σ_M represents maximum stress and σ_m minimum stress in the cycle. Also, the product $(\ln N^* - B)(\sigma_M^* - \sigma_m^* - C)$ in Eq. (1) follows the reverse Gumbel distribution.

Solving the system of functional equations according to the Appendix in Castillo et al. (2009), the Gumbel model becomes:

$$p = 1 - \exp\{-\exp[C_0 + C_1\sigma_m^* + C_2\sigma_M^* + C_3\sigma_m^*\sigma_M^* + (C_4 + C_5\sigma_m^* + C_6\sigma_M^* + C_7\sigma_m^*\sigma_M^*)\ln N^*]\} \tag{2}$$

Eq. (2) depends on eight parameters from C_0 to C_7 providing all probabilistic information for any S-N curve. In case of C_5, C_6 and C_7 non-zero simultaneously, the model features two asymptotes thanks to involved hyperbolic functions. A comparison between the models — with and without asymptotes — and experimental data confirmed that the model with asymptotes is the most suitable to represent the material fatigue strength, see Castillo et al. (2009).

Using fixed asymptotes, the general model described so far simplifies to:

$$p = 1 - \exp\{-\exp[C_0 + C_1\sigma_m^* + C_2\sigma_M^* + C_6(\sigma_M^* - \sigma_m^*)\ln N^*]\} \tag{3}$$

To be physically and statistical valid, the model with fixed asymptotes must satisfy these constraints (Castillo and Fernández-Canteli (2009)):

$$C_3 = C_4 = C_7 = 0, \quad C_5 = -C_6, \quad C_6 \geq 0, \quad \min \ln N_i \geq \max\left(\frac{C_1}{C_6}, -\frac{C_2}{C_6}\right) \text{ for } i = 1,2,\dots,n \tag{4}$$

where N_i is the number of cycles to failure of the i-th specimen tested and n is the number of tested specimens (sample size).

The model parameters are obtained by maximizing the log-likelihood function subject to the constraints in Eq. (4). The maximum log-likelihood method shows good statistical properties and the possibility of including the run-out data in the analysis. Despite this possibility, the log-likelihood function is written here without run-out terms:

$$\ln \mathcal{L} = \sum_{i=1}^{n} h(N_i) + \ln\left(C_4 + C_5\sigma_{m_i} + C_6\sigma_{M_i} + C_7\sigma_{m_i}\sigma_{M_i}\right) - \ln N_i - \exp\left(h(N_i)\right) \tag{5}$$

while the function $h(N_i)$ is given by:

$$h(N_i) = C_0 + C_1\sigma_{m_i} + C_2\sigma_{M_i} + C_3\sigma_{m_i}\sigma_{M_i} + \left(C_4 + C_5\sigma_{m_i} + C_6\sigma_{M_i} + C_7\sigma_{m_i}\sigma_{M_i}\right)\ln N_i \tag{6}$$

To simplify the analysis procedure, this paper does not consider the run-outs. However, they may be added in the log-likelihood function, see Castillo et al. (2009). Either way – with or without run-outs – the use of a non-linear optimization program is needed to obtain the model parameters.

After defining the model, the S-N curve for the model with constant stress ratio $R = \sigma_m/\sigma_M = -1$ and fixed asymptotes is obtained:

$$\ln N = \frac{\ln(-\ln(1-p)) - C_0}{C_6\Delta\sigma} + \frac{C_1 - C_2}{2C_6} \tag{7}$$

where $\Delta\sigma = \sigma_M - \sigma_m$.

Although the model allows for any probability of failure, it is customary to consider a 50% probability to perform a regression analysis on experimental fatigue data. In this case, the term $\ln(-\ln(1-p))$ in Eq. (*7*) can be replaced by $-\ln(\ln(2)) = 0.3665$.

2.2. Highly-stressed volume (HSV) and highly-stressed surface (HSS)

The highly-stressed volume concept (also known as critical volume or HSV) was originally proposed by Kuguel (1960) to investigate the size and shape effects of specimens under different type of loadings. The HSV is that volume of material subjected to a fraction (e.g. 80% and more) of the maximum stress. Kuguel (1960) discovered that, by increasing the HSV of material, the fatigue strength decreases linearly on log-log scale. This inverse relationship was detected for both smooth and notched specimens.

A similar relationship is also shown by Castillo et al. (1985) who state that the cycles to failure of wires, strands and cables decrease with increasing their length. They also state that the long wires, strands and cables have a narrower distribution of cycles to failure due to the higher probability of finding cracks or flaws in the larger volume, than in the case of shorter components. In the same paper, Castillo et al. (1985) proposed a fatigue model including the effect of different sample lengths to extrapolate the S-N curve for long components. However, the application of this model is limited to components with variation only in length.

To investigate the statistical size effects of components not only the HSV concept is usually considered, but also the highly-stressed surface (also known as critical surface area or HSS). Similar to the definition of the critical volume, the HSS is defined as the surface area subjected to a fraction of the maximum stress. This surface concept may be favourable in case of materials exhibiting less volume imperfections, which probably leads to a crack initiation at surface, see Leitner et al. (2017). For this reason, statistical fatigue models based on HSS would be preferable for some materials.

To the best of our knowledge, there is no statistical fatigue model to date that includes HSV or HSS in a general form satisfying several physical, statistical and compatibility conditions. Therefore, a statistical model combining the HSV and HSS with the general CFC model is proposed to generalize the S-N curve for large components, i.e., large volumes and surfaces.

3. Proposed fatigue model

It is well known that larger specimens, which have a high probability of finding more or larger defects, are weaker than smaller specimens. This is explained by the weakest link principle, which states that the lifetime of a system formed by k elements is the lifetime of its element having the minimum lifetime. That is, the system fails when the weakest element fails. In terms of HSV, the fatigue strength of a specimen with volume $V = k \cdot V_0$ is controlled by the lowest strength among the k reference volumes V_0.

Following the weakest link principle, the probability of failure p_V of a specimen with volume V is obtained as:

$$p_V = 1 - \left(1 - p_{V_0}\right)^{V/V_0} \tag{8}$$

where p_{V_0} is the probability of failure of a specimen with a given reference volume V_0.

By considering the general CFC model with fixed asymptotes, p_{V_0} in Eq. (8) is replaced by Eq. (3). The resulting model for a given volume V is:

$$p_V = 1 - \exp\left\{-\exp\left[\ln\frac{V}{V_0} + C_0 + C_1\sigma_m + C_2\sigma_M + C_6(\sigma_M - \sigma_m)\ln N\right]\right\} \tag{9}$$

To estimate the parameters of the model, the same maximum log-likelihood function used by Castillo et al. (2009) is also assumed here, leading to the following expression:

$$\ln \mathcal{L} = \sum_{j=1}^{m}\sum_{i=1}^{n} h\left(N_{ij}\right) + \ln\left(C_4 + C_5\sigma_{m_{ij}} + C_6\sigma_{M_{ij}} + C_7\sigma_{m_{ij}}\sigma_{M_{ij}}\right) - \ln N_{ij} - \exp\left[h\left(N_{ij}\right)\right] \tag{10}$$

where the function $h\left(N_{ij}\right)$ is:

$$h\left(N_{ij}\right) = \ln\frac{V_j}{V_0} + C_0 + C_1\sigma_{m_{ij}} + C_2\sigma_{M_{ij}} + C_3\sigma_{m_{ij}}\sigma_{M_{ij}} + \left(C_4 + C_5\sigma_{m_{ij}} + C_6\sigma_{M_{ij}} + C_7\sigma_{m_{ij}}\sigma_{M_{ij}}\right)\ln N_{ij} \tag{11}$$

The log-likelihood function refers to a general case including a reference volume V_0 and one or more different volumes $V_1, V_2, \ldots, V_m, j = 1,2, \ldots, m$, where m is the number of specimen configurations.

At this stage, the log-likelihood function is maximized with respect to parameters, but subject to constraints similar to those of Eq. (4). Indeed, the constraints are obtained by substituting N_i with N_{ij} into Eq. (4).

Following the practical application of the model to fatigue data treated in this paper, the model in Eq. (9) is rewritten to obtain the regression equation for constant $R = \sigma_m/\sigma_M = -1$:

$$\ln N = \frac{\ln(-\ln(1-p)) - C_0 + \ln V_0 - \ln V}{C_6 \Delta\sigma} + \frac{C_1 - C_2}{2C_6} \tag{12}$$

In the case of HSS, the statistical model is easily obtained by substituting volume V with surface S over all equations in this section.

4. Experimental validation

This section describes the experimental campaign performed to obtain the fatigue data of different series that are used for validating the statistical model. Eight series with different specimen geometries were designed to evaluate the size effects. They were cylindrical specimens divided into two geometry classes: solid (S1 to S4) and hollow (H5 to H8) specimens. All types of specimen series are shown in Fig. 1 with dimensions of the gauge section, which is usually the critical cross section. Those series were produced from the same lot of 42CrMo4+QT high-strength steel and machined with identical nominal surface roughness $R_a = 0.8$ µm.

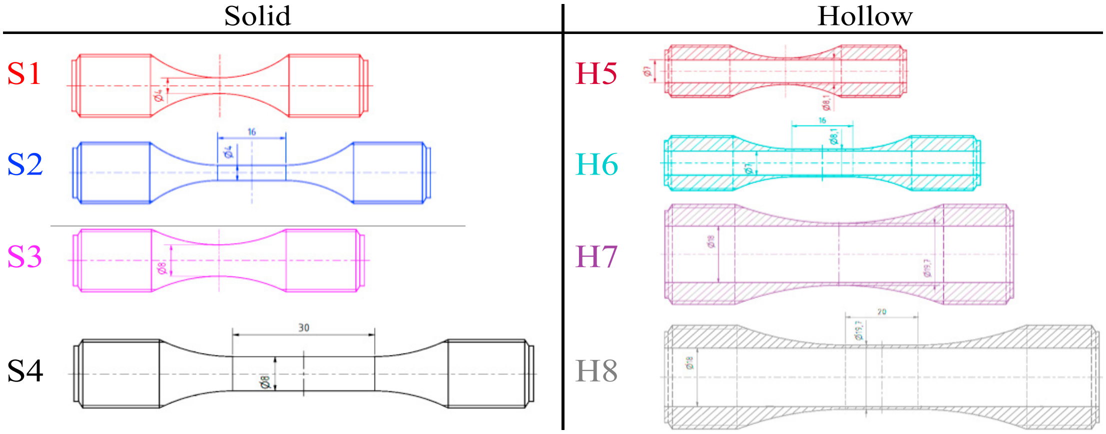


Fig. 1. Standard cylindrical specimens with dimensions of the central part: on the left solid S1 to S4 and on the right hollow H5 to H8.

Specimens were tested under fully reversed push-pull loadings using the Amsler HFP422 resonator fatigue machine, which works at the specimen's resonant frequency. The failure criterion was established by about 5% decrease of resonant frequency (for more details see Mžourek et al. (2022)).

The surface roughness, hardness and residual stresses were verified after the fatigue test. Although not documented here, some differences were observed mainly in the measured values of roughness and hardness. Since the experimental data refer to different roughness and hardness, a correction is applied to unify the data to common values of roughness and hardness in order to compare them. To make this correction, the proposal of FKM-Guideline (Rennert et al. (2012)) and Garwood et al. (1951) were applied on the fatigue limit considering the Castillo et al. (2009) regression model in Eq. (7). The experimental data after correction are shown in Fig. 2a for S1 to S4 series and in Fig. 2b for H5 to H8. Using those data, the S-N curves are estimated by applying the general CFC model as described in Section 2.1. This is a preliminary fatigue analysis that does not consider HSV and HSS concepts but forms the basis of the proposed model.

4.1. Fatigue curves when applying the CFC model

By applying the CFC model described in Section 2.1, and excluding run-outs, the S-N curve of each specimen series is obtained with 50% probability of failure. A good fitting is observed in all cases from S1 to S4 in Fig. 2a and H5 to H8 in Fig. 2b. Although the HSV and HSS concepts are not considered in the CFC model, these good fittings are important to make some considerations aimed at the application of the proposed model based on HSV and HSS.

According to the HSV concept (with 80% stress fraction), it is expected that S-N curves would shift downward as the specimen's volume increases. This trend is quite achieved in case of solid specimens from S1 to S4 in Fig. 2a, whereas it is not followed by hollow specimens from H5 to H8. In fact, H7 departs from this trend. Causes for this discrepancy are being investigated in an ongoing research. For this reason, from now on, H7 series will not be considered.

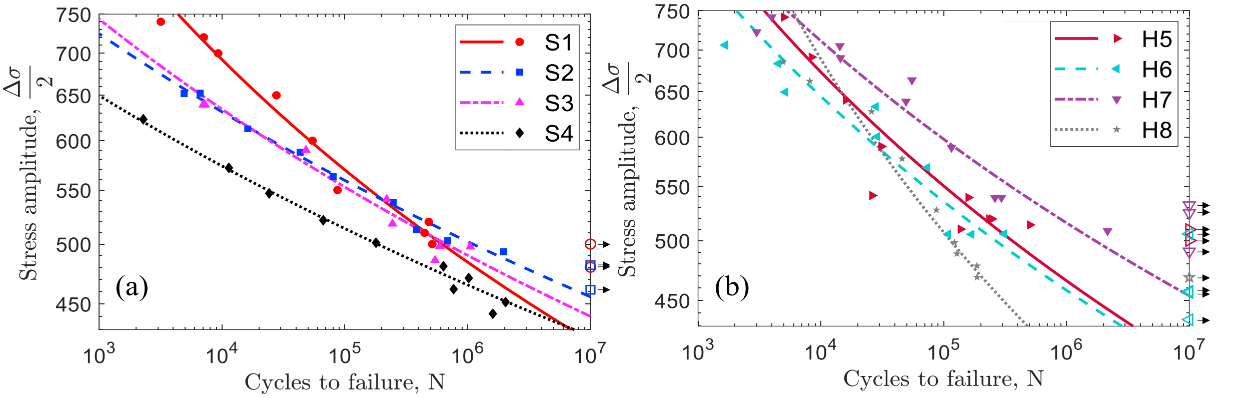


Fig. 2. Fatigue data and S-N curves without run-outs when applying CFC model: (a) S1 to S4 series and (b) H5 to H8 series.

4.2. Applying the proposed fatigue model

Experimental data were also fitted by the proposed fatigue model based on HSV and HSS. These models require the critical volumes V and surface areas S, which were computed using a finite element (FE) analysis. Axisymmetric FE models (S1 to S4 and H4 to H8) were built and run in Ansys software to obtain the stress distributions from which the values of V and S were determined as a function of the maximum stress fraction, see Fig. 3.

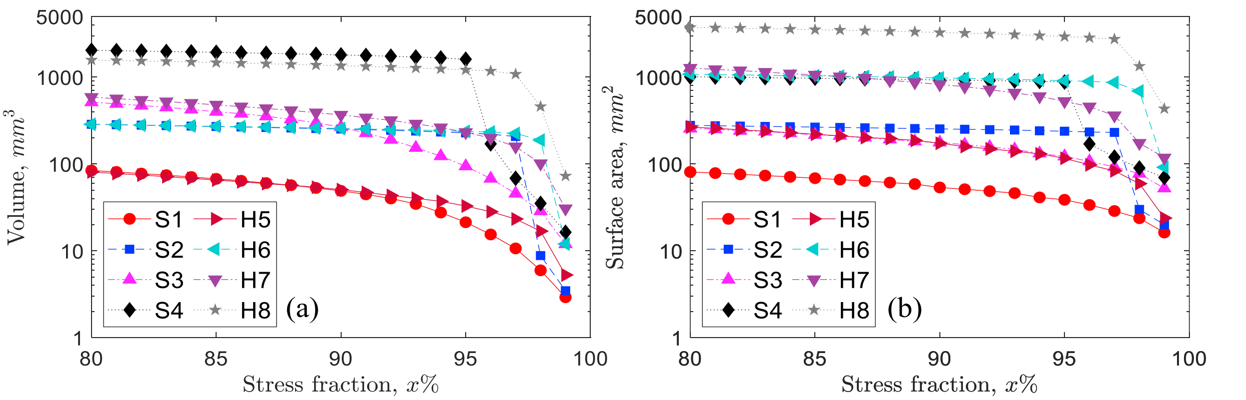


Fig. 3. (a) HSV and (b) HSS of S1 to S4 and H5 to H8 series if varying the stress fraction.

It is noted that the volume and surface area of all series decrease with stress fraction. From 95% on, a smooth decreasing trend of volume and surface is no longer observed for specimens with longer central prismatic section, see e.g. a quick decrease of V and S in case of S2 from 97% to 98% and S4 from 95% to 96%. For the case of hollow specimens, H6 and H8, this decrease in volume and surface is not as severe as for solid ones, see H6 from 98% to 99% and H8 from 97% to 98%. Although not detailed here, this rapid decrease is due to the change in the stress distribution in the gauge section.

Using volume and surface area for 80%, 85%, 90% and 95% stress fraction (values commonly used), the proposed model is checked against experimental data. This checking is needed because the model requires identically and independently distributed (i.i.d.) random variables N^* (weakest link principle). Such hypotheses were verified by applying P-P plots and Kolmogorov-Smirnov statistical test with 5% significance level (Castillo and Fernández-Canteli (2009)). Initially, all specimen series were jointly analyzed by taking as a reference the volume and surface area of S1 series ($V_0 = 49$ mm^3 and $S_0 = 53.6$ mm^2 for 90% stress fraction). The results (not shown here) did not fulfill the assumptions of i.i.d. random variables for both volume and surface case. Owing to the low probability level attributed to S1 and H5 series by the statistical test, S1 and H5 series were not analyzed together with S2, S3, S4, H6 and H8 in a second run of the test. Indeed, S1 and H5 series may be below a threshold of the critical volume or the critical surface area, see Castillo and Fernández-Canteli (2009), as they have the smallest HSVs and HSSs.

At this stage, the series S2, S3, S4, H6 and H8 were analyzed together by taking the S2 specimen's volume and surface as a reference ($V_0 = 252.4$ mm³ and $S_0 = 253.4$ mm² for 90% stress fraction). The results (not shown here) of P-P plots show now a good agreement and the results (also not shown) of Kolmogorov-Smirnov statistical test confirm that the probability levels of S2, S3, S4, H6 and H8 series are greater than 5%.

After checking that the proposed model agrees with experimental data, volume V with 90% stress fraction is chosen, such as in Sonsino and Moosbrugger (2008), to demonstrate the regression equation with $p = 0.5$:

$$\ln N = \frac{33.466 - \ln V}{1.681 \cdot 10^{-3} \cdot \Delta\sigma} - 3.381 \quad \text{for } N \geq 47 \tag{13}$$

Eq. (13) may be used for any specimen up to $V_{max} = 1802$ mm³ under the hypothesis of identically and independently distributed volumes, which may hold for large specimens ($V \geq V_0$, where $V_0 = 252.4$ mm³), but possibly not for small specimens ($V \leq V_0$) (Castillo and Fernández-Canteli (2009)).

Considering the volume V with 90% stress fraction, the proposed S-N curves with 50% probability of failure are obtained for each series by varying the volume in Eq. (13). Fig. 4a shows the results of S2, S3 and S4 series (solid specimens), while Fig. 4b shows the S-N curves of H6 and H8 series (hollow specimens).

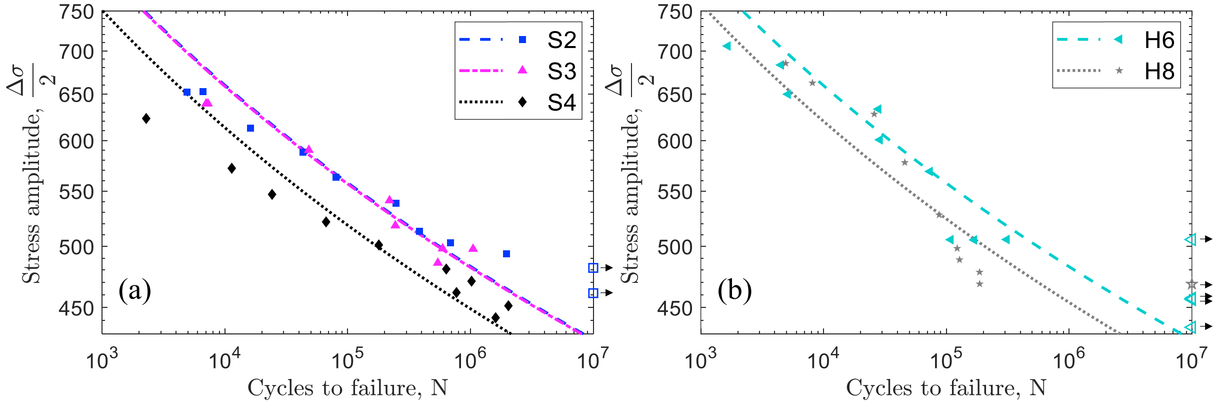
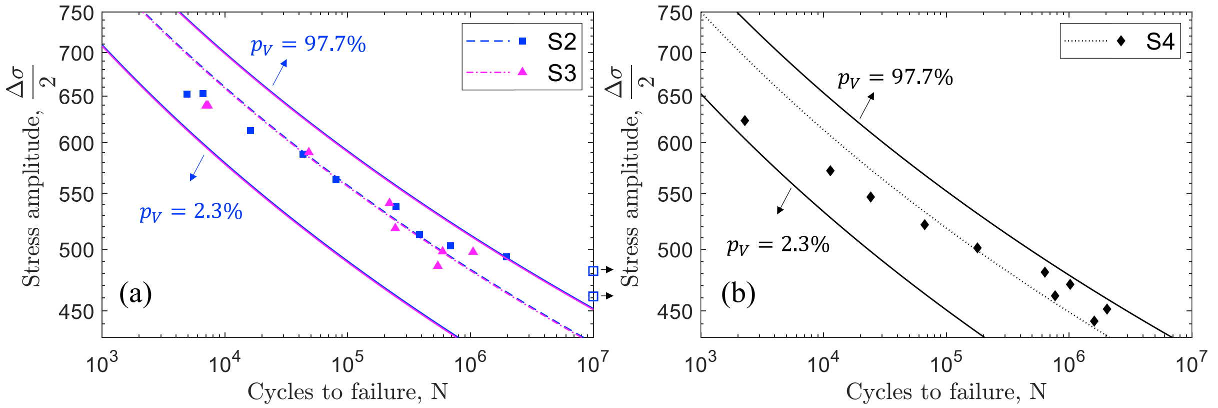


Fig. 4. Fatigue data and S-N curves without run-outs when applying the proposed model: (a) S2 to S4 series and (b) H6 and H8 series.

The fit is quite good for all S-N curves, especially in the case of S2 and S3 series, which are visually overlapped. An exception is the S-N curve of H8 series, which shows a different slope compared the one in Fig. 2. However, the data points in H8 case are not too far from the proposed S-N curve with 50% probability of failure. In any case, it is interesting to make use of the proposed model to estimate S-N curves for other percentile.

The S-N curves, estimated without run-outs and for 2.3%, 50% and 97.7% probability of failure, are presented in Fig. 5 (S2 and S3 series in Fig. 5a, S4 in Fig. 5b, H6 in Fig. 5c and H8 in Fig. 5d).



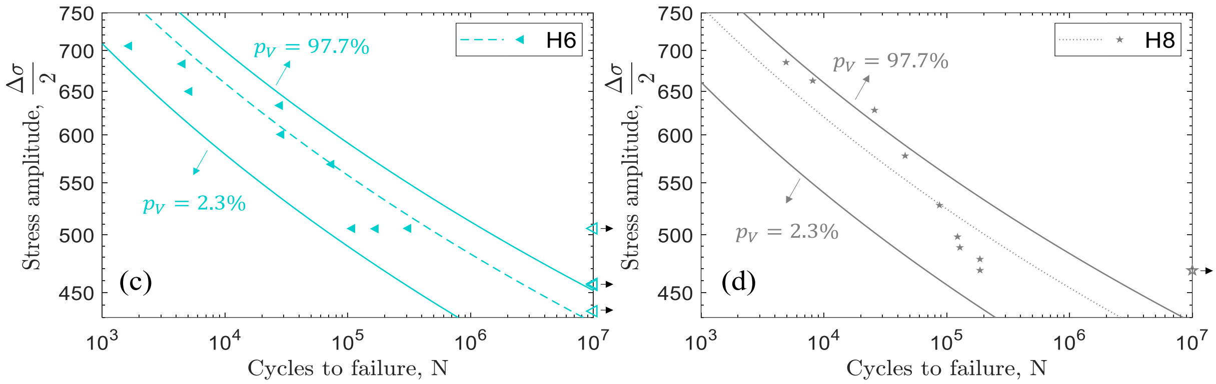


Fig. 5. S-N curves of (a) S2, S3, (b) S4, (c) H6 and (d) H8 when applying the proposed model for 2.3%, 50% and 97.7% probability of failure.

All S-N scatter bands have approximately the same width and all data points lie within the range of 2.3% and 97.7% probability of failure, except for one data point from H8 series. This data point refers to a percentile higher than $p = 0.977$. In any case, in engineering applications one typically considers a characteristic S-N curve referred to the lower probability of failure (2.3%), respect of which all data points in Fig. 5 fall on the right side – this means that the characteristics S-N curve here estimated is on the safe side.

5. Conclusions

The paper developed a statistical S-N model to account for the size effect on fatigue life. The model extended the Castillo et al. (2009) model to the case of the highly-stressed volume and the highly-stressed surface concepts. To validate the proposed model, experimental fatigue tests were performed using solid and hollow unnotched specimens made of 42CrMo4+QT steel. After checking that the proposed model agrees with experimental fatigue data, the model allowed the S-N curve to be used for any specimen of the highly-stressed volume up to 1802 mm^3. An S-N expression was also provided to be used in fatigue analysis without the necessity of performing a non-linear optimization.

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