



Review

State of the art review on rattle index formulations for geartrain applications

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ABSTRACT

Gear rattle is a major source of noise, vibration, and harshness in automotive transmissions, arising from vibro-impact excitations of unloaded gear pairs caused by engine torque fluctuations and functional backlash. Several rattle indices have been proposed to provide an objective and quantitative characterization based on physical or signal-processing formulations. However, their interpretation and comparative applicability are often limited by heterogeneous modeling assumptions and simplified transmission layouts. This survey classifies rattle indices according to their underlying physical quantities and diagnostic objectives and evaluates their performance through numerical simulations under identical excitation conditions. The analysis distinguishes between indices aimed at detecting rattle onset and those capable of tracking the evolution of tooth contact dynamics. A progressive validation framework is adopted, ranging from a single gear pair to a multi-stage transmission with multiple loose and idle gears. The results clarify the applicability of existing rattle indices and support informed metric selection for vibration and signal analysis in complex transmission systems.

1. Introduction

The continuous advancements witnessed within the automotive industry have led to an increased focus on acoustic comfort. With the widespread of electric powertrains, which are inherently quieter, gear transmissions have emerged as a prominent source of noise, both in terms of amplitude and subjective perception. In gear transmissions, the dominant noise component is typically associated with the meshing frequency. However, when gears operate in idle or lightly loaded conditions, driving torque fluctuations may promote tooth detachment thanks to the presence of backlash [24,25]. These repeated impacts excite structural resonances and generate the characteristic impulsive noise known as rattle.

Research on this phenomena, particularly the neutral or idling rattle observed in vehicles equipped with diesel engines, has advanced significantly since the early 1980s. Initial investigations primarily aimed to model the nonlinear dynamics of the meshing process [26–30] and to identify key parameters influencing rattle onset [31–34]. Sakai et al. in 1981 [35] proposed a five-degree-of-freedom torsional model that incorporated both clutch hysteresis and gear backlash nonlinearities, demonstrating that the phenomenon is mostly critical under idle conditions, with the gearbox in neutral. Seaman et al. [36] further refined the understanding of the phenomenon by distinguishing neutral rattle, occurring at standstill in neutral, from drive rattle, which appears during low-speed acceleration transients. The authors have developed the theory of “rattle threshold”, stating that rattle arises when the inertia torque on the driven gear exceeds the corresponding drag torque. Subsequent studies focused on nonlinear clutch dynamics, demonstrating

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Nomenclature

Apex

- + Instant after the impact
 - Instant before impact

Constants

- α Rayleigh damping coefficient proportional to the mass
 β Rayleigh damping coefficient proportional to the stiffness
 η_f Effective dynamic viscosity in a teeth pair flank conjunction
 η_{pet} Effective dynamic viscosity in the bearing conjunction
 A Excitation amplitude
 ω_e Excitation circular frequency
 τ Transmission ratio
 f_e Excitation Frequency
 f_s Sampling frequency
 H Amplitude of the periodic displacement of the driving gear
 J Polar inertia
 k_b Bearing stiffness
 m Mass
 N Number of scalar observations
 R_b Base radius
 t_d Defined time interval
 t_e Excitation period
 t_s Sampling period
 T_e External torque
 x_b Backlash
 Z Number of teeth

Functions

- $\ddot{\theta}$ Relative angular acceleration - according with Brancati [1]
 $\ddot{\theta}^*$ Relative angular acceleration - according with Brancati (variant) [1]
 $\Delta\tau$ Transmission ratio deviation [2]
 $\dot{q}$ Relative velocity - according with Donmez and kahraman [3,4]
 θ_i Rotation of the i-th gear
 $Contact(t)$ Contact detector function [2]
 g Phase modulation function [2]
 k_g Meshing stiffness
 TE Acronym - transmission error
 V Relative velocity - measured, according with Rigaud [5]
 W_a Acoustic power
 W_m Mechanical power induced by the impact
 x_i Radial displacement in x direction of the i-th gear
 x_r Dinamic Transmission error
 y_i Radial displacement in y direction of the i-th gear

Rattle Index

- β^* Singh - Inertia to drag torque ratio (approx) [6]
 ΔL Relative Structure-Borne Noise [7]
 Λ Rigaud - Inertial to drag force [5]
 E Wang and Shanguan [8–10]
 I_m Impulsion ratio - De La Cruz [11,12]
 $IGR2$ Brancati [1]
 $IGR1$ Brancati [13]
 $IGR3$ Brancati [14,15]
 L Structure-borne noise
 L_p Sound pressure level
 L_r Donmez - severity index [3,4,16]
 L_{β^*} Singh - Rattle Level index [6]
 $OFSCoh$ Order-Frequency Spectral Coherence [17]
 $PDNGS$ Robinette - Percent deviation normalized gear speed [18,19]

<i>PRI</i>	Pizzolante [20]
<i>RI</i>	Guo [21,22]
<i>RI1</i>	Cristofori 1 [23]
<i>RI2</i>	Cristofori 2 [23]
<i>RI3</i>	Cristofori 3 [23]
<i>TRD</i>	Transmission Ratio Deviation [2]

Subscript

0	flywheel or reference shaft
1	Driving (first) gear
2	Driven (idle) gear
<i>p</i>	Pinion
<i>rms</i>	Root Mean Square value

Variables

$\ddot{\theta}$	Angular acceleration
$\dot{\theta}$	Angular velocity
ω	Circular frequency
θ	Angular position
F_d	Drag force
I	Impulse amplitude
T_d	Drag torque
T_{drive}	Inertial Torque
X	Signal spectrum
x	Time domain signal
x_{filt}	Filtered signal
t	Time

that the torsional stiffness and hysteresis characteristics of multi-stage clutches strongly affect noise level hysteresis and jump phenomena [37–42]. In 1989 Singh et al. [6] developed both linear and nonlinear dynamic models of manual transmissions, introducing a dimensionless rattle index based on the ratio between inertia and frictional parameters. The analysis validated previous qualitative findings and provided quantitative design criteria: lower clutch stiffness, increased flywheel inertia, and higher drag torque were shown to effectively suppress neutral gear rattle. All these works originated from the definition of a proper indicator, namely the rattle index, specifically designed to detect the presence of rattle and its characteristics. Within this context, the most direct indicator is certainly represented by the Transmission Error (TE), which was used as a reference since the very first works on tooth detachment phenomena [26,31]. However, with the aim of having more general and quantitative indicators, several authors have proposed over time alternative indices capable of both detecting the presence of the phenomenon, and also estimating its severity in a continuous manner.

Starting from these very first works, the study of rattle has evolved mainly through two distinct approaches. The former focuses on identifying the operating conditions that lead to the onset of rattle, distinguishing safe regimes from those where impacts occur [39,43,44]. On the other hand, the second approach focuses on evaluating how the variation of parameters influences the intensity or severity of the phenomenon itself. As a matter of fact, in both approaches the analysis is conducted through simulations or experimental studies, in which a single parameter is varied and the system response is evaluated by means of a dedicated rattle index.

Among the first studies, Seaman et al. [36] defined the rattle threshold as the angular acceleration required for the inertial torque ($J\ddot{\theta}$) to exceed the drag torque (T_d), thus causing loss of contact. When the ratio between drag torque and inertial torque of the driven member is lower than 1, rattle occurs; otherwise, the condition is safe. Subsequent studies have extended or reformulated this criterion to include additional physical effects or quantitative measures. Satoru et al. [37] correlated the sound level with the tooth impact force through experiments and nonlinear simulations on a four-cylinder diesel engine at idle. The critical threshold was defined as the engine speed at which a sudden increase in rattle level, i.e. a jump phenomenon, occurred. Later Singh et al. [6] reformulated Seaman's criterion by introducing the parameter $\beta^*(t)$, the ratio between inertial and drag torques as a function of angular acceleration. The rattle threshold is expressed in terms of the RMS level L_{β^*} (in dB), with the transition from safe to rattle occurring when $|\beta^*(t)| = 1$ (0 dB). Barthod et al. [45] in 2007 defined the threshold as the amplitude of angular acceleration at which stable rattle occurs. Their analysis on a four-cylinder transmission showed that the onset of rattle is associated with a constant kinetic energy mainly determined by the second-order harmonic of excitation, while higher orders have minor influence. Acoustic, vibrational, and accelerometric measurements revealed a strong correlation between sound pressure and the RMS impacts on unloaded gears. De la Cruz et al. [11] incorporated tribological effects, maintaining Seaman's basic principle but reformulating it in terms of an impulsion ratio, proportional to the ratio of oil viscosities in tooth contact and between gear and shaft, allowing a more accurate description of idle gear dynamics. Around the same period, Robinette et al. [18] identified the rattle threshold experimentally,

by monitoring the Percent Deviation in Normalized Gear Speed, with the threshold corresponding to a sudden increase in its RMS value. Few years later Kadmiri et al. [46,47] adopted similar approaches. More recently Rigaud and Perret-Liaudet [5] experimentally validated Seaman's criterion using a spur gear pair under non-lubricated conditions, defining the threshold as the angular acceleration at which inertial and drag torques are equal. Donmez and Kahraman [3,16] introduced a severity index proportional to the sum of squared impact velocities over a time window, showing strong correlation with sound pressure levels and enabling noise prediction through a torsional model. The transition from safe to rattle is identified by comparing the TE with backlash limits. Finally, Cristofori and Mucchi [23] proposed three rattle indices based on external vibro-acoustic measurements-acoustic pressure and gearbox housing accelerations-allowing non-intrusive identification of the phenomenon. In run-up tests, the threshold was identified through waterfall spectra.

In parallel to these efforts, considerable research has been conducted to investigate the parameters governing rattle behavior, such as mean angular speed, applied torque, excitation amplitude, gear pair geometry [48–58]. Among the first, Brancati et al. [13] in 2005 developed a single-degree-of-freedom dynamic model for manual transmissions that incorporated the oil squeeze effect to account for lubricant damping in the meshing zone. Their simulations showed that oil presence reduces the high-frequency vibrations generated by tooth impacts. In particular, they observed a strong influence of oil viscosity on the rattle index, suggesting that higher viscosity effectively mitigates rattle severity, whereas neglecting lubrication leads to overestimation of the phenomenon. The same authors later extended their work to a complete driveline model [1] and experimental investigations comparing different lubrication regimes (oil jet, dry, and Boundary Lubrication) [14], confirming its key role. The effect of temperature on lubricant performance was subsequently analyzed by Theodossiadis et al. [59] and by De la Cruz et al. [12,59,60]. Their thermo-hydrodynamic analyses showed that increasing temperature (from 20 °C to 60 °C) reduces viscosity and hydrodynamic stiffness, promoting more irregular and severe rattle responses. More recently, Guo et al. [21,22] examined rattle through a dynamic model of a single gear pair including lubricant entrainment and squeeze effects within the backlash. These refinements revealed that higher dynamic viscosity significantly attenuates high-frequency impacts and reduces rattle severity even under double-side impacts. Torsional excitation at the input shaft is widely recognized as the primary cause of gear rattle. As early as 2009, Robinette et al. [19], demonstrated that increasing the amplitude of torsional excitation worsens rattle severity, using an analytical model of a manual transmission. Moreover, the authors found that larger backlash values delay the onset of rattle, but amplify its severity once impacts occur. In the same study, the authors observed that reducing the inertia of unloaded gears mitigates rattle onset, in agreement with the principle that rattle occurs when the inertial torque exceeds the resisting drag torque. Conversely, Bile et al. [61] found that, under established rattling conditions, lower inertia increases their proposed rattle index, indicating that higher inertia helps reduce severity. Geometric and contact parameters were also found to be influential. Bozca and Fietkau [62], studying a five-speed helical gearbox, reported that increasing parameters such as module and number of teeth leads to higher rattle noise, while Bile et al. [61] showed that higher contact stiffness, often associated with such geometric modifications, aggravates the phenomenon. A more recent study conducted by Fan et al. [63] proposes a methodology for developing high-precision digital models, improving their ability to reconstruct frequency modulations. In this study, the authors demonstrate that the model is highly sensitive to variations in bearing stiffness and eccentricity. Finally, rattle can also be self-excited by internal transmission dynamics, even in the absence of external torsional excitation. Rocca and Russo [64] incorporated the periodic oscillations of backlash limits induced by TE, confirming that it acts as an additional periodic excitation capable of triggering rattle. A recent study on multi-stage planetary systems conducted by Mo et al. [65] highlights how back-side contact becomes significant when the system is close to resonance, and how the contact stiffness and damping coefficient can effectively reduce its amplitude, thereby improving the overall stability of the system. Furthermore, Autiero et al. [66] have shown that geometric parameters such as tip relief and profile barreling play an important role in modifying Transmission Error and, consequently, the dynamic response. Most recently, D'Angelo et al. [67] proposed a framework for analysing the propagation of uncertainties arising from simplifying assumptions in multibody or lumped-parameter models, demonstrating how the integration of interdependencies between input parameters is essential to ensure the accuracy and robustness of dynamic predictions. Beyond the studies already discussed, additional investigations addressing the parametric sensitivity of rattle severity are reported in [8,53,68–72], yielding results broadly consistent with the findings of the aforementioned works.

Since rattle is a torsional phenomenon mainly, its analysis requires the evaluation of angular quantities such as rotational speed, acceleration and torque. In the experimental context, however, the majority of this information is obtained indirectly through angular velocity measurements performed on test rigs where the excitation is fully controlled [61,73]. Additionally, the design of these rigs provides ample space for the installation of sophisticated lubrication circuits [14,15,64], as well as high-speed cameras that facilitate the identification of backlash and the measurement of contact/free-flight durations [5]. Furthermore, some of these benches permit the adjustment of the wheelbase by means of spindle assemblies, thus covering a wide range of backlash values [3]. In certain instances, the original transmission is utilized, with all potential additional noise sources being eliminated through the elimination of the synchronizing mechanisms, leaving only a pair of gears [45]. Conversely, other authors have analysed parallel shaft transmissions, where it is possible to measure the instantaneous speed of each idler gear by installing Hall-effect magnetic sensors through a hole drilled in the gearbox [18,21]. In order to guarantee optimal quality of measurements, many authors adopted the use of encoders mounted directly on the gear pair under study [46,47]. However, this solution is rarely employed in industrial environments due to operating conditions or the complexity of the driveline layout. In this framework, some works have adopted the use of laser vibrometers placed directly on the input drive shaft [12]. In the aforementioned experimental approaches, the use of sensors external to the transmission is also envisaged, such as accelerometers mounted on the transmission case or microphones employed to characterise the radiated noise or validate the numerical model [74–77]. In the work by Cristofori et al. [23], for instance, the authors utilised accelerometers and microphones installed on the gearbox case to identify rattle conditions without the necessity of intervening in the

internal system. This methodology facilitates the monitoring of the phenomenon in a non-invasive manner, a particular advantage in complex applications where the installation of direct sensors on the gears would be difficult.

Given the wide range of studies conducted over the past decades, the current literature presents a remarkably diverse set of rattle indices, each developed and validated on different transmission layouts and under specific operating conditions. This heterogeneity, while valuable in terms of methodological variety, makes it difficult to establish direct comparisons among indices and to identify the most suitable ones for specific applications. The present work therefore aims not only to collect and describe the main rattle indices proposed in the literature, but also includes a dedicated numerical comparison performed within a common framework, with a view to assessing and highlighting the characteristics and limitations of each index, assessing their ability to detect the phenomenon and quantify its severity under comparable conditions. The numerical analysis therefore complements the review activity by enabling a more consistent assessment of the characteristics and applicability of the considered indices. Firstly the comparison is carried out with respect to a single gear pair, then to a three-gear train including an idler, and finally to a complete multi-speed transmission. In particular, the gear pair geometry corresponds to the configuration already investigated in several published works (e.g. [20,26,78]), selected here in order to maximize comparability with existing studies. Conversely, the multi-speed transmission model employed has been validated against experimental data concerning the angular velocities of the gears, thereby ensuring the reliability of the numerical predictions. This approach allows to evaluate the influence of system complexity on the performance and readability of each index. The ultimate goal is to provide a unified reference for researchers and practitioners, offering a clearer understanding of the relationships between the various indices and supporting a more informed selection of suitable metrics for future studies on gear rattle.

The paper is structured as follows: in the next section the theoretical framework at the basis of the various rattle index is described, while in sections 3 and 4 rattle indices properties are discussed by focusing on the results obtained from different driveline layouts, i.e. a single gear pair, a three gear branch and a complete double shaft gear transmission. Eventually, last Section 5 is devoted to concluding remarks.

2. Theoretical background: procedures, formulations and critical aspects

The present section is focused on detailing the theoretical framework and the practical features of the rattle indices defined throughout the literature survey. For the ease of the comparison, the indices are classified with respect to the physical quantity adopted for their formulation. Based on this approach, three groups are defined: rotational-quantity based RIs, i.e. rattle indices built up on processing angular displacement, velocity or acceleration; generalized force RIs, i.e. rattle indices obtained by processing the meshing force or by isolating the latter from the equations of motion and expressing it as a ratio between torques; and external measurements based RIs, i.e. rattle indices that estimate structure-borne noise (SBN) based on signal processing from the accelerometer installed on the gearbox housing.

The search was conducted in the Scopus and Web of Science databases using keywords such as “gear rattle”, “rattle noise”, “nonlinear dynamics”, or even “experimental investigation”, “numerical analysis”, “dynamic behavior”, “vibro-impact” and “gear dynamics”, with the aim of identifying the authors who have contributed to defining or using RI, thereby identifying the phenomenon; consequently, this review excludes articles that focus on characterizing the dynamics of toothed, rattle, or hammering routes unless they also involve the definition of an indicator.

2.1. Rotational quantities based rattle indices

According to the literature, the predominant cause of rattle phenomena may be attributed to external factors, such as the amplitude and frequency of the motor's torsional oscillations. Studies demonstrated that there exists an interaction between the torsional vibration generated by torque fluctuations of the motor, which are typically characterised by a low frequency content, and the vibratory behavior of idle gears, which is marked by high frequency impacts [59,70]. Acceleration, being directly related to impact force, may be an indicator of impact severity and, consequently, of the noise produced. This consideration has led several authors to assume that the ratio between the Root Mean Square (RMS) value of the angular acceleration of the idle gear with respect to the RMS value of the angular acceleration of the driving gear could provide a relative measure of the rattle impacts.

The scientific literature proposes various formulations of the rattle index based on these considerations. All these approaches, despite differing in their analytical definition, share the same basic principle: the evaluation of a ratio between RMS values of angular accelerations pertaining to different wheels. An early formulation was provided by Wang et al. [8,9], later taken up by Bile [61] and more recently by Shangguan et al. [10] in studies conducted on parallel shaft drives. This formulation defines the index as the ratio of the RMS angular acceleration of the driven gear $\ddot{\theta}_{2,\text{rms}}$ to that of the driving component $\ddot{\theta}_{0,\text{rms}}$ - typically the flywheel or the output shaft:

$$E = \frac{\ddot{\theta}_{2,\text{rms}}}{\ddot{\theta}_{0,\text{rms}}} \quad (1)$$

A variation of the same indicator is proposed in the study conducted by Brancati et al. [13] on a test bench consisting of a single gear pair. Here, the index is defined as the ratio between the RMS values of the acceleration of the driven gear and the pinion, both measured along the conjugate line of action.

$$\text{IGR1}_i = \frac{\ddot{x}_{2,\text{rms},i}}{\ddot{x}_{1,\text{rms},i}} = \frac{\ddot{\theta}_{2,\text{rms}} R_2}{\ddot{\theta}_{1,\text{rms}} R_1} \quad (2)$$

where R_1 and R_2 are the base radii of driving and driven gear, respectively. As a matter of fact, Eq. (2) differs from Eq. (1) for the scaling factor given by the base radii. A further alternative of this family is undoubtedly the one employed by Robinette et al. [18,19], focused on speed rather than acceleration, defining the Percent Deviation in Normalised Gear Speed ($PDNGS$) to quantitatively measure the instantaneous deviation from the ideal kinematic behavior.

$$PDNGS = \left(\frac{\dot{\theta}_1 - \dot{\theta}_2 \frac{Z_2}{Z_1}}{\dot{\theta}_1} \right) \cdot 100\% \quad (3)$$

where $\dot{\theta}_1$ and $\dot{\theta}_2$ are the angular speed and Z_1 and Z_2 are the number of teeth of the driving and driven gear, respectively. Specifically, the authors examine the variation of the metric as a function of the torsional excitation amplitude, showing that at low amplitudes the $PDNGS$ remains nearly constant and then exhibits an abrupt change in slope at the onset of rattle. The index then starts saturating and decreasing again when the rattle is now double-sided in nature. These differences are generally considered negligible, as the primary purpose of the rattle index is to provide a relative measure for different dynamic behaviors and assessing their magnitude. Among the available formulations, it is worth mentioning the one proposed by Brancati et al. [1], considered to be the most suitable for analysing the various gears of a transmission. This formulation keeps the angular acceleration of the flywheel in the denominator and uses the RMS value of the relative acceleration of the generic gear pair in the numerator:

$$(IGR2)_i = \frac{(\Delta\ddot{\theta}_i)_{\text{rms}}}{(\ddot{\theta}_0)_{\text{rms}}} \quad (4)$$

where

$$\Delta\ddot{\theta}_i = \ddot{\theta}_{i,2} - \left(\ddot{\theta}_{i,1} \frac{R_1}{R_2} \right) \quad (5)$$

is the relative angular acceleration of the generic gear coupling (i.e. the second derivative of the TE).

Gear rattle may also arise in the absence of external excitation, as the repeated impacts that characterise it can be driven by internal mechanisms of the transmission and generate vibrations with a high-frequency content. This led Brancati et al. [14,15] to formulate an indicator based on the Fast Fourier Transform (FFT). In particular, it focuses on the TE, measured by combining the absolute rotations of the driving and driven gear, weighting them according to the transmission ratio:

$$\Delta\theta^*(t) = \theta_1(t) - \left(\theta_2(t) \frac{R_2}{R_1} \right) = -\Delta\theta(t) \frac{R_2}{R_1} \quad (6)$$

In the experimental tests conducted to assess the influence of the type of lubrication on the contact dynamics, the TE analysis showed a behavior attributable to double-side contact, with the typical quasi-square-wave pattern due to the alternating contact between the two tooth flanks (direct and reverse). However, immediately after each contact inversion, a short interval characterised by a sequence of multiple impacts was observed before stable contact was re-established. The transition from a dry contact to an oil jet lubrication condition results in a progressive attenuation of these micro-impacts, with a consequent reduction in the high-frequency content of the relative displacement spectrum. Exploiting this correlation, the authors defined a specific index, calculated as the integral of the magnitude of the Fourier Transform of relative angular motion:

$$IGR = \int_{\omega_1}^{\omega_2} |\Delta\theta^*(\omega)| d\omega \quad (7)$$

Since the aim of the analysis was to focus on the vibrational components induced by the impacts, leaving out the low frequencies associated with the rattle cycle, the choice of integration limits assumes a crucial role. To this end, the author suggests excluding the first harmonics of the excitation, limiting the lower range of the analysis band to frequencies high enough to filter out the quasi-stationary contribution, and instead including the high-frequency components, in the order of a few kHz. The results showed that this indicator has a fair ability to discriminate between different lubrication conditions, even though it is generally insensitive to variations in speed fluctuation and it does not distinguish between single-side and double-side contact.

Among the rattle indices based on rotational quantities, two indicators proposed by A. Rivola and M. Troncossi in [2] clearly belong to this category. The authors employed two distinct indicators to compare the dynamic response of a gear timing system under different operating conditions. For the sake of clarity, the authors did not refer to the proposed parameters as rattle indicators, however, they are mentioned in the present work since they have been used to detect and quantify tooth detachment and impacts, acting therefore as proper rattle indices. The first of them is the Transmission Ratio Deviation $\Delta\tau_j$ (TRD), which directly reflects the essence of the rattle phenomenon. In the absence of rattle, the transmission ratio remains nearly constant; conversely, under double-sided impact conditions, torsional excitations cause the TE $x_{r,j}(\theta_0)$ to oscillate, producing a nearly square-wave pattern. The instantaneous transmission ratio between a generic j -th gear and a reference shaft R_0 can be expressed as:

$$\frac{d\theta_j(\theta_0)}{d\theta_0} = \tau_j + \frac{1}{z_j} \frac{dg_j(\theta_0)}{d\theta_0} \quad (8)$$

where θ_0 and θ_j denote the angular positions of the reference shaft and the j -th gear, respectively. Function $g_j(\theta_0) = z_j x_{r,j}(\theta_0)$ represents the phase modulation function associated with the gear signal. The gear TRD is therefore given by: $\Delta\tau_j = \frac{d\theta_j}{d\theta_0} - \tau_j$. In parallel, the authors proposed the Contact Time Percentage (CTP), a less conventional yet effective metric for identifying operating conditions

Table 1
Summary of acceleration-based rattle indices.

Name	Equation	Author
E	$\frac{\ddot{\theta}_{2,rms}}{\ddot{\theta}_{0,rms}}$	Wang et al. Shangguan et al.
IGR1	$\frac{\ddot{x}_{2,rms,i}}{\ddot{x}_{1,rms,i}} = \frac{\ddot{\theta}_{2,rms} R_2}{\ddot{\theta}_{1,rms} R_1}$	Brancati et al.
IGR2	$\frac{(\Delta \ddot{\theta}_i)_{rms}}{(\ddot{\theta}_0)_{rms}}$	Brancati et al.
IGR3	$\int_{\omega_1}^{\omega_2} \Delta \theta^*(\omega) d\omega$	Brancati et al.
L_r	$10 \log \left(\frac{1}{t_d} \sum_{i=1}^M \dot{q}_i^2 \right)$	Donmez and Kahraman
PDNGS	$\left(\frac{\dot{\theta}_1 - \dot{\theta}_2 \frac{Z_2}{Z_1}}{\dot{\theta}_1} \right) \cdot 100\%$	Robinette et al.
$\Delta \tau_j$	$\frac{d\theta_j}{d\theta_0} - \tau_j$	Rivola and Troncosi

in which the characteristic “free-flight” phase of rattle is most pronounced. This indicator CTP is derived from the time history of a contact detector function $Contact(t)$, which takes unit value when contact occurs and zero otherwise.

$$Contact(t) = \begin{cases} +1 & \text{if } x_r \geq \frac{x_b}{2} \\ 0 & \text{if } -\frac{x_b}{2} \leq x_r \leq \frac{x_b}{2} \\ -1 & \text{if } x_r \leq -\frac{x_b}{2} \end{cases} \quad (9)$$

The metric is obtained by integrating the contact detector over a reference period representative of the system’s operating cycle. In the specific case of a timing system, the integration is performed over a time interval corresponding to a multiple of the thermodynamic cycle, thus capturing the periodicity of the excitation and the associated contact dynamics. The superposition of the numerical signals obtained for different operating conditions confirmed the validity of the TRD as reliable indicators for the evaluation of the phenomenon.

Another way for rattle to manifests itself is through transient peaks in sound pressure, which occur immediately after each impact. Donmez and Kahraman [3,16] experimentally observed an apparent synchronism between impact events and sound pressure responses, as well as a proportionality between the amplitude of these peaks and the relative velocity between the gears $\dot{q} = R_1 \dot{\theta}_1 + R_2 \dot{\theta}_2$ at impact. To quantify this relationship, they assumed that the acoustic power W_a may be proportional to the mechanical power induced by the impact W_m . Since the kinetic energy of the system immediately before impact is proportional to $\dot{q}^2$, average mechanical power W_m can be considered proportional to the time-averaged sum of the squares of the impact velocities. In light of these considerations, they formulated an index, defined on a logarithmic scale in order to relate it directly to the sound pressure level L_p (measured in dB):

$$L_r = 10 \log \left(\frac{1}{t_d} \sum_{i=1}^N \dot{q}_i^2 \right) \quad (10)$$

where N represents the total number of impacts occurring within a defined time interval t_d . The correlation between the severity index L_r and the effective noise level L_p was confirmed through a series of parametric tests in which the amplitude of the input torque fluctuation was varied. The results showed a clear consistency between the trend of L_r and that of L_p , confirming that the proposed index faithfully reproduces the variations in the acoustic behavior of the system. In the context of a parametric investigation, the use of this indicator necessitates the establishment of a time window for its evaluation. The authors further extended this methodology to multi-mesh gear trains [4]. In such systems, an impact occurring at one mesh interface often triggers subsequent impact events across other meshes within the train. To account for this overall behavior, the authors proposed a generalized version of L_r that aggregates the impact energy across each mesh within the system:

$$L_r = 10 \log \left(\frac{1}{t_d} \sum_{k=1}^M \sum_{i=1}^{N_k} \dot{q}_{i,k}^2 \right) \quad (11)$$

where M represents the total number of gear meshes in the system. This expanded formulation was validated using a three-axis gear train experimental setup, where the predicted index demonstrated a strong correlation with measured sound pressure levels L_p across various torque fluctuation conditions. This generalization confirms that the impact velocity-based approach remains a robust metric for assessing rattle severity even in complex powertrains, enabling the evaluation of acoustic performance solely through torsional dynamic models. The indicators discussed in this subsection are summarized in Table 1 for clarity and ease of reference.

2.2. Generalised force quantities based rattle indices

Rattle between conjugate gears occurs when, in the presence of backlash (x_b), the gears are free to oscillate within, leading to a rapid detachment followed by an impact when the contact is re-established. This situation arises when dynamic TE x_r is less than $x_b/2$, typically as a result of torsional fluctuations in the resistant torque, though mounting or indexing errors may also contribute [72,79–81]. Recent studies underline how installation errors, such as shaft misalignment and center distance deviations, significantly alter the Transmission Error and the dynamic response, thereby potentially promoting the loss of contact even at lower torque fluctuations. Singh et al. [6] showed that, considering the equation of motion of a gear pair and the definition of TE (see Eqs. (12)–(14)), the rattle condition may be expressed in terms of an equilibrium between inertial torque and drag torque.

$$J_2\ddot{\theta}_2 + k_m R_2 y(x_r, x_b) = T_{d,2}(t) \quad (12)$$

where

$$y(x_r, x_b) = \begin{cases} x_r - x_b, & x_r > x_b \\ 0, & -x_b < x_r < x_b \\ x_r + x_b, & x_r < -x_b \end{cases} \quad (13)$$

and

$$x_r = R_1\theta_1 + R_2\theta_2 \quad (14)$$

Specifically, the gear pair is rattle-free if the drag torque $T_{d,2}$, which acts as resistance, is sufficient to counteract the inertial torque $J_2\ddot{\theta}_2$. So:

$$T_{d,2} - J_2\ddot{\theta}_2 : \begin{cases} \leq 0 & \text{rattle} \\ > 0 & \text{no rattle} \end{cases} \quad (15)$$

The rattle criterion can thus be expressed as the ratio between the inertial torque, which favours the loss of contact, and the drag torque, which guarantees its continuity:

$$\frac{J_2\ddot{\theta}_2}{T_{d,2}} : \begin{cases} \geq 1 & \text{rattle} \\ < 1 & \text{no rattle} \end{cases} \quad (16)$$

To obviate the experimental difficulties related to the direct measurement of $\ddot{\theta}_2$, which have already been discussed in the previous paragraphs, the author proposes a reformulation of the index based on the assumption that elastic deformations are negligible during contact. In this way, $\ddot{\theta}_2$ can be approximated to:

$$\ddot{\theta}_2(t) \approx -\frac{R_1}{R_2}\ddot{\theta}_1(t) \quad (17)$$

obtaining an indicator expressed as a function of the angular acceleration of the input gear.

$$\beta^*(t) = \left(\frac{J_2 R_1}{T_{d,2} R_2} \right) \ddot{\theta}_1(t) = \begin{cases} \leq -1 & \text{rattle} \\ > -1 & \text{no rattle} \end{cases} \quad (18)$$

The evolution of this concept has led to wider applications. For example, De la Cruz et al. [11,12] defined the Impulsion Ratio I_m , which, despite not identical to Singh's original formulation, retains the same physical principle by relating the drive torque and drag torque, including tribological considerations. In particular, it is shown that the relationship between the two torques is directly proportional to the ratio of the viscosities involved: in fact, the viscosity of the oil between the teeth of the gears contributes to a drag function, while the viscosity of the oil present in the meatus formed between the shaft and the idler gear favours the maintenance of contact.

$$I_m = \frac{T_{drive,2}}{T_{d,2}} \propto \frac{\eta_f}{\eta_{pet}} \quad (19)$$

In later studies, Rigaud et al. [5,47] also followed the same approach, but treated the two contributions no longer in terms of torques but in terms of forces:

$$\Lambda = \frac{m_2 H \omega_e^2}{F_{d,2}} \quad (20)$$

In this formulation, m_2 denotes the equivalent mass of the idler pinion, while H represents the amplitude of the periodic displacement of the driving gear. Term ω_e corresponds to the fundamental circular frequency of the excitation. Finally, $F_{d,2}$ denotes the equivalent force associated with the drag torque, defined as

$$F_d = \frac{T_d}{R_{b,2}} \quad (21)$$

where $R_{b,2}$ is the base radius of the idler gear. In their studies, the indicator was used to confirm the threshold above which rattle noise occurs. Experiments conducted at different average speeds showed that the threshold corresponds to an almost constant value

of the dimensionless parameter, regardless of the specific operating conditions. Beyond the threshold, the indicator was used to map the trend of the dynamic response. In particular, the evolution of the dimensionless pulse I as a function of Λ is plotted through Poincaré maps obtained for different operating regimes, allowing the validity of the indicator in describing the excitation level to be confirmed. This allowed the conclusion that Λ governs both the rattle threshold and the time course and pulse amplitudes above this threshold:

$$I = \frac{m_2(V^+ - V^-)}{m_2 H \omega_e} \quad (22)$$

where V is the relative velocity between gears evaluated just before and after the impact.

Subsequently, Pizzolante et al. [20] proposed an analytical procedure for the generalization of the rattle index in any type of ordinary transmission layouts, single or multiple branch, both in idle and loaded conditions. Its definition is obtained by a recursive analytical formulation of the rotational dynamics of gears. Based on the definition given by R. Singh [6], rattle in a single gear pair occurs when the TE $x_r(t)$ is less than half the backlash $x_b/2$, i.e. when in free flight conditions.

$$|x_r(t)| : \begin{cases} \leq x_b/2 : \text{rattle} \\ > x_b/2 : \text{no rattle} \end{cases} \quad (23)$$

Considering the torsional dynamics of a gear pair, the equation of motion of the driven gear can be expressed as:

$$\frac{J_2 \ddot{\theta}_2}{R_2} + \alpha \frac{J_2 \dot{\theta}_2}{R_2} + F_m = \frac{T_{e,2}}{R_2} \quad (24)$$

where α is the Rayleigh damping coefficient proportional to the mass, J_2 and R_2 are the polar inertia and the primitive radius of the idle gear, respectively, F_m is the meshing force and $T_{e,2}$ is the external torque applied to the driven gear. By isolating the gear mesh force term in Eq. (24), the criterion previously introduced in Eq. (23) can be reformulated in terms of the angular acceleration of the driven gear, yielding:

$$F_m = \frac{-J_2 \ddot{\theta}_2}{R_2} - \alpha \frac{J_2 \dot{\theta}_2}{R_2} \quad (25)$$

With this assumption the rattle criterion can be reformulated as follows:

$$\frac{-J_2 \ddot{\theta}_2}{R_2} - \alpha \frac{J_2 \dot{\theta}_2}{R_2} : \begin{cases} \leq 0 : \text{rattle} \\ > 0 : \text{no rattle} \end{cases} \quad (26)$$

And so the rattle index suitable for a gearpair, both in idle and loaded conditions, can be introduced:

$$PRI(t) = \left| \frac{J_2 \ddot{\theta}_2}{\alpha J_2 \dot{\theta}_2 - T_{e,2}} \right| \quad (27)$$

$$PRI(t) : \begin{cases} \geq 1 : \text{rattle} \\ < 1 : \text{no rattle} \end{cases} \quad (28)$$

In the transition to a single-branch layout, the analysis of the rattle associated with an idle gear becomes more complex, as each meshing involving that gear must be considered separately. The expression of the corresponding rattle index can therefore be derived as follows, with full details of the derivation available in [20].

$$PRI_{(i-1),i} = \left| \frac{\sum_{k=i}^n (-1)^k \frac{J_k \ddot{\theta}_k}{R_k}}{\sum_{k=i}^n (-1)^k \left(\alpha \frac{J_k \dot{\theta}_k}{R_k} - \frac{T_{ek}}{R_k} \right)} \right| \quad (29)$$

It is interesting to note that the methodology proposed by Pizzolante et al., although originally applied to three-branch transmission layouts, is potentially extendable to more complex configurations, including transmissions with an arbitrary number of branches and hub gears.

In the various works presented so far, the contact force has been left more or less implicit. On the contrary, Guo et al. [21,22] proposed the use of the time derivative of the contact force (often referred by the authors as Jerk) as an effective metric for evaluating gear rattle. This choice is based on the observation that, in the presence of rattle, the continuous alternation of the contact side generates impulsive fluctuations of the gear force: it goes from zero values during free flight to high values during contact, with pronounced peaks at shocks.

$$RI = \frac{\dot{F}_{m,rms}}{J_g} \quad (30)$$

Again, the joint analysis of the dynamic signals in the time domain, in particular the TE and contact force, made it possible to distinguish between single-sided and double-sided rattle conditions. The proposed index was then used as a quantitative measure of the severity of the phenomenon, useful for comparing different operating conditions. Although Guo et al. do not specify whether the rattle index value can be used as an absolute reference for comparing different transmissions, it is clearly effective in the relative comparison of parameters within the same dynamic model.

Table 2
Summary of torque-ratio based rattle indices.

Name	Equation	Author
β^*	$\left(\frac{J_2 R_1}{T_{d,2} R_2} \right) \ddot{\theta}_1(t)$	Singh et al.
L_{β^*}	$20 \cdot \log_{10} \left(\frac{\beta^*_{rms}}{0.707} \right)$	Singh et al.
I_m	$\frac{T_{drive,2}}{T_{d,2}} \propto \frac{\eta_f}{\eta_{pet}}$	De la Cruz et al.
Λ	$\frac{m_2 H \omega_c^2}{F_{d,2}}$	Rigaud et al.
PRI	$\left \frac{\sum_{k=1}^n (-1)^k \frac{J_k \ddot{\theta}_k}{R_k}}{\sum_{k=1}^n (-1)^k \left(\alpha \frac{J_k \ddot{\theta}_k}{R_k} - \frac{T_{d,k}}{R_k} \right)} \right $	Pizzolante et al.
RI	$\frac{\dot{F}_{m,rms}}{J_g}$	Guo et al.

It is worth noticing that the latter indices may be used both in the instantaneous analysis of the signal as well as in the overall evaluation of the severity of the phenomenon. When one is interested in the analysis of the instantaneous dynamic response, the index may be reported in a time history graph in which the sign variation allows one to easily distinguish between single-sided and double-sided impacts and quantify the absolute severity of the phenomenon. Furthermore, it should be remarked that indicators based on torque ratios inherently provide also a criterion for the onset of the phenomenon, i.e. when the excitation exceeds the resistance. This feature helps reducing the tuning of the post-processing, that can be easily generalized and straightforwardly applied to different gear trains. On the other hands, approaches based on experimental measurements, e.g. [7,23], and rotational quantities, e.g. [5,18] often requires dedicated procedures to define rattle thresholds that may differ depending on the gear train layout. Although the formulations presented varies in their mathematical structure, they are based on the same physical principle. Differences are not to be considered fundamental to the physical reasoning behind the phenomenon, but rather represent complementary approaches: the indicator proposed by Rigaud et al. is effective for defining and validating the excitation threshold, while those proposed by the other authors are also well suited for analysing the instantaneous dynamic response. The generalisation proposed by Pizzolante, in particular the recursive formulation for single-branch systems and its explicit extension to loaded gears, is crucial for its applicability to complex transmission layouts, overcoming the limitations of initial models focused on idler dynamics. The indicators discussed in this subsection are summarized in Table 2 for clarity and ease of reference.

2.3. External measurements based rattle indices

In this section, rattle indicators specifically developed for experimental applications are presented. This brief survey is included for the sake of completeness, even though the present work is focused on the analysis of indices that were mainly born for the assessment of computational predictions.

Several authors have sought to quantify rattle noise through approaches that can be implemented even in complex transmission architectures [82–85]. Initially, Zhou et al. [7] provided a comprehensive experimental investigation of gear rattle in Dual-Clutch Transmissions (DCTs), with particular emphasis on how different pre-selection strategies affect noise generation. The study aimed to assess the rattle sensitivity of a DCT and compare it with that of a manual transmission featuring a similar architectural layout. The two transmissions were mounted on a dedicated test bench and excited by a prime mover capable of reproducing the internal combustion engine torque oscillations up to 300 Hz. Structure-borne noise (SBN) was qualitatively quantified at two locations on the transmission housing using triaxial accelerometers, while the nominal input torque, input speed, and the pre-selection strategy applied in the DCT were systematically varied. Rattle sensitivity was quantified through the relative SBN level, ΔL , defined as the contribution of vibro-impacts from internal clearances:

$$\Delta L = L_e - L_0, \quad (31)$$

where L_e and L_0 denote the band-pass SBN levels at a given excitation angular acceleration $\ddot{\theta}$ and in the absence of angular acceleration, respectively. The latter represents the background vibration induced by rig dynamics, rolling bearings, and loaded gear meshing. A band-pass filter was applied and SBN level L was computed as:

$$L = \sqrt{\frac{1}{t_e} \int_{t_e}^{t_e+t_d} A_{avg}^2 dt_s} \quad (32)$$

with $t_e = 1/f_e$ being the excitation period, $t_s = 1/f_s$ the sampling interval, t_d the evaluation window and A_{avg} is the average of the measured acceleration signals within the selected frequency band. Results reveal a distinctive jump-up phenomenon in the DCT: when a gear is preselected, ΔL remains low until the input angular acceleration $\ddot{\theta}_0$ exceeds a threshold, after which ΔL suddenly increases. This behavior contrasts with manual transmissions and DCT configurations without pre-selection, where the rattle response grows smoothly with excitation amplitude.

The study conducted by Axel Baumann and Bernd Bertsche [86] investigates the influence of different gearbox lubricants on rattle noise in manual transmissions, with the primary objective of comparing their effectiveness in mitigating the phenomenon. The experimental setup consisted of a test bench equipped with a single gear pair, where gear backlash could be systematically varied. The

system was driven and excited by a three-phase synchronous motor capable of reproducing the torsional oscillations characteristic of internal combustion engines. Rattle severity was assessed using the continuous sound-pressure level, L_p , measured in dB(A). By plotting L_p against the excitation amplitude, the authors obtained rattle curves from which the onset threshold of the phenomenon could be clearly identified. Sound-pressure measurements were performed under sound-absorbing conditions, approximating an anechoic environment, using microphones positioned at three predefined locations at a distance of 1 m from the transmission housing.

The work of Baudin et al. [17] introduces a non-intrusive and industrially applicable methodology for detecting and quantifying gear rattle in automotive transmissions under non-stationary operating conditions. The study specifically addresses scenarios such as acceleration and deceleration run-ups, in which the non-stationarity of the signals makes conventional analysis techniques ineffective. The proposed indicator, based on the Order-Frequency Spectral Coherence (OFSCoh), was designed to operate on both vibration signals and instantaneous angular velocity measurements. The experimental setup consisted of a dedicated test bench equipped with an automotive gearbox. The torsional excitation was mechanically filtered through a torsion bar and two inertia wheels. The configuration ensured that only a single gear pair—the instrumented one—remained unloaded. Vibratory response was captured using an accelerometer mounted on the transmission housing. The core indicator proposed by the authors is the squared magnitude of the Order-Frequency Spectral Coherence, $|\gamma_{xx}(a_\theta, f)|^2$, computed from the vibration signal $x(t)$:

$$\gamma_{xx}(a_\theta, f) = \frac{S_{xx}(a_\theta, f)}{\left[S_{xx}(0, f), S_{x_{a_\theta} x_{a_\theta}}(0, f) \right]^{1/2}} \quad (33)$$

where a_θ denotes the cyclic order and S_{xx} represents the order-frequency spectral correlation (OFSC). This indicator provides a quantitative measure of rattle relative to background noise, behaving essentially as an order-synchronous signal-to-noise ratio:

$$|\gamma_{xx}(a_\theta, f)|^2 : \begin{cases} = 0 : \text{rattle is absent or fully masked by background noise} \\ = 1 : \text{rattle is dominant in the measured band} \end{cases} \quad (34)$$

The OFSCoh is shown to be particularly effective because it is synchronised with the cyclic order of the rattle (a_θ), enabling clear distinction between gear rattle and unrelated impulsive noise. It must be emphasised that this metric does not quantify the absolute level of rattle noise but its prominence relative to background excitation, which is the relevant criterion for perceptual assessment. The final outcome was presented as a two-dimensional map representing the evolution of rattle intensity across the entire speed run-up. This visualisation allowed the authors to identify the onset of rattle and its growth as a function of operating conditions, confirming the effectiveness of OFSCoh in detecting rattle during non-stationary gearbox operation.

More recently, Cristofori and Mucchi [23] proposed three different indicators that avoid the need to measure rotational quantities. The first index, referred to as RI1, is based on the assumption that torsional irregularities are directly correlated with the occurrence of gear rattle. An increase in these irregularities should result in a greater number and amplitude of sidebands. To capture this effect, RI1 is defined as the standard deviation of the signal filtered around the excitation frequency (Eq. (35)). The application of a band-pass filter is essential to isolate the spectral components of interest, eliminating low-frequency contributions and structural resonances, which might otherwise distort the index.

$$RI1 = \sqrt{\frac{1}{N-1} \sum_{k=1}^N \left| X_{k,f_e} - \frac{1}{N} \sum_{k=1}^N X_{k,f_e} \right|^2} \quad (35)$$

Where X_{k,f_e} is the signal spectrum filtered around the excitation frequency (f_e) and N is the number of scalar observations. For the operating conditions tested, RI1 index was ineffective when applied to acoustic pressure signals, since the low frequencies were easily masked by background noise, rattle detection was achieved when applied to acceleration signals.

The second index, namely RI2, is grounded on the observation that rattling noise originates from vibro-impacts between the teeth of idle or lightly loaded gears. This behavior, which resembles that observed in the presence of bearing faults, motivates the use of diagnostic techniques commonly employed in bearing condition monitoring. Specifically, RI2 is based on the envelope spectrum analysis originally introduced by Randall et al. [87,88] for the detection of bearing defects and it is defined as the maximum amplitude of the envelope spectrum. The effectivity of this indicator was found to be mostly significant when applied to sound pressure signals measured perpendicular to the gearbox.

The last index, referred to as RI3, is based on the assumption that tooth impacts behave similarly to a series of hammer tests, producing a response characterized by a flat spectral distribution. On this basis, RI3 is defined as the Root Mean Square (RMS) value of the signal filtered in the high-frequency domain (see Eq. (36)). The filter cutoff frequency must be selected to lie sufficiently far from the meshing frequencies, which could otherwise obscure the rattling components.

$$RI3 = RMS(x_{f_{ilt}}(t)) \quad (36)$$

In contrast to previous indicators, RI3 has been found to be especially effective in both cases, whether applied to the accelerometer signal, regardless of sensor position, or to the microphone signal. The indicators discussed in this subsection are summarized in Table 3 for clarity and ease of reference.

3. Assessment of rattle indices: application to literature data

The present section provides a comparative assessment of the rattle indices introduced in Section 2. In order to maximize repeatability of the results and comparability with future works, the analysis is carried out on two different gear train layouts taken from the

Table 3
Summary of external measurements based rattle indices.

Name	Equation	Author
ΔL	$\sqrt{\frac{1}{t_e} \int_{t_e}^{t+t_d} A_{e,avg}^2 dt_s} - \sqrt{\frac{1}{t_e} \int_{t_e}^{t+t_d} A_{0,avg}^2 dt_s}$	Zhou et al.
$OFSCoh$	$\frac{[S_{xx}(0,f), S_{xy}(0,f), S_{yy}(0,f)]^{1/2}}{\sqrt{\frac{1}{N-1} \sum_{k=1}^N X_{k,f_e} ^2 - \frac{1}{N} \sum_{k=1}^N X_{k,f_e} ^2}}$	Baudin et al.
$RI1$	$\sqrt{\frac{1}{N-1} \sum_{k=1}^N X_{k,f_e} ^2 - \frac{1}{N} \sum_{k=1}^N X_{k,f_e} ^2}$	Cristofori and Mucchi
$RI2$	$\max(FFT(envelope(x_{filt})))$	Cristofori and Mucchi
$RI3$	$RMS(x_{filt}(t))$	Cristofori and Mucchi

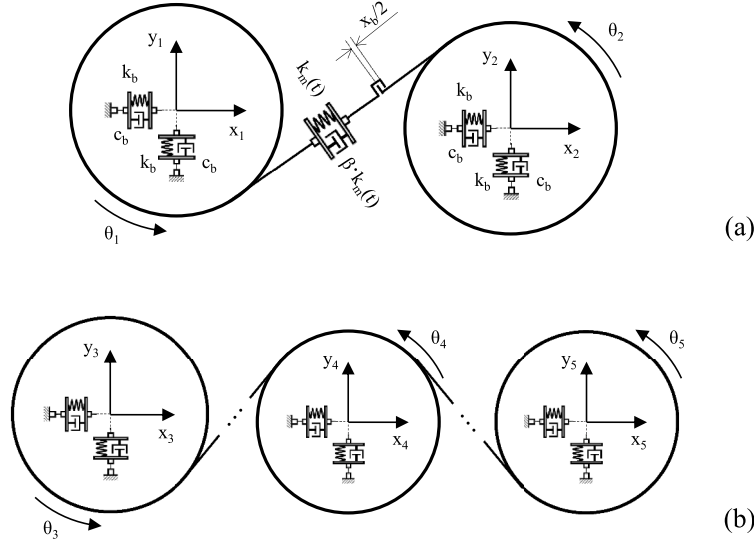


Fig. 1. Lumped parameter model of a gearpair (a) and a single branch (b).

work reported in [20]. The first configuration consists of a single gear pair with the driven gear operating in idle conditions, while the second corresponds to a single-branch, three-gear train with a central idle gear. This choice allows to quantitatively correlate the behavior of each index with respect to the others on classical single pair schemes as well as the transition to more complex transmission architectures. The section is organised into two subsections, one for each gear train layout. In the first subsection, three complementary analyses are carried out on the single gear pair: i) the ability of the indices to discriminate between operating conditions with and without rattle, ii) their sensitivity to the amplitude of the excitation, and iii) their capability to capture transient impact phenomena in time domain. The second subsection, devoted to the single-branch configuration, focuses on the assessment of the ability of each index to identify rattle occurrence (i) and on their instantaneous behavior in the time domain (iii).

3.1. Single gear pair model

In this section the steady state idle gear dynamics of a single gear pair is investigated on the basis of the classic 6-DOFs dynamic model for a single gear pair as shown in Fig. 1. Gears are mounted on rolling bearings. Three degrees of freedom are associated to each gear, namely θ_i , x_i , y_i . According to the reference frame, θ_i is the rotation around z axis, positive in counterclockwise direction, while x_i and y_i are the radial displacement in x and y directions. The motion of gear pair is formulated by following the analytical framework established in previous literature [6,26,31], leading to the following set of governing equations:

$$\begin{cases} m_1 \ddot{x}_1 + F_{mx} + k_b x_1 + \alpha m_1 \dot{x}_1 = 0 \\ m_2 \ddot{x}_2 + F_{mx} + k_b x_2 + \alpha m_2 \dot{x}_2 = 0 \\ m_1 \ddot{y}_1 + F_{my} + k_b y_1 + \alpha m_1 \dot{y}_1 = 0 \\ m_2 \ddot{y}_2 + F_{my} + k_b y_2 + \alpha m_2 \dot{y}_2 = 0 \\ J_1 \ddot{\theta}_1 + \frac{1}{R_1} + \alpha J_1 \dot{\theta}_1 + F_m = \frac{T_{e1}}{R_1} \\ J_2 \ddot{\theta}_2 + \frac{1}{R_2} + \alpha J_2 \dot{\theta}_2 + F_m = \frac{T_{e2}}{R_2} \end{cases} \quad (37)$$

Table 4
Design parameters for single branch geartrain composed of three gears.

Parameters	Gear 1	Gear 2	Gear 3
Mass [kg]	0.12	0.181	0.243
Inertia [kg·m ²]	8.49e - 5	1.6e - 4	1.07e - 4
Module [mm]	3	3	3
Number of teeth	25	32	17
Pressure angle [deg]	20	20	20
Gear mesh stiffness (max) [N/m]		40.8e + 6	26.8e + 6
Gear mesh stiffness (min) [N/m]		26.91e + 6	17.38e + 6
k_b bearing stiffness [N/m]	2e + 8	2e + 8	2e + 8
α damping coefficient [s ⁻¹]	5	5	5
β damping coefficient [s]	8e - 5	8e - 5	8e - 5
Backlash [μ m]		115	115
f_e [Hz], freq. of acc. excitation $\ddot{\theta}_0$	30		
$\hat{A}$ [rad/s ²], amp. of acc. excitation $\ddot{\theta}_0$	1000		

where m_i , J_i , R_i denote mass, polar mass moment of inertia, and the base radius of gear i , respectively. α is the Rayleigh's damping mass proportionality coefficient. T_{ei} is the external torque applied to gear i . k_b is the constant bearing stiffness and F_m represents the gear mesh force, that is modeled by taking into account backlash non-linearity. The Rayleigh's damping model was adopted and, for the sake of clarity, damping terms proportional to stiffness are included in F_m definition as described in Eq. (38):

$$F_m(t) = \begin{cases} k_m(t)[(x_r - x_b/2) + \beta \dot{x}_r] & \text{if } x_r > x_b/2 \\ 0 & \text{if } -x_b/2 \leq x_r \leq x_b/2 \\ k_m(t)[(x_r + x_b/2) + \beta \dot{x}_r] & \text{if } x_r < -x_b/2 \end{cases} \quad (38)$$

where k_m denotes the gear mesh stiffness, which is considered to vary according to the kinematic definition of the meshing course, x_r the dynamic TE and x_b the backlash clearance. To determine the meshing stiffness, the Weber-Banaschek formulation for two-dimensional problems [89] was adopted. Its analytical nature makes it particularly well-suited for integration into fast computational codes that require the evaluation of dynamic transmission error without the computational costs of a full FEM simulation [90,91]. The definition of the dynamic TE depends on which side of the tooth is actually in mesh at each time instant. When contact is occurring on the tooth working flank, TE x_r is defined on the Direct Line of Action (DLA). On the other hand, when contact takes place on the tooth back flank, x_r is defined on the Inverse Line of Action (ILA):

$$x_r = \begin{cases} R_1 \theta_1 + R_2 \theta_2 - x_1 \sin \phi - y_1 \cos \phi + x_2 \sin \phi + y_2 \cos \phi & (DLA) \\ -R_1 \theta_1 - R_2 \theta_2 - x_1 \sin \gamma + y_1 \cos \gamma + x_2 \sin \gamma - y_2 \cos \gamma & (ILA) \end{cases} \quad (39)$$

where ϕ and γ are the angles between y axis and the DLA and ILA respectively. Starting from these terms, the rattle condition can be defined as follows:

$$x_r(t) : \begin{cases} \leq x_b/2 : \text{rattle} \\ > x_b/2 : \text{no rattle} \end{cases} \quad (40)$$

The gear design parameters adopted in this study, as well as the mathematical formulation, are consistent with those reported in the literature [20] and are summarized in Table 4 for clarity. Rayleigh's proportional damping coefficients were calibrated to achieve a damping ratio (ζ) within the range of 2 – 5% over the frequency band of interest. This range is considered representative for similar mechanical power transmissions and aligns with empirical observations for such systems.

The numerical analysis has been conducted utilising the lumped parameter model that was developed by the authors in Simcenter Amesim environment. The solution to each model is achieved by employing a fixed time step integrator based on the Runge-Kutta method of order 4. The temporal resolution of the numerical study is set to 10⁻⁶ s.

The steady-state idle gear dynamics of the gear pair is investigated. In this condition, the system is subjected to an angular velocity that varies around an average value of 1000 rpm, and no external loads are applied to the driven gear, except for the drag torque due to friction, which is proportional to the velocity of the related gear. The duration of the simulation is 1 second. During the initial 0.3 seconds, the rotational velocity remains constant, while subsequently a non-zero fluctuation of $\dot{\theta}_1$ is imposed on the driver gear (see Eq. (41)) to simulate the speed fluctuation and induce tooth detachments.

$$\ddot{\theta}_1 = \begin{cases} 0 & \text{if } t \leq 0.3 \text{ s} \\ \hat{A} \sin(2\pi f_e t) & \text{if } t > 0.3 \text{ s} \end{cases} \quad (41)$$

where $\hat{A}$ and f_e are the amplitude and the frequency of the angular acceleration excitation, respectively. Numerical values are shown in Table 4.

By observing the results in Fig. 2a, it is noticeable that, as long as the acceleration excitation is zero, gears are in meshing on the front side of the tooth and no detachment events are recognised. Fig. 2b, on the other hand, shows that double-sided impacts occur when the acceleration excitation induces an oscillating rotating speed. Specifically, the drive gear tooth collides with the driven gear tooth and remains in contact for several gearings before detaching.

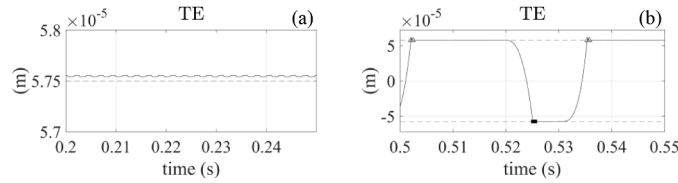


Fig. 2. TEs of the gearpair under full contact conditions(a) and rattle conditions (b).

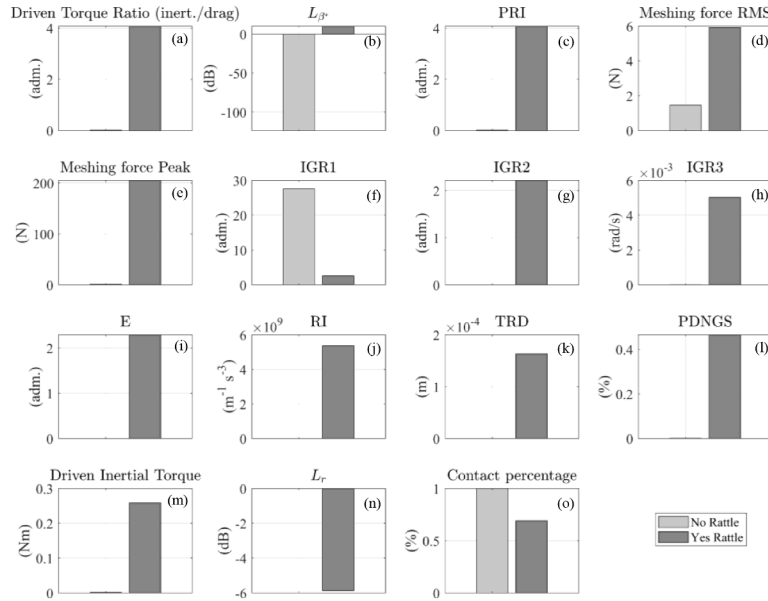


Fig. 3. Rattle index level comparison for the single gear pair.

A preliminary analysis is conducted to evaluate the performance of the aforementioned rattle indices under the presence and absence of rattle. The indices are quantified in terms of Root Mean Square (RMS) values or decibel (dB) levels over a specific time window, with the primary objective of assessing their sensitivity in discriminating between these two dynamic regimes. Given the purely numerical nature of this study, external measurements based rattle indices have been omitted. Furthermore, certain indicators were excluded to avoid conceptual redundancy; specifically, the Λ index by Kadmiri and Rigaud [46,47] was bypassed due to its equivalence with De La Cruz's impulse ratio I_m [11]. Similarly, Rivola's TRD [2] was not considered, as it was originally formulated to quantify tooth detachment and impacts within general dynamic analyses rather than serving as a dedicated rattle indicator. Nevertheless, to provide a more comprehensive overview, Rivola's percentage contact [2] was incorporated as a complementary metric. A time window of 0.2 s was defined for the evaluation of both RMS values and dB levels. The corresponding results are reported in Fig. 3. It is evident that, despite differences in scaling, most of the proposed indicators effectively discriminate between the safe regime and that characterised by impacts.

Two exceptions are observed: $IGR1$ and the Contact Percentage. The latter exhibits an apparently anomalous trend which, however, is physically consistent. Under safe conditions, tooth contact is continuous (100%), whereas during rattle the alternate engagement along the direct and inverse lines of action introduces intervals of free flight, thereby reducing the overall contact ratio (approximately 70%). The behavior of $IGR1$ is less straightforward to interpret. As shown in Fig. 4, under safe operating conditions the driving gear exhibits negligible acceleration, since it is numerically imposed as an input. The vibration response is therefore dominated by the motion of the driven gear which, although subjected only to the drag torque, shows low-amplitude ($\approx 10^0 m/s^2$) fluctuations at relatively high frequencies, close to the meshing frequency. Conversely, when rattle occurs, the excitation frequency dominates the response, and the acceleration amplitudes, though higher ($\approx 10^3 m/s^2$), become comparable for both gears. It is worth noting that $IGR1$ was not originally employed to characterise the absence of rattle, but rather to distinguish between different rattle regimes (e.g., single-sided vs. double-sided), consistent with its sensitivity to lubrication parameters.

A further simulation campaign was conducted by varying the amplitude of the external excitation, exploring amplitude values lower than those adopted in the first test, with the purpose to investigate the onset of single-side and double-side impacts and to assess the response of the different rattle indices. Recalling that the reference test was performed with an excitation amplitude of 1000 rad/s^2 , which produced a clear double-side condition, additional runs were carried out at 54%, 58%, 62%, 66% and 70% of this value. Fig. 5 illustrates the evolution of the TE for all five excitation levels considered, highlighting the progressive change in dynamic behavior. From this analysis, it emerges that tooth separation does not occur as long as the excitation amplitude stays below 52%

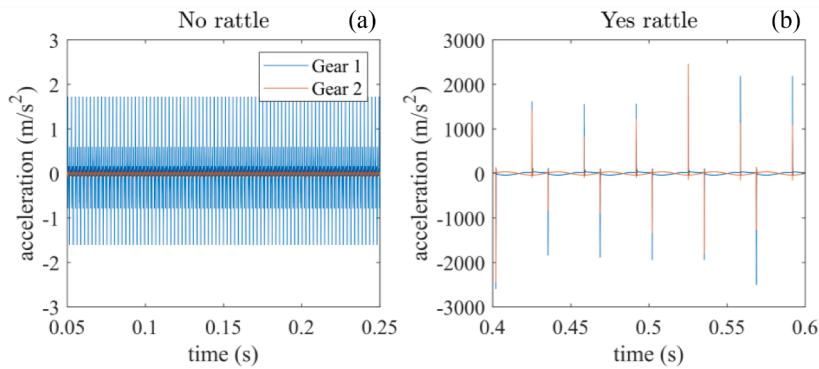


Fig. 4. Focus on gear pair acceleration utilised in IGR1.

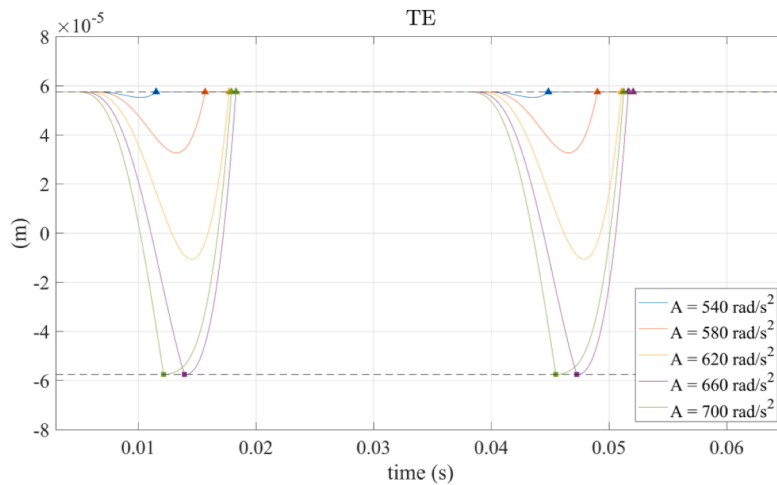


Fig. 5. TE for gear pair at different excitation amplitudes.

of the reference value ($\approx 520 \text{ rad/s}^2$). Above this threshold, the system initially exhibits single-side impacts, which persist until the excitation amplitude reaches roughly 650 rad/s^2 ; beyond this value the response transitions to a double-side condition. Fig. 6 reports the RMS levels of the rattle indicators under investigation for the different excitation amplitudes, enabling a direct comparison of the indicators throughout the transition between regimes. Most indices display a monotonic increasing trend with respect to the amplitude, coherently tracking the growth in dynamic severity. However, some notable deviations from this trend are observed. Specifically, $IGR1$, $IGR2$ [1,13] and E [8–10] exhibit irregular, non-monotonic behavior, suggesting a lack of direct correlation with the excitation amplitude.

Finally, it is worth noting that indices originally conceived to provide an overall severity measure, such as $IGR3$, L_r , $L_{\beta s}$, prove to be effective in quantifying the intensity of the rattle phenomenon, but do not offer information on the underlying impact condition (single- or double-side). Indeed, these indices exhibit a monotonic increase proportional to the excitation levels, without displaying any qualitative shifts or changes in slope that would signal a transition. Their sensitivity seems related to the macroscopic intensity of the phenomenon, a limitation that becomes particularly evident in the transition region investigated here.

Considering the outcomes of the preliminary analyses, a final investigation is devoted to the time-domain behavior of the rattle phenomenon, with particular emphasis on the evolution of tooth contact and on the instantaneous response of the individual rattle indices. To this end, the transition between contact along the Direct Line of Action (DLA) and the Inverse Line of Action (ILA), previously illustrated in Fig. 2b, is examined in detail. The investigation is restricted to the excitation-driven phase, during which the imposed angular acceleration results in a double-sided impact condition. The analysis includes both indices explicitly conceived for time-domain interpretation of contact transitions, such as $\beta^*(t)$ [6], PRI [20], I_m [11,12] and TRD [2], as well as indices that are more commonly employed in RMS-based formulations. The latter group, comprising $IGR1$ [13], $IGR2$ [1], RI [21,22], $PDNGS$ [18,19] and E [8–10], can nevertheless provide meaningful insight when examined in their instantaneous form, owing to the structure of their definitions. For the sake of completeness, additional physically meaningful quantities are also considered, namely the Impact Velocity $\dot{q}$ [3], the Driven Inertial Torque [11,12], and the Meshing Force [2]. Fig. 7 reports the instantaneous time histories of the considered indicators for the gear-pair meshing. Three distinct families of indices may be clearly identified, each characterised by a markedly different dynamic response.

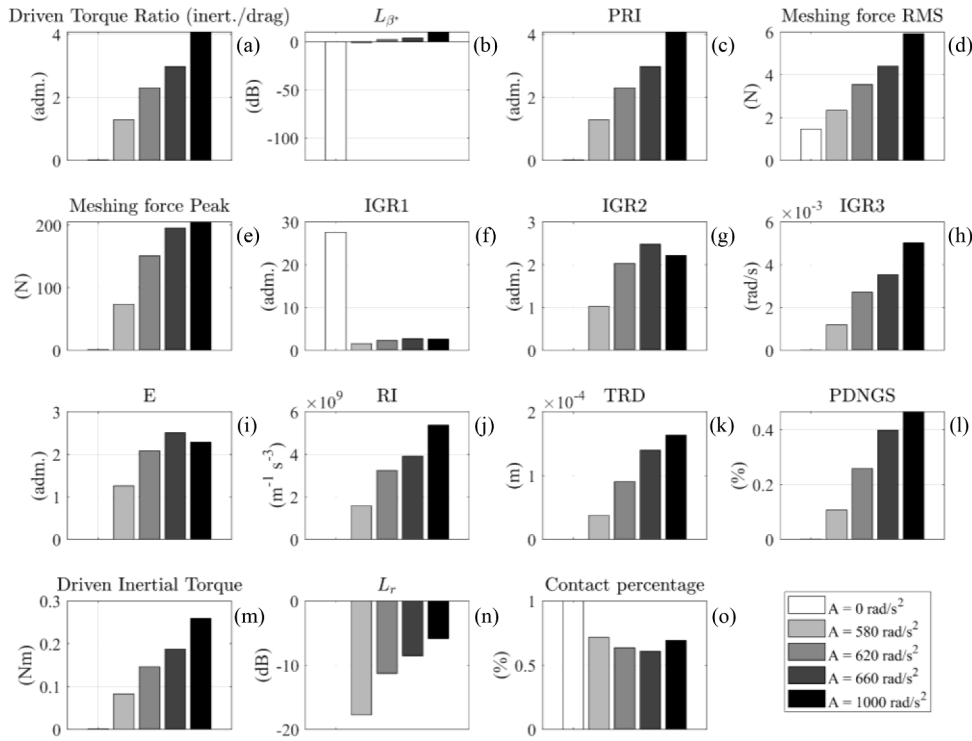


Fig. 6. Rattle index level comparison for Gear Pair in single- and double-side impact behavior.

The first family, involving RI , β^* , PRI , I_m , comprises those indicators that assume values close to zero during both contact and free-flight phases, exhibiting sharp peaks only at impact events. These indices reflect the evolution of the contact force (Fig. 7f) or the inertia torque (Fig. 7b). In particular, RI [21,22] (Fig. 7i) magnifies this behavior by incorporating the first derivative of the meshing force. It is worth noting that the two impacts occurring along the DLA exhibit nearly identical amplitudes for all indices belonging to this family, with the exception of β^* (Fig. 7h). The slight discrepancy observed in this case stems from the approximation introduced in Singh's original derivation [6], whereby the inertia torque of the driven gear is evaluated by approximating its angular acceleration through that of the driving gear, i.e. $(\ddot{\theta}_2(t) \approx -\frac{R_1}{R_2}\ddot{\theta}_1)$. As it may be seen from Fig. 7h, the two impacts in β^* shows the same order of magnitude, thus confirming that the adopted approximation is acceptable. It is also noteworthy that the overall shape of the signals within this first family of indices remains largely unaffected by the specific formulation. Whether the index is based on contact force or on inertial torque, the resulting trends are very similar. Indeed, higher inertial torque values are directly associated with more severe impacts in terms of contact force. The second family, which includes the Impact Velocity (Fig. 7c), the $PDNGS$ (Fig. 7j) and the TRD (Fig. 7l), is characterised by smooth, ramp-like trends during the free-flight phase, which are abruptly interrupted at the moment of impact. This behavior may be interpreted as follows: during free flight, the driven gear is influenced by its inertia effects exclusively, undergoing an acceleration that follows an approximately exponential law governed by the viscous damping of the lubricant. The relative velocity between the gears increases until contact is re-established with the tooth flank of the driving gear. Contact is then maintained over several successive meshing courses, with their number mainly governed by the ratio between meshing and excitation frequencies, resulting in an almost zero relative velocity. The third family includes $IGR1$ and $IGR2$ proposed by Brancati [1,13] and E proposed by Wang [8,9]. The original purpose of these formulations was to evaluate the RMS value of the signal over an extended time window, typically chosen as a multiple of the excitation period. In the present study, however, the results shown in Fig. 7 were obtained without applying RMS post-processing. Within a time-domain analysis, this leads to the appearance of sharp spikes at instants where the denominator approaches zero. To mitigate this numerical issue, a filtering procedure was introduced to set the index value to zero in those regions. This correction is evident just before the last impact (around 0.534 s) in each plot (Fig. 7a, d and g). All the indices reported in Fig. 7 are normalised with respect to their respective maximum values for the ease of the comparison. Under this normalisation, $IGR2$ and E exhibit identical functional forms. When the normalisation is not applied, however, a clear difference in amplitude emerges, as illustrated in Fig. 8. This discrepancy stems from the different scaling inherent in the two formulations: $IGR2$ is expressed in terms of relative velocity, whereby the transmission ratio acts as a normalising factor, whereas E is directly based on the angular acceleration of the driven gear, leading to systematically higher absolute values. Furthermore, since they are formulated with respect to the flywheel acceleration, these indicators display a smoother and more regular time evolution compared to $IGR1$, whose definition relies on the angular acceleration of the driving gear.

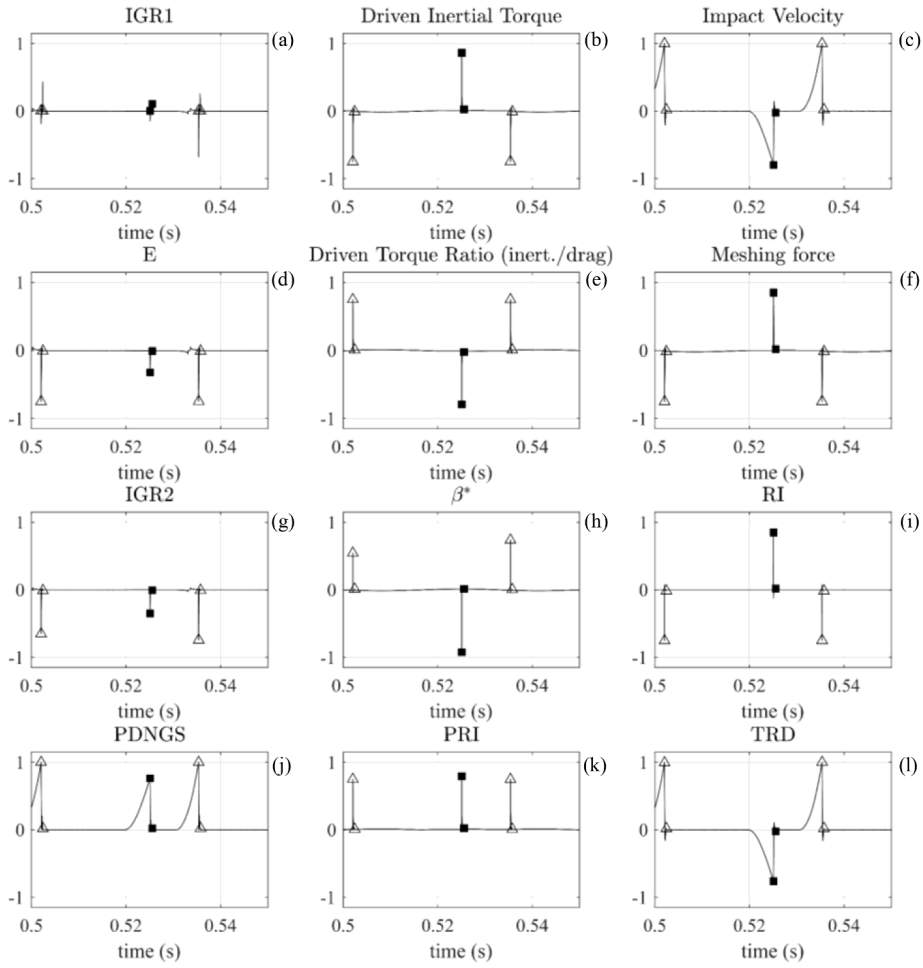


Fig. 7. Rattle index instantaneous comparison for Gear pair.

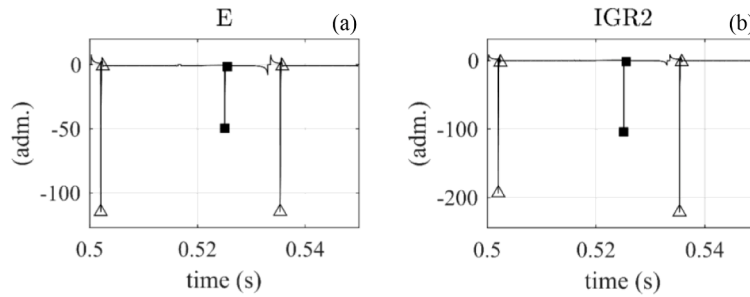


Fig. 8. IGR2 and E comparison for Gear pair.

3.2. Single branch model

In this section, the analysis is extended to the second model, namely a single-branch, three-gear train with a central idle gear. The objective is to investigate the steady-state dynamics of the idle gear within a single-branch configuration and to assess the behavior of the proposed rattle indices under these conditions. The adopted model, already introduced in Fig. 1, consists of the previously analysed gear pair augmented by an additional output gear. The geometric and inertia properties of the gears are reported in Table 4, and all gears are assumed to be supported by rolling bearings. The governing equations of motion are those detailed in the previous section (Eqs. (37)–(40)), while the system excitation is defined by Eq. (41). The total simulation time is 1 s; during the initial 0.3 s the rotational speed is kept constant in order to eliminate transient effects, then, the transmission is subjected to a rotational speed oscillating around an average value of 1000 rpm. No external loads are applied to the driven gears, with the exception of the drag

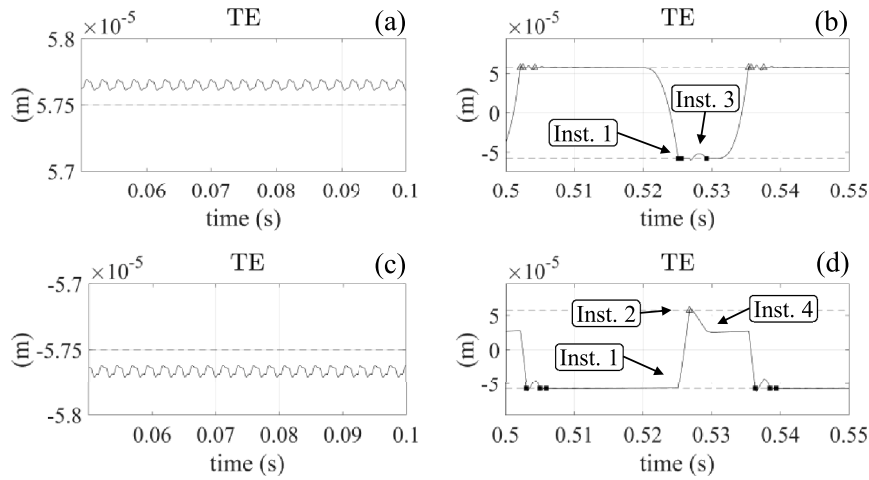


Fig. 9. TE of the single-branch gear train under full-contact and rattle conditions: (a,b) meshing between gears 3–4 and (c,d) meshing between gears 4–5; (a,c) full contact and (b,d) rattle.

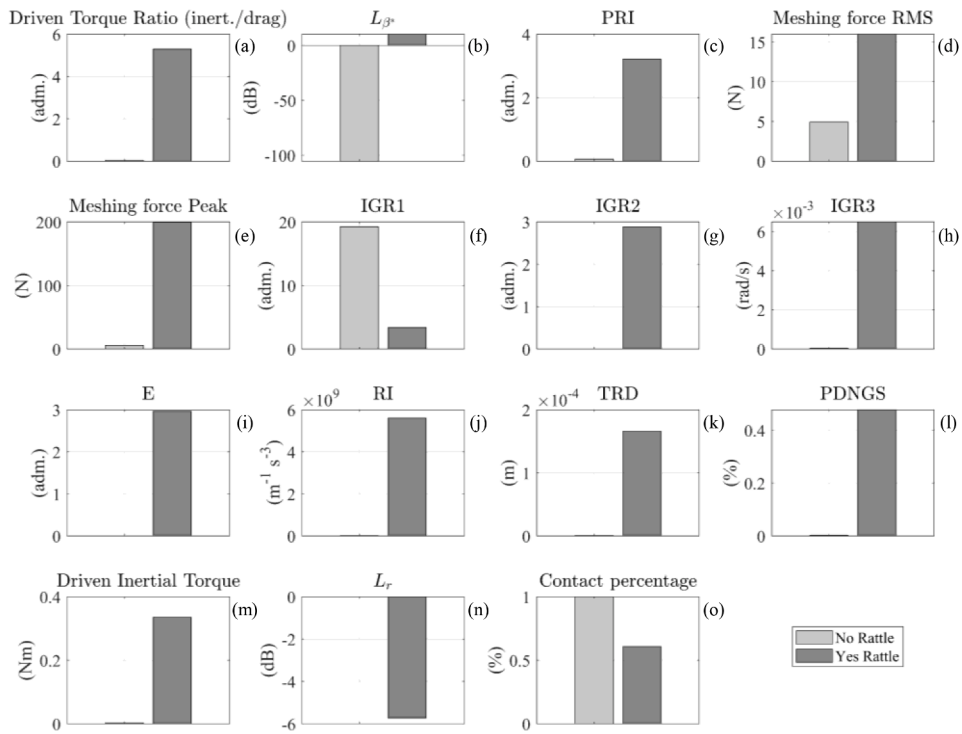


Fig. 10. Rattle index level comparison for Single branch (gears 3–4).

torque due to friction. Fig. 9a refers to the initial phase, in which the excitation is absent and the meshing between gears 3 and 4 occurs under rattle-free conditions, while Fig. 9b refers to the subsequent phase, in which the applied excitation leads to the onset of rattle. The corresponding operating conditions for the meshing between gears 4 and 5 are reported in Fig. 9c and d.

By observing Fig. 9, it is noticeable that, as seen for the gear pair in the previous section, when the acceleration excitation is zero all the gears are in contact on the front side of the tooth and no detachment events are recognized. Conversely, when the acceleration excitation induces an oscillating rotating speed, dual impacts are observed in both meshings.

Initially, gears 3 and 4 operate in contact along the DLA, while the meshing between gears 4 and 5 occurs along the ILA. When gear 3 undergoes a flank transition in its engagement with gear 4, an impact takes place on the ILA (Inst. 1). This event triggers an immediate loss of contact between gears 4 and 5, resulting in a subsequent impact on the DLA (Inst. 2). The latter induces a short rebound in the first mesh (Inst. 3), after which both gear pairs enter a free-flight phase. During this interval, the TE between gears 4

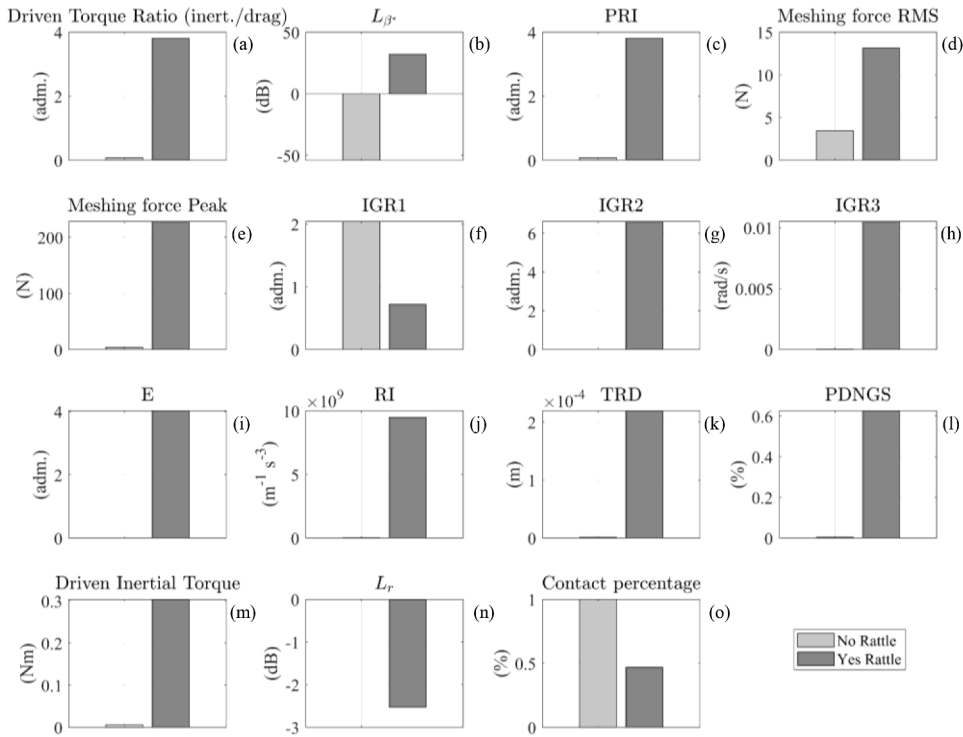


Fig. 11. Rattle index level comparison for single branch (gears 4–5).

and 5 remains temporarily constant (Inst. 4), until contact is re-established on the ILA by the impact that terminates the free flight of the first mesh. This observation indicates that while both meshes are subject to double side impact, the terminal gear of the branch (namely gear 5) exhibits significantly greater mobility and it is more prone to altering its contact side with greater impulsiveness, as evidenced by the TE slope. This suggests that gear 5 reaches a higher speed during free flight and it is possible that impacts between gears 4 and 5 may be more energetic than those between gears 3 and 4. These factors will be revisited once the rattle indices are presented. A comparison between the TEs of the gear-pair configuration (Fig. 2b) and of the first mesh of the single branch (Fig. 9b) reveals no substantial differences, aside from the presence of additional micro-rebounds in the latter. These rebounds originate from the dynamics of the second mesh (gears 4–5), which influence the first through the idle gear. As a consequence, markedly higher values of the rattle indices are not expected for the mesh 3–4 with respect to the gear pair.

The same preliminary analysis introduced for the single gear pair is here extended to the single-branch configuration. For consistency, the set of rattle indices under investigation and the post-processing methodology are kept unchanged. In particular, the indices are again evaluated in terms of RMS values or decibel levels over an identical time window, with the aim of assessing their capability to discriminate between rattle-free and impact-dominated regimes. The corresponding results are reported in Fig. 10 for mesh 3–4 and Fig. 11 for mesh 4–5. At a qualitative level, the overall trends observed for the single branch largely mirror those discussed in the previous subsection. This agreement is not trivial, as the presence of an additional gear and the dynamic coupling introduced by the intermediate idle gear modify the dynamics of impacts. The fact that most of the indices retain a comparable detection capability provides an initial indication of their robustness when applied to more complex gear train layouts. Notable deviations from this general behavior are nevertheless observed. In particular, the considerations previously made for IGR1 become even more evident when comparing Figs. 10f and 11f. While most indicators exhibit higher levels for the 4–5 gear pair, IGR1 displays the opposite trend. This behavior is again attributable to its formulation, which normalises the response with respect to the driving gear rather than a reference shaft such as the flywheel. A similar, albeit less pronounced, deviation is observed for the Driven Torque Ratio, which assumes slightly lower values for the 4–5 gear pair (Fig. 11a) compared to the 3–4 pair (Fig. 10a). This difference may be primarily due to the reduced driven inertial torque (Figs. 10m and 11m), resulting from the lower inertia of gear 5 relative to gear 4, as reported in Table 4. With these limited exceptions, the majority of the proposed indicators clearly distinguish between the rattle-safe regime and the regime characterised by impacts.

Building on the results of the preliminary analyses, attention is now shifted to the time-domain characterisation of the rattle phenomenon. The objective is to examine the dynamic evolution of tooth contact and to assess how the individual rattle indices respond instantaneously to contact transitions and impact events. For consistency, the same post-processing procedure adopted in the previous subsection is retained. Figs. 12 and 13 report the instantaneous time histories of the selected indicators for the two gear meshes within the single-branch configuration. Inspection of the time histories reveals that all indicators based on relative velocity, namely the impact velocity $\dot{q}$, PDNGS, and TRD, identify the meshing between gears 4 and 5 as more severe than that between

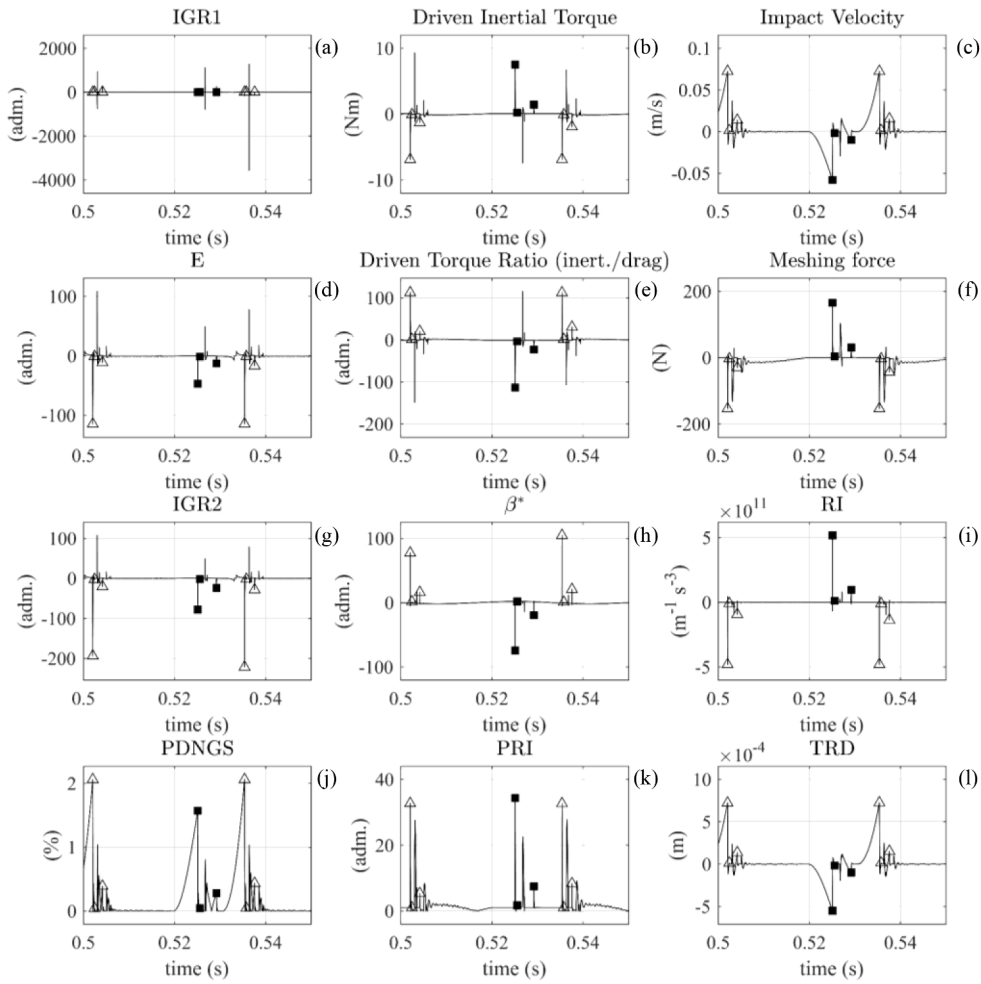


Fig. 12. Rattle index instantaneous comparison for single branch (gears 3 and 4).

gears 3 and 4. This distinction emerges not only from the higher amplitude levels, but also from the qualitative features of the signals. In Fig. 12, the indicators still exhibit the smooth, ramp-like evolution during the free-flight phase previously observed for the single gear pair. By contrast, this behavior is no longer discernible in Fig. 13, where the signals display sharper transitions, indicating a more abrupt reversal of motion during the wobble phase.

Given the close similarity between their mathematical formulations, another meaningful comparison may be made between PRI and I_m . PRI reports a more severe engagement for gears 4 and 5 (Fig. 13k) than for the mesh 3–4 (Fig. 12k), whereas I_m (subplot “e” in both figures) exhibits an opposite trend. This discrepancy stems from the different physical content of the two indicators. While I_m evaluates the driven torque ratio without accounting for the global dynamic role of the gear within the branch, PRI is explicitly formulated to isolate the local contribution of each meshing pair. As a result, I_m treats gear 4 equivalently to gear 5, despite gear 4 being an intermediate idle gear rather than the terminal element of the branch. As a consequence, PRI captures the greater severity of the interaction associated to engagement 4–5.

A similar consideration applies to the Driven Inertia Torque indicator: as well as the Driven Torque Ratio, this index does not recognize the last meshing of the branch as the most severe. This confirms how crucial the choice of indicator is in the analysis of gear engagements of an idle gear involved in multiple pairs simultaneously. With regard to β^* , larger pulses are observed for mesh 4–5 compared to mesh 3–4; however, the difference remains relatively limited. This behavior does not reflect an intrinsic limitation of the indicator, but rather the context in which it was originally formulated, namely gear configurations involving a single driving gear and a single idle gear, without the dynamic coupling introduced by a single-branch layout. As the system complexity increases, such indices therefore retain their effectiveness in identifying the prevailing rattle regime (single- or double-sided contact), while their capability to quantify the relative severity of different meshing conditions becomes less pronounced.

As a final remark, this analysis suggests that, in the case of idle gears involved in multiple meshes, rattle indicators based on quantities that are effectively representative of the single contact such as the meshing force or its derivative (e.g. RI [21,22]) should be preferred. An additional observation emerges when comparing the indicators of gears 1–2 (Fig. 7) with those of gears 3–4 (Fig. 12).

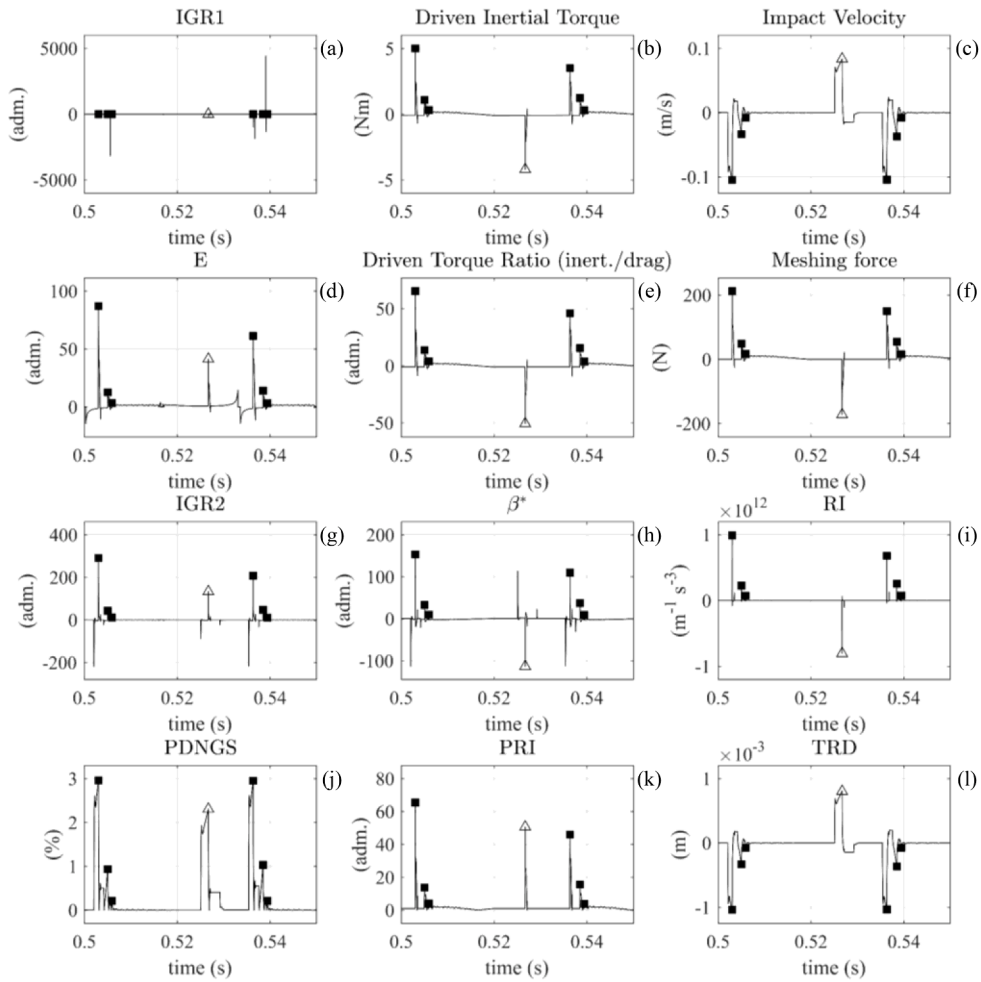


Fig. 13. Rattle index instantaneous comparison for single branch (gears 4 and 5).

The presence of multiple gears within the same branch causes mutual interference between impacts occurring in different stages, which makes the corresponding indices inherently less regular than in the isolated gear-pair case. This effect will be further discussed in [Section 4](#) in relation to the results of the multi-speed transmission model.

4. Application to a multi-speed transmission model

This section presents the application of the previously introduced rattle indices to a lumped-parameter model of a multi-speed transmission employed in agricultural machinery. This additional analysis has the purpose to highlight the behavior of each index when applied to multi-speed transmission model characterized by high complexity and several meshings. As noted earlier, most studies in the literature rely on test rigs consisting of either a single gear mesh or, at most, transmission layouts featuring two main shafts. In contrast, the present work investigates a more complex architecture, providing a real context for evaluating the robustness and practical relevance of the indices. The analysis follows the same structure as in the previous section. First, the ability of each index to reliably discriminate between operating conditions with and without rattle is assessed. Subsequently, attention is directed to the dynamic evolution of tooth contact and to the time-resolved behavior of the individual indices.

The schematic representation of the transmission system is illustrated in [Fig. 14](#). The system is composed of two primary shafts (first and second), with each shaft supporting a coaxial shaft via journal bearings (third and fourth). Downstream of the second shaft, a gear branch that connects the transmission to both the differential (via gear 17) and the 4WD engagement (via gear 19). The transmission contains five synchronisers (denoted by the letters *a*, *b*, *c*, *d* and *e*), which facilitate the engagement of the various gears. Clutches are not reported in the schematic, being out of the scope of the present study. The transmission facilitates a maximum of twelve distinct ratios, however, due to the inherent complexity of the system, the analysis hereinafter focuses exclusively on the condition depicted in [Fig. 14](#), corresponding to fourth gear in fast range.

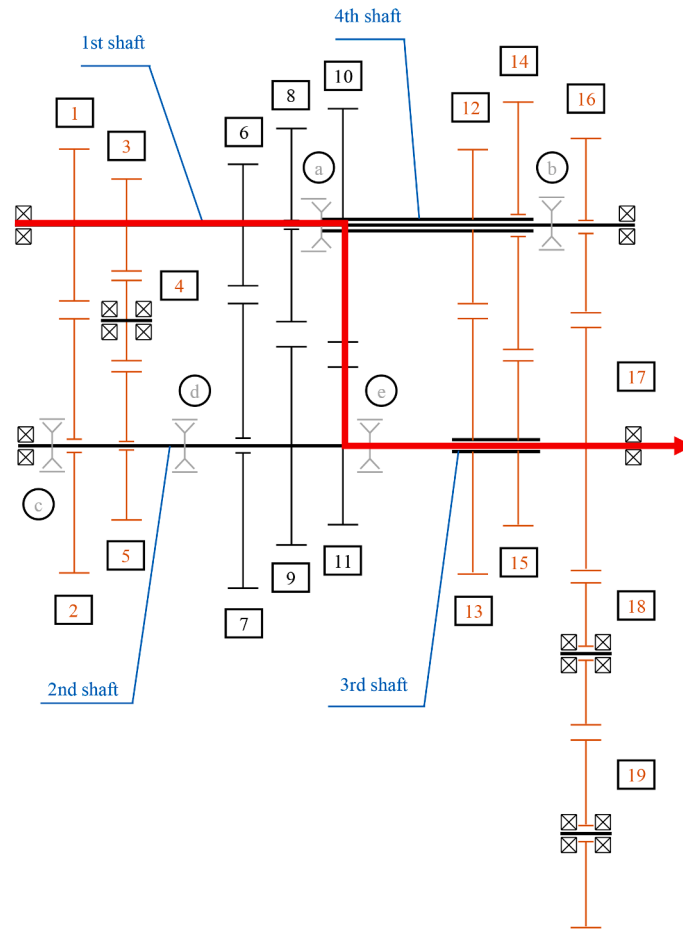


Fig. 14. Schematic representation of the transmission system.

Table 5
Design parameters for multi-speed transmission.

Lumped Mass Name	Gear Number	Mass (kg)	Inertia (kg·m ²)
Shaft 1	1	-	9.65E-03
Shaft 1	3	-	-
Shaft 1	12	-	-
Shaft 2	17	10.88	1.52E-02
Gear 2	2	1.4	2.40E-03
Gear 4	4	0.66	4.26E-04
Gear 5	5	1.92	4.01E-03
Shaft 3	13	3.19	2.75E-03
Shaft 3	15	-	-
Gear 14	14	1.63	3.09E-03
Gear 16	16	1.06	1.05E-03
Gear 18	18	0.79	8.41E-04
Gear 19	19	1.15	1.79E-03

When this gear is engaged, synchroniser *a* rigidly connects the 1st and 4th shafts, so that power flows from the 1st shaft through the meshing between gears 10 and 11, reaches gear 17 and it is then transmitted to the gear chain consisting of gears 16, 18 and 19. Following the modelling approach introduced in the preceding sections, a lumped-parameter representation is employed, assigning three degrees of freedom (θ_i , x_i , y_i) to each gear. The system dynamics are governed by the equations of motion presented in Section 3.1. In the context of the nine loose gears available (2, 4, 5, 7, 8, 14, 16, 18, 19), the present analysis will focus on two specific subsets: gears from 1 to 5 have been retained as representative examples of gear pairs and single branches, while gears from 16 to 19 are also include as they are recognized as the area most susceptible to vibration phenomena. The geometrical and inertial properties of these gears are illustrated in Table 5.

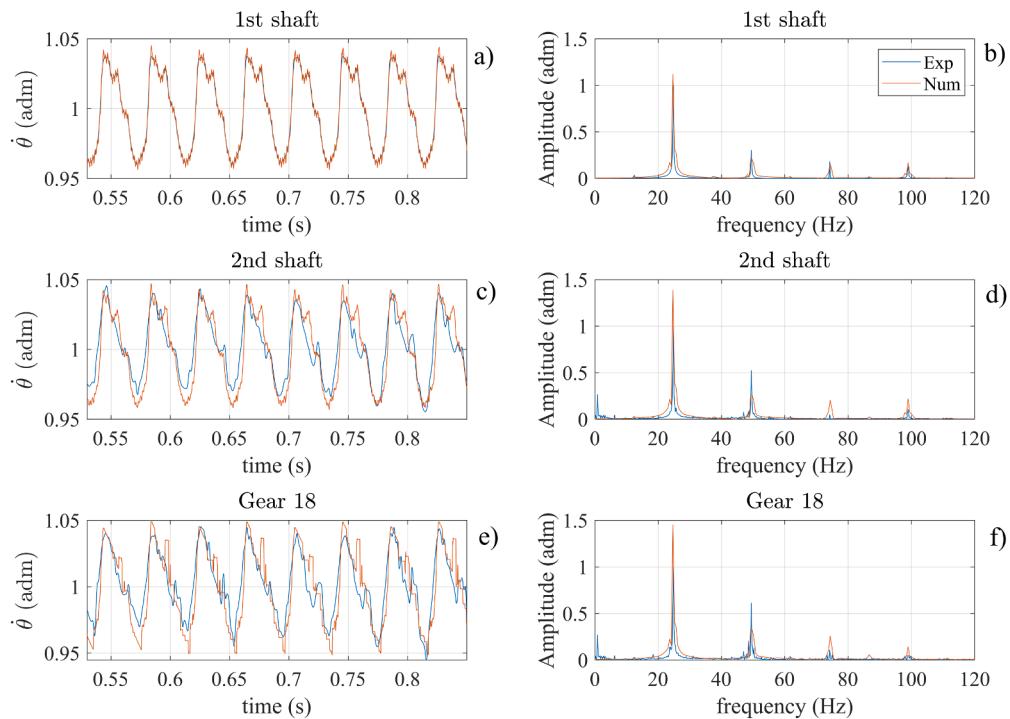


Fig. 15. Validation results for the lumped parameter model in time domain (a,c,e) and frequency domain (b,d,f).

In the course of the experimental campaign, the transmission was coupled to the crankshaft of a four-stroke, four-cylinder engine specifically adopted to promote rattle, while two dynamometers were connected to the output axes. For the validation of the lumped parameter model, a test was conducted under stationary conditions, setting a nominal motor speed of 750 rpm and imposing a torque of 135 Nm on each output axes. In order to acquire the instantaneous speed of the gears, the primary shaft, the secondary shaft (on which gear 17 is derived) and the idler gear (gear 18) were instrumented with Hall effect velocity transducers. The signals collected were used to validate the model: in particular, a synchronous average was performed on the signal relating to the primary shaft, which was useful for reducing fluctuations and background noise. The signal thus filtered was then applied to the model as input excitation. The steady-state simulation was conducted in the Simcenter AMESim environment, using a time step of $2 \cdot 10^{-5}$ s for a total duration of 2 s, while the experimental data were acquired with a sampling frequency of 40 kHz. The comparison was limited to angular velocities, as these are the only quantity that can be directly compared with numerical results. The results of the validation are shown in Fig. 15 in terms of angular velocity: the blue curves represent the experimental measurements, while the orange curves show the numerical predictions. The analysis shows good agreement between model and experimental data in both time and frequency domains.

Once the predictive capability of the model had been assessed, the first analysis is focused on investigating rattle indices behavior by comparing two distinct operating conditions: a rattle-safe condition, and a rattle-prone condition. A numerical simulation divided into two phases was carried out: during the initial 0.3 s, the rotational speed was kept constant in order to eliminate transient effects and to isolate the intrinsic dynamics of the transmission. In the subsequent phase, the transmission is subjected to a rotational speed oscillating around an average value of 1500 rpm. The excitation frequency was set to 30 Hz, while its amplitude was fixed at 1000 rad/s². A resistance torque of 88 Nm was applied to gear 17 as for the validation run. Fig. 16 reports the TE time histories for the considered idle gear meshes, where the blue curves refer to the excitation-free phase and the orange curves to the subsequent phase in which rattle develops.

The discussion is first focused on the rattle-safe interval ($t \leq 0.3$ s), during which no external torsional excitation is applied and the observed dynamics are therefore representative of the intrinsic behavior of the transmission. The single-branch configuration formed by gear pairs 3–4 (Fig. 16b) and 4–5 (Fig. 16c) largely maintains continuous contact along the line of action, showing a stable meshing behavior over the same interval. A similar stability is observed for the double-branch assembly involving gear pairs 17–16, 17–18 and 18–19 (Fig. 16f,g and h respectively), which preserves full contact in most operating conditions, with only occasional contact losses in the 17–16 mesh. Furthermore, gear pairs 12–13 and 15–14 (Fig. 16d and e) constitute a single branch articulated over two stages, with shaft 3 acting as an intermediate idle element. It is evident that both gears of this single branch operate under single side impact conditions, though a more pronounced intermittency is observed in the downstream pair 15–14. These results highlight that, in a real industrial transmission, contact stability under rattle-safe conditions is strongly influenced by the overall layout and by the dynamic coupling among multiple meshing stages. As a consequence, even meshes that are kinematically analogous to the simplified cases analysed in the previous section may exhibit substantially different behaviors when embedded within a complex gear train. During the excitation-driven operating condition, in which the imposed angular acceleration induces oscillations in the rotational

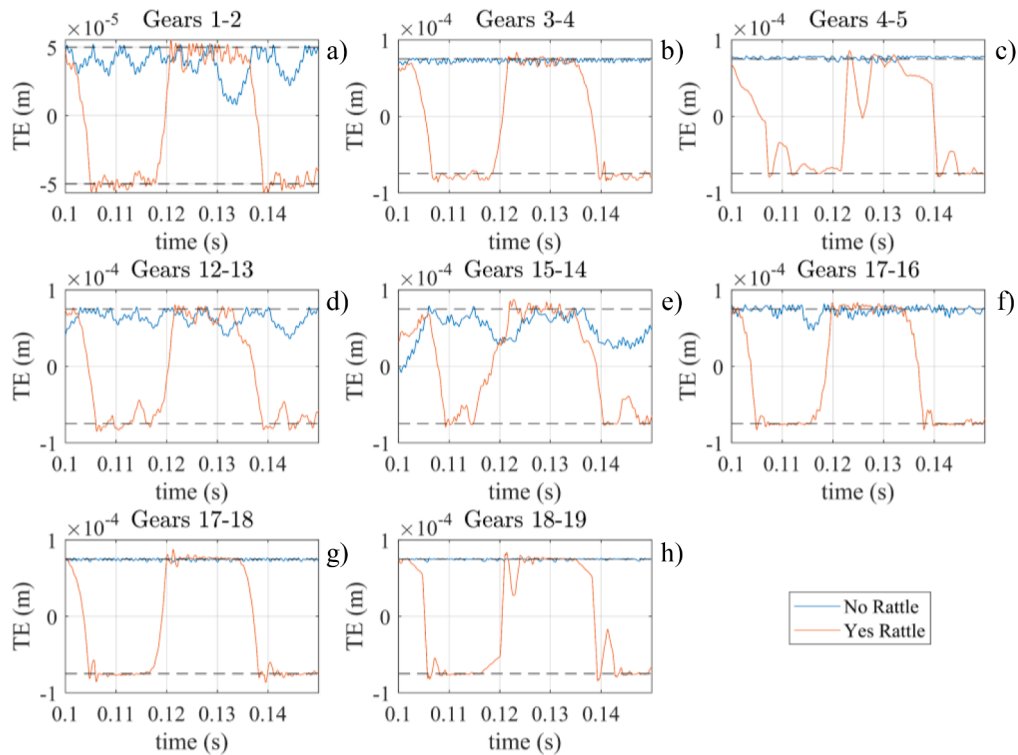


Fig. 16. TE of the different multi-speed transmission gear meshes without excitation (blue) and with excitation (orange). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

speed, all gear meshes operate in a double-sided contact condition. From a theoretical standpoint, the contact is expected to alternate between the DLA and the ILA and to be preserved over several consecutive mesh cycles. However, inspection of Fig. 16 reveals that this ideal behavior is not uniformly achieved across the transmission. In particular, gear pairs 4–5 and 15–14, and to a lesser extent 18–19 (Fig. 16c, e and h respectively), exhibit a reduced ability to sustain contact along the line of action, as evidenced by more frequent interruptions in the TE. This observation highlights a non-uniform sensitivity of the different meshes to excitation-induced speed fluctuations, which ultimately governs the local appearance of double-sided rattle.

Following the qualitative assessment of the contact behavior, the investigation is extended to a quantitative evaluation of the rattle indices. In line with the methodology established in the previous sections, the same set of indicators is considered and post-processed accordingly. The indices are therefore evaluated in terms of RMS values or decibel (dB) levels over the same time interval. The corresponding results are reported in Fig. 17. Overall, despite differences in scaling, most of the proposed indicators exhibit a clear capability to discriminate between the rattle-safe regime and the impact-dominated condition. A notable exception is represented by the Contact Percentage (Fig. 17o), which displays a trend opposite to that observed for both the gear-pair and the single-branch models. This behavior, although potentially counterintuitive, is fully consistent with the TE analysis reported in Fig. 16. In the excitation-free condition, several gear pairs already operate under single-sided contact, leading to reduced Contact Percentages. When excitation is introduced, these same meshes transition to double-sided contact; although dynamically more severe, this regime results in longer cumulative contact durations, and therefore in higher percentage contact values.

This effect is further amplified by the fact that the meshes under consideration involve idle gears, which are unable to guarantee stable contact even in nominally safe operating conditions owing to the absence of a significant resisting torque. Consequently, what may appear as permanent contact in the TE, as for instance in gear pair 17–18 in Fig. 16g, actually corresponds to a sequence of high-frequency micro-impacts. These phenomena are intrinsic to the transmission layout and stiffness characteristics, rather than being directly induced by the external excitation. A brief clarification is required for the E and $IGR2$ indicators, as Fig. 17 may otherwise lead to a misleading interpretation. Contrary to what might be inferred from the figure, these indices do not assume null values under rattle-safe conditions; instead, they formally diverge. This behavior follows directly from their definition, since both indicators are normalised by the RMS value of the imposed angular acceleration (see Table 1). In the absence of excitation, the denominator vanishes and the indicators tend to infinity. For clarity, these divergent values were therefore omitted from the plots. When a low-amplitude excitation is applied in order to induce a mild loss of contact, corresponding to a single-sided rattle condition, E and $IGR2$ take finite and relatively small values. This behavior, already observed for the single gear-pair configuration (Fig. 6g and 6i), confirms that these indicators are well suited to quantify the intensity of the phenomenon. However, they do not provide direct information on the nature of the impact regime, such as the distinction between single- and double-sided rattle. With this

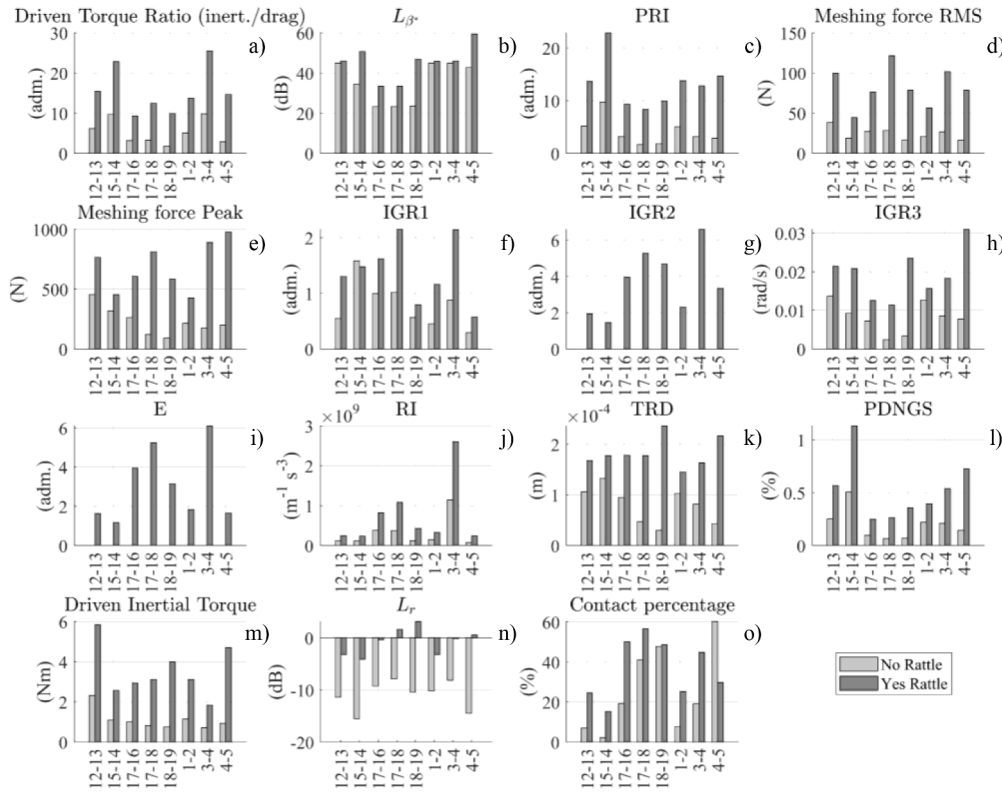


Fig. 17. Rattle index level comparison for multi-speed transmission.

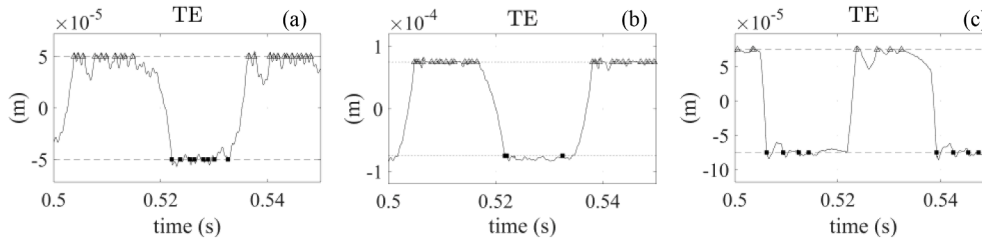


Fig. 18. TE of the multi-speed transmission model under rattle conditions: (a) meshing between gears 1–2; (b) meshing between gears 3–4; (c) meshing between gears 4–5.

limited exception, almost all the proposed indicators clearly distinguish between the rattle-safe and impact regime, confirming their applicability also in more complex transmission layout. However, it is not possible to define a detection threshold a priori: if only rattle-free conditions are observed, the value of the indicator alone does not allow the operating mode to be identified. The threshold value is inherently dependent on the system and must be established by testing the transmission under various operating conditions.

Considering the outcomes of the preliminary analyses, the discussion is now extended to the time-domain characterisation of the rattle phenomenon. The analysis focuses on gear pair 1–2, representative of a simple gear pair located at the input of the gearbox, and on the meshing pairs 3–4 and 4–5, which together constitute a single-branch configuration. The corresponding transitions between contact along the Direct Line of Action (DLA) and the Inverse Line of Action (ILA) are illustrated through the TE time histories reported in Fig. 18. The same set of rattle indices and the same post-processing procedure adopted in the previous subsections are retained to ensure consistency.

Before analysing the various indicators for the gearings under examination, the frequency spectrum of the TE shown in Fig. 19 is considered in order to highlight the main differences among the meshing conditions. To this end, the frequency band in the vicinity of the meshing frequency of the engaged gears (Gears 10 and 11) is magnified. For the meshing operations involving shaft 1, namely gearings 1–2 and 3–4 (Fig. 19a and b), a clear meshing frequency component can be identified, accompanied by several sidebands, which indicate the presence of modulation effects induced by the excitation. In contrast, the spectrum of gearing 4–5 (Fig. 19c) does not exhibit either the meshing frequency or the associated sidebands. Considering these spectral features, together with the

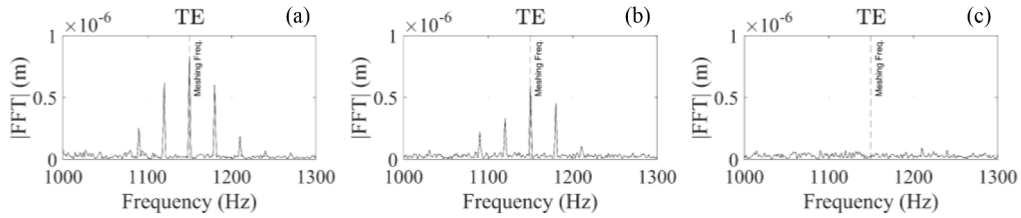


Fig. 19. TE FFT of the multi-speed transmission model under rattle conditions: (a) meshing between gears 1–2; (b) meshing between gears 3–4; (c) meshing between gears 4–5.

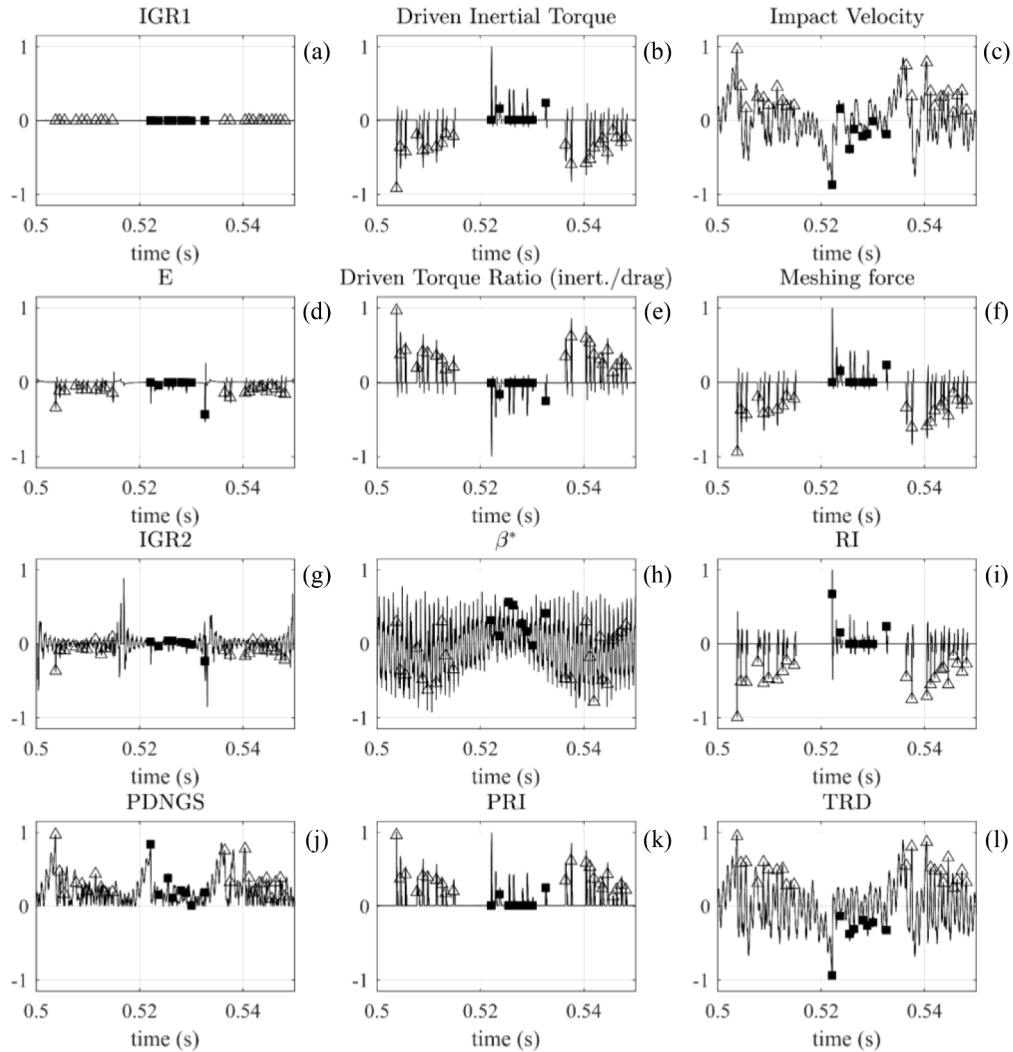


Fig. 20. Rattle index instantaneous comparison for multi-speed transmission (gears 1 and 2).

occurrence of numerous micro-impacts, the considered indicators are expected to show a higher variability with respect to the cases in the previous section.

Fig. 20 reports the instantaneous time histories of the considered indices for the engagement between gears 1 and 2. All indicators are normalised with respect to their respective maximum values in order to facilitate a direct comparison of their temporal trends. A first, evident feature is the presence of low-amplitude-high-frequency oscillations, which make the analysis more challenging than in the simplified gear-pair model discussed in Section 3.1.

As shown in Fig. 18a, what appears macroscopically as a double-sided impact regime actually conceals a dense sequence of micro contact losses. This behavior reflects the strong dynamic interaction among the multiple gear meshes mounted on shaft 1, which

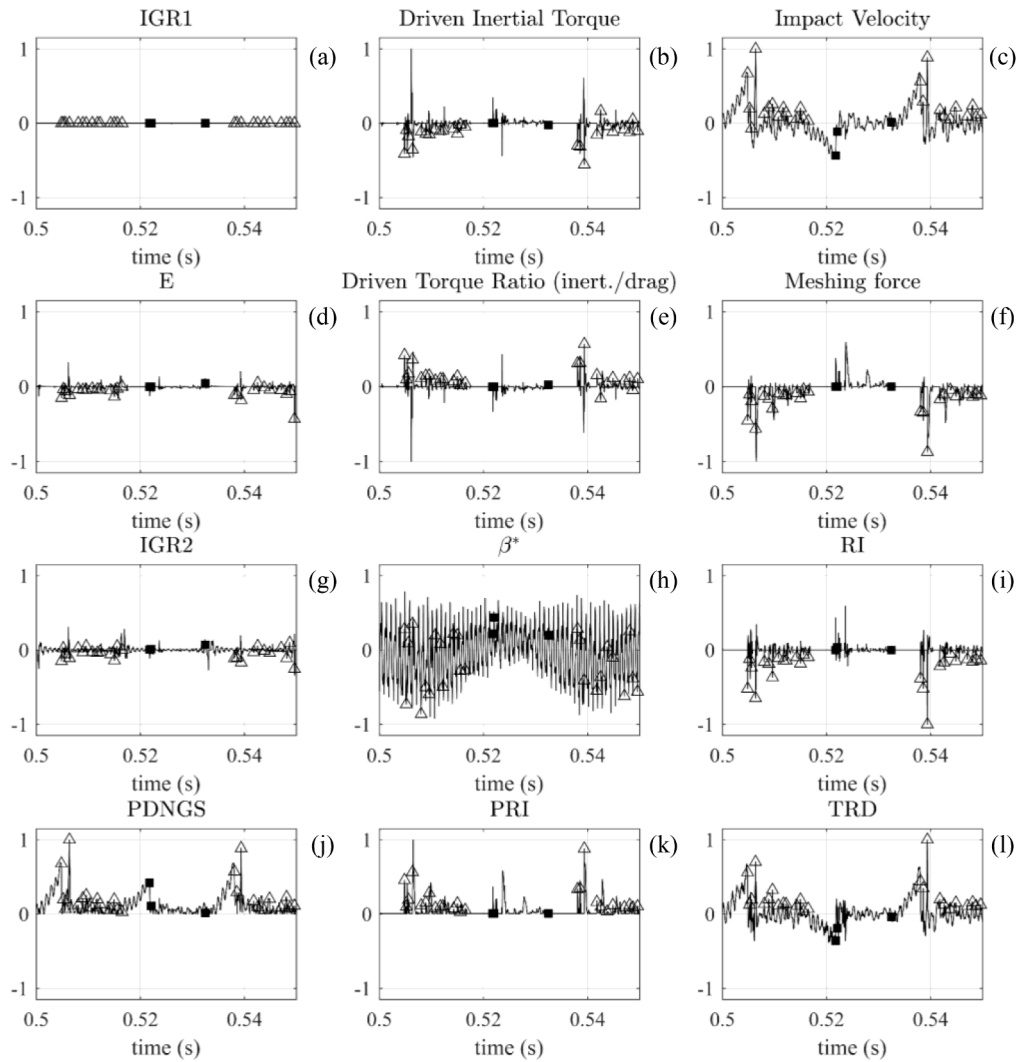


Fig. 21. Rattle index instantaneous comparison for multi-speed transmission (gears 3 and 4).

modulates the local contact conditions even in the absence of large-scale detachments. This complexity primarily affects *IGR1*, *E*, and *IGR2*, already identified as belonging to the third family in 3.1, whose responses (subplots a, d and g in Fig. 20) are strongly influenced by the cumulative dynamics of the transmission. Their temporal evolution appears less directly correlated with individual contact events, confirming that these indices are better suited for comparative assessments over extended time windows rather than for tracking the instantaneous evolution of tooth contact. In contrast, several indices from the first family retain a clear physical signature. With the exception of β^* (Fig. 20h), indicators such as *RI* (Fig. 20i), *PRI* (Fig. 20k) and I_m (Fig. 20e) exhibit pronounced peaks immediately following the initial impact associated with a change in contact direction, while subsequent minor impacts give rise to pulses of progressively lower amplitude. This behavior is consistent with their formulation in terms of contact force and with their original intent of distinguishing different rattle regimes. A similarly robust response is observed for the indices of the second family. Impact velocity $\dot{q}$ (Fig. 20c), *PDNGS* (Fig. 20j) and *TRD* (Fig. 20l) preserve the characteristic features already discussed for the simpler configurations. In particular, comparison with the TE trend highlights low values during contact phases, followed by smooth ramp-like increases during free flight, culminating in sharp peaks at the impacts associated with DLA-ILA transitions. Overall, these results indicate that, even within the complexity of a multi-speed transmission, indices based on contact force or relative velocity are capable of faithfully capturing the dynamic evolution of tooth contact when the analysis is confined to a gear pair directly driven by the motor, without additional idle gears upstream or downstream.

The behavior observed for the meshing between gears 3 and 4 (Fig. 21) largely confirms the trends identified for the pair 1–2, while exhibiting a noticeably cleaner temporal structure. This difference can be directly linked to the underlying contact dynamics. As shown by the corresponding TE evolution in Fig. 18b, this gear pair operates predominantly under stable double-sided contact, without the high-frequency bouncing and repeated micro-detachments that characterise the engagement of gears 1–2. In this configuration,

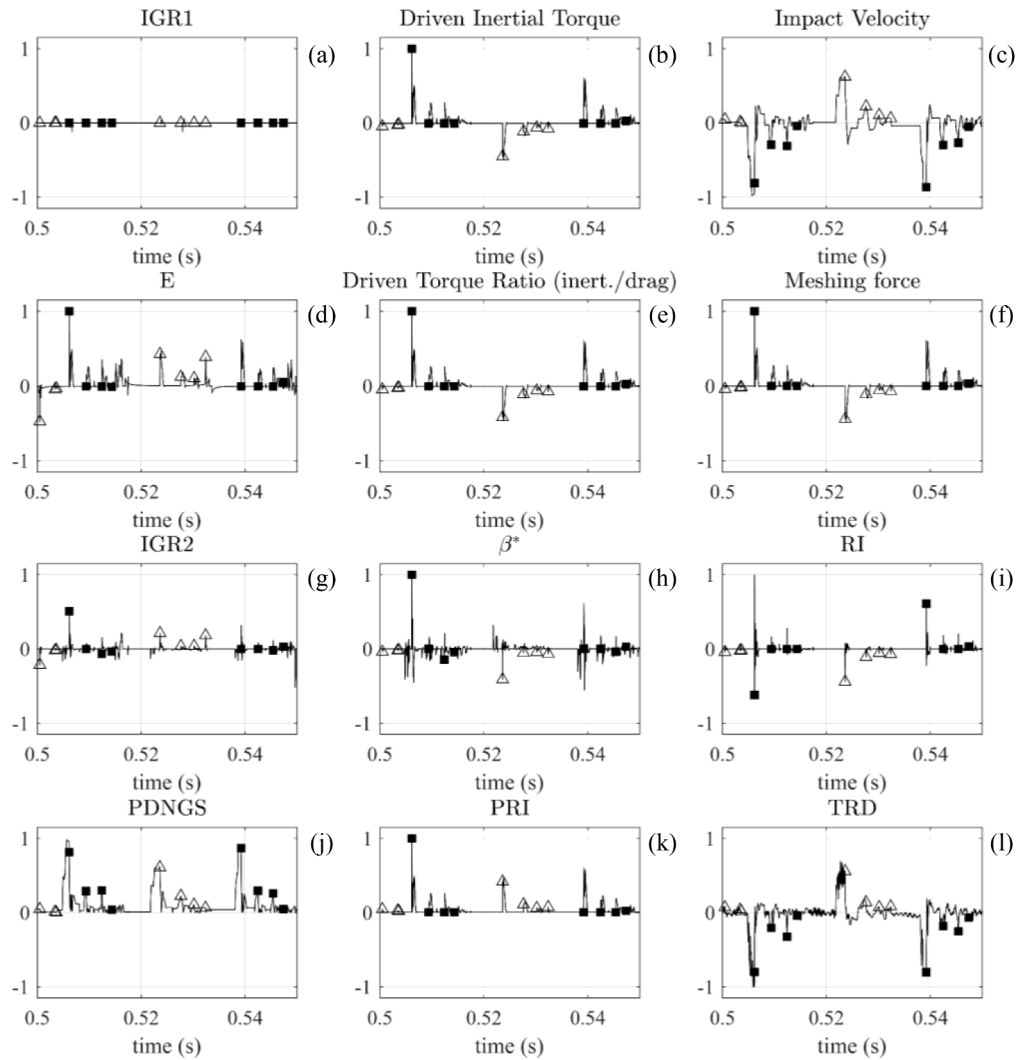


Fig. 22. Rattle index instantaneous comparison for multi-speed transmission (gears 4 and 5).

contact is transferred from the DLA to the ILA and remains sustained until the subsequent detachment event. As a result the indices based on relative velocity, namely the impact velocity $\dot{q}$ (Fig. 21c), *PDNGS* (Fig. 21j) and *TRD* (Fig. 21l), exhibit temporal trends that closely resemble those obtained for the isolated single-branch layout discussed in Section 3.2. This indicates that, despite the overall complexity of the multi-speed transmission, the dynamic evolution of the first meshing within the branch is only marginally affected by the presence of the surrounding gear train. A consistent picture also emerges from the indices based on contact force. Indicators such as *RI* (Fig. 21i), *PRI* (Fig. 21k) and I_m (Fig. 21e) clearly identify the dominant impacts associated with changes in the contact line, effectively distinguishing them from the lower-amplitude residual micro-impacts induced by the global transmission dynamics. Compared to the gear pair 1–2, the improved regularity of the contact conditions for gears 3–4 therefore results in a more interpretable and physically coherent response across all indicator families.

The analysis of the meshing between gears 4 and 5 consolidates the overall interpretation of the single-branch dynamics within the full multi-speed transmission layout. By looking at Fig. 22, almost all indices continue to exhibit pronounced peaks associated with changes in the contact direction, confirming that the core rattle-related events remain clearly identifiable despite the increased transmission complexity. At the same time the interaction with the remaining gear meshes in the transmission reveals a background disturbance that, despite clearly visible, do not obscure the dominant impact events, which remain consistently captured by the indicators. In line with the observations previously reported for the isolated single-branch configuration in Section 3.2, the relative-velocity-based indicators, namely *PDNGS* (Fig. 22j), *TRD* (Fig. 22l) and the impact velocity $\dot{q}$ (Fig. 22c), do not exhibit the characteristic ramp-like evolution during the free-flight phase when applied to the second meshing of the branch. This behavior is therefore confirmed as an intrinsic feature of the single-branch layout rather than a influence of the global multi-speed transmission dynamics. Overall, the response of the indices for the 4–5 gear pair reinforces the conclusion that, while the surrounding transmission

Table 6

The most suitable indices depending on the application.

	Rattle Onset	Analysis of severity and Dynamic Evolution
Single Gear-pair	$I_m, L_\beta, PRI, F_m, IGR3, IGR2, E, RI, TRD, PDNGS$, Contact percentage, Driven Inertial Torque	$I_m, L_\beta, PRI, IGR3, RI, TRD, PDNGS, L_r, F_m, \beta^*$ Driven Inertial Torque, Impact Velocity ($\dot{q}$)
Idler Gears engaged in Multiple Mesh- ing	$I_m, L_\beta, PRI, F_m, IGR3, IGR2, E, RI, TRD, PDNGS$, Contact percentage, Driven Inertial Torque	$I_m, PRI, IGR2, E, RI, TRD, PDNGS, F_m, \beta^*$ Driven Inertial Torque, Impact Velocity ($\dot{q}$)
Complex Multi- stage Systems	$I_m, PRI, F_m, IGR3, IGR2, E, RI, TRD, PDNGS, L_r$ Driven Inertial Torque	$I_m, PRI, RI, PDNGS, F_m, TRD$ Driven Inertial Torque

introduces measurable perturbations, the essential rattle mechanisms and their temporal signatures remain robustly detectable, even at the downstream end of the branch.

5. Concluding remarks

This work has presented a structured review and comparative assessment of rattle indices proposed in the literature for the analysis of gear rattle phenomena. The considered formulations were classified according to the physical quantities on which they are based and were subsequently applied within a consistent numerical framework, allowing a direct comparison of their behavior under identical operating conditions. By adopting a unified modelling approach, the study aimed at clarifying similarities, differences, and complementarities among indices that were originally developed and validated in heterogeneous contexts.

The results confirm that rattle indices do not all serve the same diagnostic purpose. Indices based on torque ratios or force balance formulations prove particularly effective in identifying the transition between non-rattling and rattling regimes, providing clear and physically interpretable yes/no discrimination criteria. Conversely, indices derived from rotational quantities or contact-related variables show a higher sensitivity to the temporal evolution of the phenomenon, enabling a more detailed description of tooth detachment, impact sequences, and contact inversion dynamics. These latter indicators are therefore better suited for analysing rattle severity and for tracking the evolution of tooth contact conditions once rattle has been established, rather than for defining sharp onset thresholds. In light of the comparisons presented in this article, Table 6 sets out the most appropriate indices based on the application and the type of study to be conducted.

A distinctive aspect of the present study is the application of the reviewed rattle indices to an unconventional transmission architecture. Unlike the majority of previous works, which focus on isolated gear pairs or simple parallel-shaft layouts, the analysis was extended to a multi-speed transmission characterised by the presence of loose and idle gears distributed over several stages. This configuration represents a more realistic and challenging scenario, in which the interaction between multiple meshes and inertial elements may significantly affect the interpretability of rattle indicators. The results show that several indices retain their qualitative meaning even in this increased level of complexity, while others exhibit limitations that would not emerge in simpler layouts, thus highlighting the importance of evaluating rattle metrics beyond canonical test-bench configurations.

Based on these findings, the study suggests that the selection of a rattle index should be driven by the specific objective of the analysis, distinguishing between detection of rattle occurrence and investigation of its dynamic characteristics. Moreover, the extension to complex, multi-stage transmissions indicates that further studies may benefit from a combined use of complementary indices, as well as from additional validation on experimental data, particularly in architectures where direct access to rotational measurements is limited. Within this perspective, the present work provides a unified reference framework that may support both future methodological developments and the informed application of rattle indices to realistic transmission systems.

CRedit authorship contribution statement

Fabio Bortoluzzi: Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis, Data curation; **Mattia Battarra:** Writing – review & editing, Supervision, Project administration, Methodology, Funding acquisition, Conceptualization; **Giulia Cristofori:** Visualization, Resources; **Emiliano Mucchi:** Project administration, Funding acquisition.

Data availability

Data will be made available on request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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