"Quaderni di Ricerca in Didattica" N.5, 2022
G.R.I.M. (Department of Mathematics and Computer Science, University of Palermo, Italy)

Cooperative learning processes in solving tasks related to a game theory activity

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Abstract. The purpose of this paper is focused on analysis of the implementation of problem solving and argumentation processes in game theory activities. In particular, we studied the effectiveness of groupwork solution processes and the planning of strategies to solve the tasks. Moreover, we studied group and individual performances. It was possible to highlight how collective work can help produce a better performance and how it influences the emotions of students. Collective activities make it possible to build a shared knowledge and to support the development of individual metacognition.

Sunto. Questa ricerca ha come scopo l'analisi dell'implementazione dei processi di problem solving e di argomentazione in attività di teoria dei giochi. In particolare, è stata studiata l'efficacia del lavoro di gruppo nei processi risolutivi e nella pianificazione di strategie per risolvere i problemi. Inoltre, sono state analizzate le performance di gruppo e quelle individuali: è stato possibile evidenziare come il lavoro collettivo porti a una resa migliore e in che modo le emozioni degli studenti vengano influenzate dal lavoro di gruppo. Le attività collettive permettono di costruire una conoscenza condivisa e di sostenere lo sviluppo della metacognizione individuale.

1. Introduction

The focus of this research is to analyse how group work can influence problem solving and argumentation processes. These are essential elements in many educational systems (i. e., OECD PISA, 2021) but sometimes they are undervalued in school activity. Di Martino's research shows that students usually apply repetitive procedures to solve mathematical tasks. He believes that it is very important to change this approach and to propose non-standard problems to the students (Di Martino, 2017). Moreover, he highlights three criteria for determining whether a problem is non-standard for students:

- not knowing the resolution strategy is to be used during solving;
- the choice of multiple strategies to solve the problem;
- an unfamiliar context.

One aim of this work is to study the students' ability to apply their knowledge in an unfamiliar setting. Therefore, we chose game theory as the setting for our project, an activity which is not usually found in school curricula (Antonini, 2019). This unfamiliar setting allows development of the problem-solving and argumentation processes (Cramer, 2014).

We created two game theory tasks, a cooperative game and a non-cooperative game. In the first one, the players of the game can reach agreements with each other, while in the second one the players cannot collaborate. There was no single strategy to solve the tasks, which allowed us to explore and to analyse different approaches activated by the students. Moreover, the tasks were solved in groups and individually; it was possible to study the different methods used to tackle them. In particular, we studied the metacognitive activities that are triggered during the problem solving. We analysed how the collective metacognition induces a better performance than individual metacognition. Thanks to comparing and explaining different points of view, students were able to understand better and find more solutions and more strategies to tackle tasks. Our attention is focused on the way students planned the strategy to solve the tasks and their ability to justify their answers. For this purpose, we analysed the argumentations of the students during the group and the collective discussion and planning actions carried out during the problem solving. We used the theory of Kagel and Cooper to interpret the group and individual performances and the Habermas' construct to analyse the students' argumentations. Moreover, thanks to Di Martino and Zan's research, we were also able to study the impact of unfamiliar activities on students' view of mathematics and their emotional disposition, an aspect which was analysed in depth in a previous paper (Viola & Gambini, 2022).

2. Theoretical framework

In this research, the attention was on problem solving and argumentation processes, which are also objectives of the PISA (Programme for International Student Assessment) survey. PISA is an international survey conducted by the OECD (Organization for Economic Cooperation and Development) every three years. The purpose of the PISA survey is to assess a student's ability to apply their knowledge to real-life problems. This survey focuses on "mathematical reasoning", which is the ability to produce logical reasoning and arguments that can justify actions taken. According to the PISA 2022 Mathematics Framework (OECD PISA, 2021), tasks were designed to stimulate students in the formulation of strategies and the production of explanations.

To analyse the argumentation process, we used Habermas' construct based on three elements: "epistemic rationality", "teleological rationality" and "communicative rationality" (Habermas, 1998).

- Epistemic rationality: regards an individual's knowledge and his/her ability to explain why certain statements are true. This rationality is linked to actions, and use of language is needed to represent the knowledge.
- Teleological rationality: regards actions and objectives, in particular the intentionality of the actions and reflective judgement about these. An individual acts rationally when he/she is aware of why he/she was successful and of the actions that led to success.
- Communicative rationality: regards the ability to argue and to communicate choices to others. Intentionality and a reflective attitude are essential elements of this rationality.

Habermas' theory allows us to analyse the argumentation process of students and to identify their intentional and reflective attitudes, as well as the elements of rationality, present in their discussions (Boero & Planas, 2014). This is why the Habermas' construct was used to analyse the argumentations linked to the activity of problem solving (Martignone & Sabena, 2014). In game theory activities, the decisions of the players affect the final outcome and are important in determining the winning strategy. In this context, it is possible to analyse the key characteristics of the problem-solving process, such as the planning and control processes. Planning skill utilises an individual's ability to recognise and imagine. Thanks to this ability, it is possible to envisage the same fact/event in the past and in the future. In the case of a future fact, it is possible to identify the knowledge of the event (semantic future thinking) and the projection of the event into the future (episodic future thinking) (Atance & O'Neill, 2001).

Control processes, introduced by Schoenfeld, are essential in the problem-solving activities. He defines a "good solver" who is able to manage his/her abilities to achieve a goal, using the control processes. These processes are:

- checking the text of the problem for comprehension;
- planning a strategy;
- managing time;
- managing one's resources.

Moreover, Schoenfeld emphasises the importance of identifying (thorough the decision-making process) the objectives to be achieved. He highlights that individuals need to be aware of their capabilities in order to implement control processes. Analysing the causes of failure or successful, he introduces the individuals' beliefs, their perception of themselves and of mathematics. Beliefs, derived from the subject's past experiences with mathematics, can influence the individual's performance during the problem-solving process (Schoenfeld, 1983). The individual's perception of themselves, beliefs and attitude play an important role in the performance of an individual. These elements are affected by previous experiences and influence their approach to this discipline.

Research by Di Martino and Zan underlines the link between emotional and cognitive processes; they used a model to analyse the data collected from their studies. The model is composed of three dimensions: emotional disposition towards mathematics, vision of mathematics and perception of one's own abilities (Di Martino & Zan, 2011).

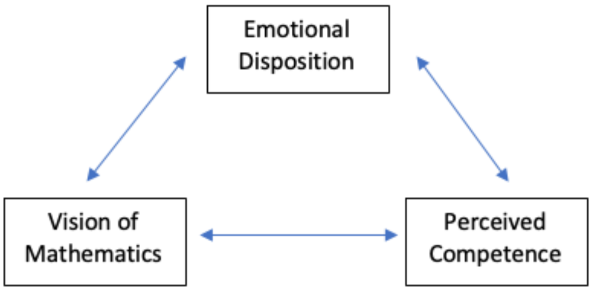

Figure 1: Di Martino-Zan three-dimensional model

Authors highlight that the emotional component of beliefs is very important because different individuals may feel different emotions arising from the same beliefs. The link between emotions and beliefs is essential, because a negative approach to mathematics can be due to a negative emotional disposition. Moreover, they observed a link between emotional disposition, vision of mathematics and the perception of one's own competences. These features are influenced by the idea of success in mathematics. Di Martino and Zan underline three concepts of success:

- successful in terms of knowledge of the rules and correct application;
- successful in terms of perception of knowledge of rules' meaning and their connection;
- successful in terms of the scholar's success.

The idea of failure is often linked to negative emotional disposition and low perceived competences. In accordance with Weiner's theory (Weiner, 1986), we can divide the causes of failure into:

- local internal/local external: the subject influences local internal causes, while external characteristics affect local external causes;
- stable/instable: stability is given by the possibility of these causes changing over time;
- controllable/uncontrollable: these causes are influenced by the beliefs of the subjects.

Some students have failed in mathematics because they have a negative emotional disposition towards this discipline, but low perceived competence and negative emotional disposition are not dependent dimensions. In fact, there are students who have a low perceived competence, but a positive emotional disposition and vice versa.

Arzarello and Sabena analysed control processes in the argumentation processes. They use the term "semiotic control" to identify the process regarding knowledge and decisions that play a role during choice, for example in the representation of a problem. Meanwhile,"theoretic control" indicates the process that has to be acted upon when there is a theoretical reference during an activity, such as property or theorem. In particular, students have to be able to interpret symbols, to find a link between symbols and the context of the problem and to produce an argumentation that can justify their actions, supported by theoretical foundations (Arzarello & Sabena, 2011).

An important aspect of the problem-solving process is metacognition, and how it differs from group to individual activities. Metacognition is the individual’s awareness of his/her learning process. The collective metacognition is provided by comparison and discussion between group members. They share ideas, identify a common goal and then select multiple strategies for action. Thanks to the contribution of each individual, it is possible to have a better understanding of the problem and to find more solutions.

The model of Tuckman and Jensen is composed of five phases, which are important in order to implement a good problem-solving process (Tuckman & Jensen, 1977):

- forming: happens when group members decide how to work
- storming: occurs when different ideas emerge
- norming: happens when the conflict between ideas is solved and there is a common strategy to adopt
- performing: occurs when group members work together to achieve the objective
- adjourning: occurs when the group check their work

To be successful in the group activity, the students need to develop a shared knowledge. The production of this knowledge is enabled by mental representations and shared ideas put forward by each group member. Shared knowledge can be built if the following actions are activated: orientation, planning, monitoring, evaluation and execution. Orientation precedes the planning activity; orientation and planning are essential to prevent students from performing by trial and error, allowing them to reflect on the activities and apply their knowledge to solve them. Orientation and planning include the identification of a strategy for action, time management, and planning objectives. Monitoring and evaluation help the students to analyse their actions during the resolution process and to interpret the results in the context of the problem. These activities allow control of the actions of each member of the group, and help to reduce errors. At the end, it is possible to elaborate and explain the processes carried out (Meijer, et al., 2006; Van der Stel, et al., 2010).

The collective metacognition is an extension of the individual metacognition: it divides the metacognitive responsibility among group members, improves cognitive processes of the individuals and facilitates the learning (Chiu, et al., 2009). During the group activity, group members may have different roles and each person’s work is reviewed by all group members. Therefore, through the communication between group members, the visibility of the cognitive and metacognitive processes is improved. The distribution of the metacognitive responsibilities allows for a reduction of errors and a focus on individual strengths, which can lead to an increase in problem-solving efficiency. Indeed, during the exploration of different points of view, students take part in socio-cognitive experiences which they address through metacognitive evaluations. Through socio-cognitive conflicts, students learn to evaluate their strategies and to find new ones.

During the group activities, group members have to reflect not only on their own mental processes, but also on the mental processes of the other members, because collaboration is essential to achieve the goal. Questions and explanations from group members help to build a shared knowledge and allow individual difficulties to be identified. In this way, the group work helps to develop individual metacognition; in fact, the groups perform better than individuals (Frith, 2012). Moreover, collective metacognition helps to reduce individual anxiety about failure, by dividing it among all group members. The existence of shared solutions increases the motivation to carry out mathematical tasks (Chiu, et al., 2009). During the group work, it is possible to have difficulties in communication and interaction between group members. These difficulties can damage the collective metacognition.

Behavioural Game Theory is a part of Behavioural Economics, which uses psychological features to anticipate the behaviour of players. In fact, individual features are able to influence the strategies to be applied. In particular, in game theory the analysis of individual behaviour aims at identifying the strategic evolution of the game. Kagel and Cooper's research shows that groups are able to act more strategically than individuals. This may be due to the communication between group members, which allows the thought processes of each individual to be made explicit (Cooper & Kagel, 2005).

3. Methodology

The tasks designed for this work are located in game theory. These tasks are characterised by the absence of one single strategy to solve them. It is not necessary to know advanced mathematics to solve these problems, which is why the tasks are proposed to high school students of different levels. The objectives that have guided the formulation of these tasks are:

- Stimulating the problem-solving process
- Production of argumentations to justify the strategies chosen
- Encouraging different points of view
- Promotion of group work
- Stimulating peer discussion
- Promotion of non-standard mathematical activities

This context should encourage a vision of mathematics that does not aim to produce a result but, rather, to activate problem-solving and the argumentation processes.

This experiment is composed of two tasks: a cooperative game and a non-cooperative game. In the first task, players can cooperate and find an agreement to reach a solution; students have to take into account different points of view to solve it. In the second task, the players cannot cooperate, because the objectives are in conflict. In accordance with the PISA 2022 Mathematics Framework (PISA, 2021), the tasks are designed to stimulate the production of argumentations and strategies in real-life contexts.

At the end of each task, an interview was conducted to identify difficulties encountered by the students during the resolution process, and to understand the motivations that guided their choice of strategy. Two questionnaires were proposed to the students: one questionnaire was administered before the first task, and the other after the second task. The purpose of the questionnaires is to investigate the relationship between students and mathematics, their view of this discipline and their emotions about mathematics. Moreover, we analysed the skills that students believe they need in order to be good at mathematics and which ones they think they actually have. The questionnaires adopt some open questions to give students the opportunity to express their emotions and thoughts, while other questions used rating scales from 1 to 10 on certain statements. The questionnaires were based on research studies by Di Martino and Zan (Di Martino & Zan, 2010; Di Martino & Zan, 2011), which highlight the fact that through the answers expressed by students, it is possible to find more features that can influence their beliefs, vision of mathematics and approach to problematic situations.

The tasks were submitted to three classes of high school students in grades 9, 11 and 13 of the Italian education system, and a group composed of mathematicians and non-mathematicians. There were 81 high school students, divided as follows:

- One grade 13 class (18-year-old students) consisting of 29 students. In this class, half of the students carried out tasks individually
- One grade 11 class (16-year-old students) consisting of 25 students;

- One grade 9 class (14-year-old students) consisting of 27 students.

The group of mathematicians and non-mathematicians was composed of 16 individuals: 10 students from a mathematics master's degree course and 6 university students not following any mathematical course. This group carried out the tasks individually, but also as part of the grade 13 class. In fact, due to the Covid-19 pandemic regulations, the class was divided into two groups, half present in the school and half at home. Those who were remotely located solved the tasks in groups, while those who were in the classroom worked individually. In the second task, the roles were reversed.

Due to the Covid-19 pandemic, we interacted with participants in the experiments entirely remotely, using platforms such as Google Meet and Zoom. Thanks to these platforms, virtual rooms were created, and it was possible to leave the students free to collaborate and work together.

The experiment can be summarised as follows:

1. first questionnaire: to analyse the relationship between students and mathematics, their view of mathematics and their emotions about this discipline
2. task: students solved the task in groups or individually in one hour
3. collective discussion: to analyse the proposed solutions and stimulate comparison between pairs
4. interviews: to understand the difficulties encountered during the resolution process, the strategies implemented and the building of the argumentations

   Steps 2, 3 and 4 were repeated for the second task, after which step 5 was conducted.

5. final questionnaire: to analyse any possible change in their view of mathematics, their emotional disposition after these activities.

The participants were not experts in game theory; we merely informed them about the type of game activity (cooperative or non-cooperative). This choice is justified by our wish to investigate their approach in real settings. During the resolution process, we observed the students' behaviour and noted their statements and actions. All participants were interviewed in order to understand the strategies chosen and the motivations that guided their resolution processes. The questionnaires were only proposed to high school students, because we wanted to analyse how emotional components affect resolution processes.

The questionnaires are structured in two parts: open questions and closed questions. In the first questionnaire, the open questions analysed the following aspects: their relationship with mathematics, emotions towards this discipline, the abilities they believe they have and the most interesting aspects of mathematics. In the second questionnaire, the open questions investigated: how easy or difficult students found the tasks solved, changes in their view of mathematics, interesting aspects of the tasks, usefulness of this experience in future situations and the interaction between the groups. The closed questions focused on the following aspects: usefulness of mathematics, creativity in mathematical activity, the aim of mathematical activity, influence of good results on the judgment of this discipline and the idea of mathematics linked only to rules. Questionnaires were submitted via the Google Modules platform

4. Analysis of the students' solutions

First task

The text is the following: *"In a shopping centre, there are three shops, AltaModa, BlueJeans and CookLover,* *which need a new lighting contract. They have been offered several alternatives: if they take out the contract individually, they will pay €250, €200 and €350 per month respectively; if they decide to take out an overall contract, they will pay €600; alternatively, if they agree in pairs, the prices will be €350 per month for AltaModa and BlueJeans, €450 per month for AltaModa and CookLover and €420 per month for BlueJeans and CookLover.*

Try to explain the offer and how the three shops could agree on the best offer. Give reasons for your answers."

In the table we can observe the solutions proposed by the subjects of this experiment.

Proposed solutions	**A**	**B**	**C**	**% groups**	**% individuals**
Equal division of the total contract	200	200	200	47%	65.5%
Proportional division of the total contract	187.50	150	262.50	31.5%	24%
A and C in pair, B individually	225	200	225	0%	3.5%
Saving of 25% to everyone	185	150	265	0%	3.5%
Division of the total contract: A and B in pair, C pays the rest	175	175	250	6%	0%
Division of the total contract: proportional division of the pair contract between A and B, C pays the rest	195	155	250	6%	0%
Equal division of the overall saving	183	134	283	6%	0%
A and B in pair, C individually	175	175	350	3.5%	3.5%

Table 1: Proposed solutions. A, B and C represent expenses for A, expenses for B and expenses for C. % groups and % individuals indicate the percentage of students' solutions.

We can observe that the majority of the students chose equal division of the total contract. In particular, this choice is more common among individuals than groups. These results are in agreement with the research of Kagel and Cooper (2005), who stated that individuals act less strategically than groups. During the first phase, students divided the total contract and pairwise contracts equally, in order to highlight the price for each shop. The equal division of the overall contract is the most intuitive answer and all identified this solution in the first stage. Afterwards, some individual students and more than 50% of the groups were able to move away from this idea, thinking of other solutions to this task.

Let us see what drives individuals to distribute expenses equally and what, instead, drives them to seek other solutions.

The choice of an equal division of the overall contract is often justified with the following statements:

Student 1: "I thought they were trying to meet each other's needs";
Student 2: "They all pay less than or equal to the single contract";

Student 3: "I thought that the best solution should be found for everyone and not for the single companies".

Such a solution does not guarantee that each shop’s savings will be optimised. There are, in fact, other solutions which give the second shop the possibility of making a greater saving than with the single contract.

In the case of the other solutions, however, questions and doubts arose which led to the implementation of different strategies, for example:

Student 4: "Should they behave in a selfish way?".

With this expression, the students wondered whether each shop should try simply to optimise its own savings or act by taking into consideration the other two.

Student 5: "BlueJeans does not like the overall contract, it might decide not to accept it";
Student 6: "It is not fair that everyone pays the same amount".

These expressions made it possible to question the choice of dividing the overall contract equally and led to the formulation of new hypotheses for solving this task. As reported in Schoenfeld’s research, those who approach the problem correctly set the goals to be achieved and choose the strategy to be adopted. In this case, the students understand that if the aim is to minimise the expense per shop, it is not possible to divide equally because BlueJeans would not obtain any saving with this solution. In this approach, students were able to analyse the problem from different points of view to understand which solution would be best.

Second task

The text is the following: *"In 2022, there will be the volleyball World Cup, and two television broadcasters (SuperVolley and LiveSport) have to divide television rights. The World Federation has created two television packages; each has a certain number of matches. A closed envelope method is used to allocate the packages. Supervolley has a budget of 5 million, while LiveSport has 2 million. We denote with (x,y) the division of the budget, x is the expected sum for the first package and y for the second; for example, we identify with (3,2) that 3 million will be bet on the first package and 2 million on the second. If the amount expressed by two broadcasters is the same, the package will not be assigned and a new allocation phase will be necessary. We indicate in a table the SuperVolley winnings:*

- *1 if it manages to take one package;*
- *2 if it manages to take two packages;*
- *0 if it manages to take one package and LiveSport takes the other one*

Down the column we have the division of the SuperVolley budget and the top row represents the division of the LiveSport budget.

S I L	(2,0)	(1,1)	(0,2)
(5,0)	1	0	0
(4,1)	2	1	0
(3,2)	2	2	1
(2,3)	1	2	2
(1,4)	0	1	2
(0,5)	0	0	1

Table 2: Representation of SuperVolley winnings.

Indicate if there is any move that Livesport and SuperVolley should avoid and which one should be more convenient. Justify your answers. "

We summarise the proposed solution by the students involved in the experiment.

Proposed solutions	**Convenient moves for S**	**Inconvenient moves for S**	**Convenient moves for L**	**Inconvenient moves for L**
Equal distribution per S and invariant for L	(3.2), (2.3)	(5.0),(0.5)	None	None
Equal distribution per S and not for L	(3.2), (2.3)	(5.0),(0.5)	(2.0), (0.2)	(1.1)
Equal distribution per S and for L	(3.2), (2.3)	(5.0),(0.5)	(1.1)	(2.0), (0.2)
Non equal distribution for S and for L	(4.1), (1.4)	(5.0),(0.5)	(2.0), (0.2)	(1.1)
Non equal distribution for S and for L (different inconvenient moves for S	(4.1), (1.4)	(3.2), (2.3)	(2.0), (0.2)	(1.1)

Table 3: Students solutions

The majority of the groups and individuals identified the best situations for two broadcasters: the allocation of two packages is the best situation for Supervolley; LiveSport has a smaller budget and therefore can only hope to be allocated two packages while another goes to SuperVolley. In this case the subjects of the experiment tried to optimise the profit of each broadcaster.

Almost all participants highlighted that the most inconvenient choice for SuperVolley is to bet the whole budget on one package. Regarding these considerations, some students stated:

Student 7: "LiveSport cannot win the two packages, whatever it does";
Student 8: "We have to look at the 0 in the table to see what LiveSport's winnings are".
Student 9: "If SuperVolley bets the whole budget on one package, it has no chance to take the second one".

Some subjects used probability to calculate the chance of receiving packages, in this way, they pointed out that LiveSport has the same probability to acquire one package. In this case, the goal of optimisinga win by the broadcasters leads to assessing the chance of winning in each situation. Other students, however, stated:

Student 10: "Moves (2.0) and (0.2) are efficient if you want to avoid the allocation of two packages";
Student 11: "SuperVolley will not put the whole budget on one package, therefore for LiveSport the moves (2.0) and (0.2) are more convenient";
Student 12: "LiveSport can spendtheir whole budget to avoid allocating one package to SuperVolley".

This approach is based on the idea of maximising one's own winning while trying to minimise the second player's chance of winning. Students highlight the strategy for players to optimise his/her own result, knowing that the other player will try to minimise his/her own success.

Different points of view were considered during the group discussions. Peer comparison played an important role because students explored different ways to achieve the goals and different possible solutions. In fact, the interaction between group members allowed them to identify more strategies to solve the tasks.

5. Discussion

During group work, it is possible to observe the communication capacity of each individual, particularly the ability to justify one's choices and to be able to make people understand. During the resolution of the tasks, students used some examples to reveal the strategy chosen or to explain the text of the problem.

While carrying out the tasks, the students showed that they were able to rationally justify the strategy used (epistemic rationality), referring to their knowledge, and aware of why they were successful with a certain strategy (teleologic rationality), as can be seen from these statements:

Student 13: "The proportions maintain the same initial disparity";
Student 14: "We started from the fact that it was not fair to make everyone pay the same price, so we decided, instead of dividing the expenses, to divide the overall saving";
Student 15: "We need to see which moves can cause a loss for broadcasters or give them a high probability of being losers".

For example, according to the statement by student 14, a fixed objective and the awareness that this strategy would allow a better offer for each shop are evident.

Many students also showed that they were able to explain their ideas and make them understood by others (communicative rationality), as in the case of using examples to make understanding easier for their peers.

Students used their knowledges to carry out the tasks, such as probability or proportions; this aspect regards the theoretical control. We can observe some examples of this through the statements of the students:

Student 16: "We need to calculate the SuperVolley's probability of winning and LiveSport's probability of winning";
Student 17: "If we use proportions to see how much they would pay in this case, instead of dividing by three…".

It can be seen that students were able to implement the process of "episodic future thinking" to project the event into the future through the theoretical tools. The choice of the strategy and its implementation belong to the process of semiotic control. Students, in fact, have to be able to justify the chosen strategy with logical reasoning. While solving the tasks, students implemented the following semiotic actions:

- Interpretation of the symbology used, especially in the second task
- Identification of the link between symbols and data presented with the context of the problem
- Justification of the strategies implemented, through theoretical foundations

The analysis of the group work highlighted some interesting aspects. Thanks to answers to the interviews and the questionnaires, it is possible to see that the possibility of collaboration was appreciated by the students. In the grade 13 class, we were able to compare the impressions of students regarding the individual approach and the group activity. For example, when we asked the students to describe the interaction between the groups, they stated:

Student 18: "The group work helps to stimulate discussion and create different points of view";
Student 19: "I think it is possible to achieve more through group work than individually";
Student 20: "Initially I did not understand very well, but thanks to my classmates I understood the task";
Student 21: "Some ideas we had, though individually, were changed by the analysis of the whole group, and others emerged thanks to the collective activity".

We can observe that debate with others is useful to understand the tasks and find a shared solution. In fact, during the group discussions, those who understood the task explained it to the others. Some students stated that the initial idea of solving the task changed thanks to dialogue with their peers. This emerged from the results of the tasks and the group discussion. This element is very important to allow the production of different strategies and solutions. Moreover, it is possible to see how the collective activity helps to stimulate collaboration between students in order to reach the common goal.

From statements of student 21, it is possible to highlight how discussion between peers leads to more strategies and better performance during task resolution, as shown by Frith in his study (Frith, 2012). In fact, in the group activity we can observe the application of metacognitive activities, for example, planning, monitoring and evaluation. These elements allow students to build a shared knowledge.

As reported in Kagel and Cooper's research, we can underline that the groups acted more strategically than individually (Cooper & Kagel, 2005). This is evident in the discussion between group members, which highlights different ways of interpreting the tasks and different possible strategies. In the groups, the strategy proposed by each group member was analysed in depth in terms of its validity and eventual critical points. Moreover, the solutions proposed were interpreted in the problem context.

The groups were able to implement the stages of Tuckman and Jensen's model (Tuckman & Jensen, 1977). They, in fact, organised their group work in the following way: collective reading of the text, conflict between different ideas, planning of the solution strategy, construction of a shared solution and evaluation of the work done.

Furthermore, we analysed the emotional components; from questionnaire answers, we can observe that collective work has a positive impact on the emotional sphere. We asked the students which emotions they felt about mathematics and what was their view of mathematics before and after the tasks. Some students, before the task, stated:

Student 22: "I get satisfaction when I am able to solve the exercises";
Student 23: "I have contrasting emotions about mathematics, like love and hate".

We can observe that there is a link between the idea of success and a positive emotional disposition, as reported by Di Martino and Zan in their research (Di Martino & Zan, 2010; Di Martino & Zan, 2011). In fact, 75% of the students gave a grade of agreement greater than or equal to 6 to the statement: "I like mathematics, because I have good results".

After the tasks, the students with contrasting emotions about mathematics expressed a positive judgment regarding the group work. In particular, the students' view of mathematics has changed. For example, some students stated:

Student 24: "I changed my approach, I acquired a different way of performing the tasks";
Student 25: "I had the opportunity to see that there are problems with different solutions";
Student 26: "We often think that mathematics is a science with certain rules, but in these tasks, we saw that there are different possible strategies to implement"

We can see that the students changed their view of mathematics after these tasks. A good view of mathematics and a positive emotional disposition are essential elements to achieve a good performance in the problem-solving process. The unfamiliar activities helped the students to see a different way to do mathematics and to see problems without one single strategy to solve the task or one single possible solution. These tasks allow them to have a greater perception of the useful of mathematics in a real-life context. They, in fact, believe that these tasks will be useful in the future, because they reveal different ways of reasoning.

6. Conclusion

In this research we analysed the ability of students to implement problem-solving and argumentation processes in unfamiliar activities. It was possible to observe their capacity to plan and to implement strategies, identify goals, organise work and manage group activity. In particular, we studied the effectiveness of group work and how collective activity can help them to perform better during the resolution process. In fact, through discussion among group members and explanation of different points of view, in most cases the group work allowed them to find more strategies to solve the task. During the group work, individual metacognition was developed thanks to the collective metacognition, because students have to be aware not only of their own reasoning process but also of the processes of their group members. Difficulties encountered by the students were overcome with collaboration between pairs, and the explanations and examples of their classmates. At the end of the group work, through the activations of collective metacognition activities, students were able to build a shared knowledge.

Moreover, emotions and beliefs involved in mathematical activities were analysed in this work. This is justified by the desire to understand the influence of emotional components on the approach to mathematics. Indeed, one aim of this research is to study how collective activity can improve the performance of students in mathematical tasks. Group work helps to reduce anxiety and the fear of failure, distributing responsibility among group members. This can create a positive emotional disposition, which increases the motivation to carry out mathematical tasks. In fact, we can observe that also students with contrasting emotions evaluated the group work positively.

The comparison of groups and individuals allowed us to study the use of different approaches. In particular, individuals showed more difficulties in understanding the task and finding different strategies, stopping at the initial intuition. On the other hand, in the groups each member outlined his/her own ideas, allowing discussion and pursuit of a common solution that was often different from the first intuition. The students themselves stated that the work group fosters a better performance than individual activity:

Student 24: "I think that by working well in groups you can do more than you would do alone";

From the student's statement, we can highlight how the group work (when carried out efficiently) guided the students to reach a common goal and build a shared knowledge, which is essential for the metacognitive process. In some groups, due to low student participation, group work did not lead to the construction of shared knowledge and the metacognitive process was compromised.

These results show that cooperative learning, if done correctly, can improve the students' approach to mathematics and their performance in this discipline. Moreover, it can improve their emotional disposition and view of mathematics.

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