



Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education

Editors: Uffe Thomas Jankvist, Marja van den Heuvel-Panhuizen, Michiel Veldhuis
Organised by: Freudenthal Group & Freudenthal Institute, Utrecht University
Year: 2019

e r m e
european society for research in mathematics education



Editors

Uffe Thomas Jankvist, Danish School of Education, Aarhus University, Denmark.

Marja van den Heuvel-Panhuizen, Freudenthal Group & Freudenthal Institute, Utrecht University, Netherlands; Nord University, Norway.

Michiel Veldhuis, Freudenthal Group, Utrecht University, Netherlands; iPabo University of Applied Sciences, Amsterdam, Netherlands.

Editorial Board

Mario Sánchez Aguilar; Paul Andrews; Pedro Arteaga; Fatma Aslan-Tutak; Michal Ayalon; Jonas Bergman Årlebäck; Anna Baccaglioni-Frank; Anette Bagger; Arthur Bakker; Berta Barquero; Johannes Beck; Aline Bernardes; Angelika Bikner-Ahsbahs; Irene Biza; Laura Black; Laurinda Brown; Orly Buchbinder; Àngela Buforn; Susana Carreira; Renata Carvalho; Patrick Capraro; Esther Chan; Maria Chimoni; Renaud Chorlay; Anna Chronaki; Alison Clark-Wilson; Jason Cooper; Jenny Cramer; Seán Delaney; Javier Diez-Palomar; Ana Donevska-Todorova; Beatriz Vargas Dorneles; Ove Gunnar Drageset; Viviane Durand-Guerrier; Andreas Eckert; Kirstin Erath; Ingvald Erfjord; Nataly Essonnier; Eleonora Faggiano; Marie Therese Farrugia; Fiona Faulkner; Janne Fauskanger; Carla Finesilver; Marita Friesen; Daniel Frischemeier; Eirini Geraniou; Imène Ghedamsi; Inés M. Gómez-Chacón; Orlando Rafael González; Alejandro S. González-Martín; Koeno Gravemeijer; Ghislaine Gueudet; Tanja Hamann; Çiğdem Haser; Dave Hewitt; Jeremy Hodgen; Kees Hoogland; Jenni Ingram; Paola Iannone; Eva Jablonka; Britta Jessen; Heather Johnson; Keith Jones; Sibel Kazak; Beth Kelly; Jeta Kingji; Iveta Kohanová; David Kolloosche; Evelyn H. Kroesbergen; Sebastian Kuntze; Ana Kuzle; Aisling Leavy; Esther Levenson; Peter Liljedahl; Božena Maj-Tatsis; Francesca Martignone; Michela Maschietto; Maria Mellone; Christian Mercat; Vilma Mesa; Siún Nic Mhuirí; Morten Misfeldt; Joris Mithalal; Miguel Montes; Hana Moraová; Francesca Morselli; Reidar Mosvold; Andreas Moutsios-Rentzos; Reinhard Oldenburg; Samet Okumus; Antonio M. Oller-Marcén; Chrysi Papadaki; Birgit Pepin; Alon Pinto; Núria Planas; Joao Pedro da Ponte; Caterina Primi; Elisabeth Rathgeb-Schnierer; Miguel Ribeiro; Ornella Robutti; Helena Roos; Kirsti Rø; Charalampos Sakonidis; Libuse Samkova; Judy Sayers; Petra Scherer; Stanislaw Schukajlow; Priska Schöner; Marcus Schütte; Nathalie Sinclair; Karen Skilling; Michelle Stephan; Hauke Straehler-Pohl; Heidi Strømskag; Andreas Stylianides; Gabriel J. Stylianides; Osama Swidan; Michal Tabach; Rukiye Didem Taylan; Melih Turgut; Marianna Tzekaki; Behiye Ubuz; Anita Valenta; Olov Viirman; Hanna Viitala; Katrin Vorhölter; Geoff Wake; Hans-Georg Weigand; Kjersti Wæge; Constantinos Xenofontos; Stefan Zehetmeier.

Publisher

Freudenthal Group & Freudenthal Institute, Utrecht University, Netherlands and ERME

ISBN 978-90-73346-75-8

© Copyright 2019 left to the authors

Recommended citation for the proceedings

Jankvist, U. T., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (Eds.). (2019). *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*. Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Recommended citation for single entries in the proceedings

Authors (2019). [Title of paper/poster](#). In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. xxxx-yyy). Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

TABLE OF CONTENTS

Preface: CERME 11 in lovely Utrecht historic sites <i>Susanne Prediger, Ivy Kidron</i>	1
Introduction	
Introduction to the Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11) <i>Uffe Thomas Jankvist, Marja van den Heuvel-Panhuizen, Michiel Veldhuis</i>	3
Plenary lectures	
Embodied instrumentation: combining different views on using digital technology in mathematics education <i>Paul Drijvers</i>	8
History and pedagogy of mathematics in mathematics education: History of the field, the potential of current examples, and directions for the future <i>Kathleen M. Clark</i>	29
Extensions of number systems: continuities and discontinuities revisited <i>Sebastian Rezat</i>	56
Plenary panel	
ERME anniversary panel on the occasion of the 20th birthday of the European Society for Research in Mathematics Education <i>Konrad Krainer, Hanna Palmér, Barbara Jaworski, Susanne Prediger, Paolo Boero, Simon Modeste, Tommy Dreyfus, and Jana Žalská</i>	81

TWG01: Argumentation and proof	99
Introduction to the papers of TWG01: Argumentation and Proof <i>Gabriel J. Stylianides, Orly Buchbinder, Jenny Cramer, Viviane Durand-Guerrier, Andreas Moutsios-Rentzos, and Anita Valenta</i>	100
Understanding geometric proofs: scaffolding pre-service mathematics teacher students through dynamic geometry system (dgs) and flowchart proof <i>Lathiful Anwar and Martin Goedhart</i>	104
Initial participation in a reasoning-and-proving discourse in elementary school teacher education <i>Kristin Krogh Arnesen, Ole Enge, Kirsti Rø and Anita Valenta</i>	112
Epistemological obstacles in the evolution of the concept of proof in the path of ancient Greek tradition <i>Miglena Asenova</i>	120
Teachers' perspectives on mathematical argumentation, reasoning and justifying in calculus classrooms <i>Sabrina Bersch</i>	128
Integrating Euclidean rationality of proving with a dynamic approach to validation of statements: The role of continuity of transformations <i>Paolo Boero and Fiorenza Turiano</i>	136
The role of the teacher in the development of structure-based argumentations <i>Fiene Bredow</i>	145
Prospective teachers enacting proof tasks in secondary mathematics classrooms <i>Orly Buchbinder and Sharon McCrone</i>	147
Developing Pre-service Mathematics Teachers' Pedagogical Content Knowledge of Proof Schemes: An Intervention Study <i>Fikret Cihan and Hatice Akkoç</i>	155
Games as a means of motivating more students to participate in argumentation <i>Jenny Cramer</i>	163
Towards an interactional perspective on argumentation in school mathematics <i>Markos Dallas</i>	171
A pilot study on elementary pupils conditional reasoning skills and alternatives generation skills in mathematics <i>Anastasia Datsogianni, Stefan Ufer and Beate Sodian</i>	179
What do prospective mathematics teachers mean by definitions can be proved? <i>Merve Dilberoğlu, Çiğdem Haser and Erdinç Çakıroğlu</i>	181
The justification of conjectures in the study of the congruence of triangles by 5th grade students <i>Marisa Gregório and Hélia Oliveira</i>	189
Fundamental task to generate the idea of proving by contradiction <i>Hiroaki Hamanaka and Koji Otaki</i>	191

Student teachers' argumentation in primary school mathematics classrooms <i>Helene Hauge</i>	199
The interplay of logical relations and their linguistic forms in proofs written in natural language <i>Kerstin Hein</i>	201
Mathematical Arguments in the Context of Mathematical Giftedness Analysis of Oral Argumentations with Toulmin <i>Simone Jablonski and Matthias Ludwig</i>	209
"Using geometry, justify (...)". Readiness of 14-year-old students to show formal operational thinking <i>Edyta Juskowiak</i>	217
On the effect of using different phrasings in proving tasks <i>Leander Kempen, Petra Carina Tebaartz and Miriam Krieger</i>	225
Task design for developing students' recognition of the roles of assumptions in mathematical activity <i>Kotaro Komatsu, Gabriel J. Stylianides and Andreas J. Stylianides</i>	233
Primary students reasoning and argumentation based on statistical data <i>Jens Krummenauer and Sebastian Kuntze</i>	241
Questions and answers & but no reasoning! <i>Dorte Moeskær Larsen and Camilla Hellsten Østergaard</i>	249
Factors influencing teachers' decision making on reasoning-and proving in Hong Kong <i>Chun Yeung Lee</i>	257
On teachers' experiences with argumentation and proving activities in lower secondary mathematics classrooms <i>Silke Lekaas</i>	265
An epistemological study of recursion and mathematical induction in mathematics and computer science <i>Nicolás León</i>	274
Explanatoriness as a value in mathematics and mathematics teaching <i>Eva Müller-Hill</i>	276
Proof, reasoning and logic at the interface between Mathematics and Computer Science: toward a framework for analyzing problem solving <i>Simon Modeste, Sylvain Beauvoir, Jonathan Chappelon, Viviane Durand-Guerrier, Nicolás León and Antoine Meyer</i>	284
A systemic investigation of students' views about proof in high school geometry: the official and shadow education systems in a school unit <i>Andreas Moutsios-Rentzos and Eleni Plyta</i>	292
Meta-knowledge about definition: The case of special quadrilaterals <i>Shogo Murata</i>	300
Proof comprehension of undergraduate students and the relation to individual characteristics <i>Silke Neuhaus and Stefanie Rach</i>	302

Abduction in argumentation: Two representations that reveal its different functions <i>Chrysi Papadaki, David Reid and Christine Knipping</i>	310
Some linguistic issues on the teaching of mathematical proof <i>Yusuke Shinno, Takeshi Miyakawa, Tatsuya Mizoguchi, Hiroaki Hamanaka and Susumu Kunimune</i>	318
Planning for mathematical reasoning Surprising challenges in a design process <i>Helén Sterner</i>	326
Metacognitive activities of pre-service teachers in proving processes <i>Nele Stubbemann and Christine Knipping</i>	328
Features of mathematics teacher argumentation in classroom <i>Jorge A. Toro and Walter F. Castro</i>	336
Beyond direct proof in the approach to the culture of theorems: a case study on 10th grade students' difficulties and potential <i>Fiorenza Turiano and Paolo Boero</i>	338
Identifying key ideas in proof: the case of the irrationality of \sqrt{k} <i>Xiaoheng (Kitty) Yan and Gila Hanna</i>	346
TWG02: Arithmetic and number systems	354
Introduction to the work of TWG 2: Arithmetic and number systems <i>Elisabeth Rathgeb-Schnierer, Renata Carvalho, Beatriz Vargas Dorneles and Judy Sayers</i>	355
Use of conceptual metaphors in the development of number sense in the first years of mathematical learning <i>Natividad Adamuz-Povedano, Elvira Fernández-Ahumada, Teresa García-Pérez and Rafael Bracho-López</i>	359
Development of an intervention program in counting principles for first-grade students - Pilot study <i>Évelin Assis and Luciana Vellinho Corso</i>	367
Intervention in counting principles with first-grade students <i>Évelin Assis and Luciana Vellinho Corso</i>	369
Design of a board game around the notion of fractions <i>Alix Boissière</i>	377
Exploring students adaptive use of domain specific knowledge <i>Joana Brocardo, Catarina Delgado, Fátima Mendes, Jean Marie Kraemer</i>	379
Mental computation: An opportunity to develop students' strategies in rational number division <i>Renata Carvalho and João Pedro da Ponte</i>	387
Intervention in counting principles: teachers' perceptions on students learning profile <i>Luciana Vellinho Corso and Évelin Assis</i>	395
Teaching and learning decimal numbers: the role of numeration units <i>Lalina Coulange and Grégory Train</i>	403

The offer of tasks to work on multiplication in grades 2 and 3 <i>Sandra Gleißberg and Klaus-Peter Eichler</i>	411
Considerations on developmental stage models, learning trajectories and maybe better ways to guide early arithmetic instruction <i>Michael Gaidoschik</i>	419
Rational number operations: What understandings do children demonstrate? <i>Sofia Graça, João Pedro da Ponte and António Guerreiro</i>	427
Pairing numbers: An unconventional way of evaluating arithmetic expressions <i>Robert Gunnarsson and Ioannis Papadopoulos</i>	435
Interactions between pupils' actions and manipulative characteristics when solving an arithmetical task <i>Doris Jeannotte and Claudia Corriveau</i>	443
An analysis of understanding the algebraic structure in school mathematics: focusing on the extension of number sets <i>Kazuhiro Kurihara</i>	451
Strategy choices and formal algorithms: A mixed methods study <i>Ems Lord and Andreas J. Stylianides</i>	453
Number line estimation and quantitative reasoning: two important skills for mathematical achievement <i>Camila Peres Noguez and Beatriz Vargas Dorneles</i>	461
Mental calculations with rational numbers across educational levels <i>Ioannis Papadopoulos, Styliani Panagiotopoulou and Michail Karakostas</i>	469
Students' ability to compare fractions related to proficiency in the four operations <i>Pernille Ladegaard Pedersen and Peter Sunde</i>	477
Flexible mental calculation skills of freshmen and graduates <i>Charlotte Rechtsteiner</i>	485
A tool to evaluate students' performance in solving fraction word problems <i>Maria T. Sanz Garcia, Olimpia Figueras, and Bernardo Gómez</i>	487
Development and variance components in single-digit addition strategies in year one <i>Pernille B. Sunde, and Peter Sunde</i>	495
Learning fractions using a teaching model designed with applets and the number line: The cases of Alvaro and Fernanda <i>Carlos Valenzuela García, and Olimpia Figueras</i>	503
Reflecting on a series of studies on conceptual and procedural knowledge of fractions: Theoretical, methodological and educational considerations <i>Xenia Vamvakoussi, Maria Bempeni, Stavroula Pouloupoulou and Ioanna Tsiplaki</i>	511
Metacognition in non-routine problem solving process of year 6 children <i>Aikaterini Vissariou and Despina Desli</i>	519
TWG03: Algebraic Thinking	527
Algebraic Thinking <i>Maria Chimoni, Dave Hewitt, Reinhard Oldenburg, and Heidi Strømskag</i>	528

Teachers Conceptions of Algebra and Knowledge of Task Purposes <i>Burcu Alapala and Isil Isler-Baykal</i>	532
Strategies exhibited by good and average solvers of geometric pattern problems as source of traits of mathematical giftedness in grades 4-6 <i>Eva Arbona, María José Beltrán-Meneu and Ángel Gutiérrez</i>	534
Investigating early algebraic thinking abilities: a path model <i>Maria Chimoni, Demetra Pitta-Pantazi and Constantinos Christou</i>	542
Students expected engagement with algebra based on an analysis of exams in Norway from 1995 till 2018 <i>James Gray, Bodil Kleve and Helga Kufaas Tellefsen</i>	550
Never carry out any arithmetic: the importance of structure in developing algebraic thinking <i>Dave Hewitt</i>	558
Algebraic thinking in the shadow of programming <i>Cecilia Kilhamn and Kajsa Bråting</i>	566
Linear figural patterns as a teaching tool for preservice elementary teachers the role of symbolic expressions <i>Iveta Kohanová and Trygve Solstad</i>	574
Graphing formulas to give meaning to algebraic formulas <i>Peter Kop, Fred Janssen, Paul Drijvers and Jan van Driel</i>	582
Relational thinking and operating on unknown quantities. A study with 5 to 10 years old children <i>Denise Lenz</i>	590
Enhancing students' generalizations: a case of abductive reasoning <i>Joana Mata-Pereira¹ and João Pedro da Ponte²</i>	598
Body Motion and Early Algebra <i>Ricardo Nemirovsky, Natividad Adamuz-Povedano, Francesca Ferrara, and Giulia Ferrari</i>	606
Color-coding as a means to support flexibility in pattern generalization tasks <i>Per Nilsson and Andreas Eckert</i>	614
A classification scheme for variables <i>Reinhard Oldenburg</i>	622
Transforming equations equivalently? theoretical considerations of equivalent transformations of equations <i>Norbert Oleksik</i>	630
How students in 5th and 8th grade in Norway understand the equal sign <i>Hilde Opsal</i>	638
Fifth-grade students solving linear equations supported by physical experiences <i>Mara Otten, Marja van den Heuvel-Panhuizen, Michiel Veldhuis and Aiso Heinze</i>	646

Representational variation among elementary school students. A study within a functional approach to early algebra	654
<i>Eder Pinto, Bárbara M. Brizuela, and María C. Cañadas</i>	
Unpacking 9th grade students algebraic thinking	662
<i>Marios Pittalis and Ioannis Zacharias</i>	
Portuguese and Spanish prospective teachers functional thinking on geometric patterns	670
<i>Irene Polo-Blanco, Hélia Oliveira and Ana Henriques</i>	
When a number moves across, it changes its sign Investigating Teachers' Conceptual Understanding of Algebra	672
<i>Mark Prendergast, Aoibhinn Ní Shúilleabháin, Paul Grimes and Niamh O'Meara</i>	
Dealing with quantitative difference: A study with second graders	674
<i>Margarida Rodrigues and Lurdes Serrazina</i>	
Short note on algebraic notations: First encounter with letter variables in primary school	682
<i>Anna Susanne Steinweg</i>	
Structures identified by second graders in a teaching experiment in a functional approach to early algebra	690
<i>María D. Torres, María C. Cañadas and Antonio Moreno</i>	
Comparing the structure of algorithms: The case of long division and log division	698
<i>Christof Weber</i>	
Relationships between procedural fluency and conceptual understanding in algebra for postsecondary students	706
<i>Claire Wladis, Jay Verkuilen, Sydne McCluskey, Kathleen Offenholley, Dale Dawes, Susan Licwinko and Jae Ki Lee</i>	
Forming basic conceptions in dealing with quadratic equations	714
<i>Alexander Wolff</i>	
Provoking students to solve equations in a content-oriented fashion and not using routines by solving slightly modified standard tasks	716
<i>Simon Zell</i>	
TWG04: Geometry Teaching and Learning	724
Introduction to the papers of TWG04: Geometry Teaching and Learning	725
<i>Keith Jones, Michela Maschietto, Joris Mithalal-Le Doze and Chrysi Papadaki</i>	
A computer-based environment for argumenting and proving in geometry	729
<i>Giovannina Albano, Umberto Dello Iacono and Maria Alessandra Mariotti</i>	
The evolution of 9-year-old students' understanding of the relationships among geometrical shapes	737
<i>Melania Bernabeu, Salvador Llinares and Mar Moreno</i>	
Measuring area on the geoboard focusing on using flexible strategies	745
<i>Øyvind Jacobsen Bjørkås and Marja van den Heuvel-Panhuizen</i>	
Justifying geometrical generalizations in elementary school preservice teacher education	753
<i>Lina Brunheira and João Pedro da Ponte</i>	

Professional actions of novice teachers in the context of teaching and learning geometry <i>Caroline Bulf</i>	761
A model of the instrumentation process in dynamic geometry <i>Madona Chartouny, Iman Osta and Nawal Abou Raad</i>	769
Spatial structuring in early years <i>Joana Conceição and Margarida Rodrigues</i>	777
The development of geometrical knowledge starting from arts education <i>Catarina Delgado, Fátima Mendes and Filipe Fialho</i>	779
“Going straight”: discussions and experience at primary school <i>Maria Elena Favilla, Maria Luppi and Michela Maschietto</i>	781
Mathematical creativity and geometry: The influence of geometrical figure apprehension on the production of multiple solutions <i>Panagiotis Gridos, Athanasios Gagatsis, Iliada Elia and Eleni Deliyianni</i>	789
In-service teachers' conceptions of parallelogram definitions <i>Aehsan Haj Yahya, Wajeeh Daher and Osama Swidan</i>	797
Children s use of spatial skills in solving two map-reading tasks in real space <i>Cathleen Heil</i>	805
A step in the development of an evidence based learning progression for geometric reasoning: focus on shape and angle <i>Marj Horne and Rebecca Seah</i>	813
Task design with DGEs: The case of students’ counterexamples <i>Keith Jones and Kotaro Komatsu</i>	821
The connection between angle measure and the understanding of sine <i>Valentin Katter</i>	823
To figure out more than a solution to a geometric problem: What do prospective teachers? <i>Fátima Mendes, Ana Maria Boavida, Joana Brocardo and Catarina Delgado</i>	831
Personal geometrical work of pre-service teachers: a case study based on the theory of Mathematical Working Spaces <i>Alain Kuzniak and Assia Nechache</i>	833
Students’ reasons for introducing auxiliary lines in proving situations <i>Alik Palatnik and Tommy Dreyfus</i>	841
Construction of triangles: some misconceptions and difficulties <i>Paula Vieira da Silva and Leonor Santos</i>	849
Grade 3/4 students’ understanding of geometrical objects: Australian case studies on (mis)conceptions of cubes <i>Ann Downton, Sharyn Livy and Simone Reinhold</i>	857

TWG05: Probability and Statistics Education	865
Introduction to the work of TWG5: Probability and Statistics Education	866
Formalizing students informal statistical reasoning on real data: Using Gapminder to follow the cycle of inquiry and visual analyses	870
<i>Martin Andre, Zolt Lavicza and Theodosia Prodromou</i>	
Design of statistics learning environments	878
<i>Dani Ben-Zvi, Koeno Gravemeijer and Janet Ainley</i>	
Informal statistical inference and permutation tests	886
<i>Øyvind Jacobsen Bjørkås, Kjetil B. Halvorsen and Dag Oskar Madsen</i>	
Unravelling teachers' strategies when interpreting histograms: an eyetracking study	888
<i>Lonneke Boels, Arthur Bakker and Paul Drijvers</i>	
The importance of high-level tasks in the access of statistical errors: A study with future teachers of the first years	896
<i>Ana Caseiro, Ricardo Machado and Tiago Tempera</i>	
Design principles for short informal statistical inferences activities for primary education	904
<i>Arjen de Vetten, Aisling Leavy and Ronald Keijzer</i>	
Chilean primary school difficulties in building bar graphs	912
<i>Danilo Díaz-Levicoy and Pedro Arteaga</i>	
How Realistic Mathematics Education approach influences 6th grade students' statistical thinking	914
<i>Bedriye Doluzengin and Sibel Kazak</i>	
Repurpose and extend: making a model statistical	922
<i>Michal Dvir and Dani Ben-Zvi</i>	
The role of decision-making in the legitimation of probability and statistics in Chilean upper secondary school curriculum	930
<i>Raimundo Elicer</i>	
The challenge of constructing statistically worthwhile questions	938
<i>Daniel Frischemeier and Aisling M. Leavy</i>	
Investigating teachers' pedagogical content knowledge for statistical reasoning via the real life problem scenario	946
<i>Rukiye Gökce and Sibel Kazak</i>	
<i>It's a good score! Just looks low: Using data-driven argumentation to engage students in reasoning about and modelling variability</i>	954
<i>Orlando González and Somchai Chitmun</i>	
Prospective teachers' interdisciplinary learning scenario to promote students' statistical reasoning	962
<i>Ana Henriques, Hélia Oliveira and Mónica Baptista</i>	
The development of a domain map in probability for teacher education	964
<i>Judith Huget</i>	

Testing the negative recency effect among teacher students trying to generate random sequences <i>Olav Gravir Imenes</i>	972
Statistical Graphs Semiotic Complexity, Purpose and Contexts in Costa Rica Primary Education Textbooks <i>Maynor Jiménez-Castro, Pedro Arteaga and Carmen Batanero</i>	980
Building up students' data analytics skills to solve real world problems <i>Sibel Kazak, Taro Fujita and Manoli Pifarre Turmo</i>	988
Interweaving probability games and other mathematical areas in Tamás Varga's spirit in Hungary <i>Anna Kiss</i>	990
Pitfalls and surprises in probability: The battle against counterintuition <i>Signe Holm Knudtzon</i>	992
Why does statistical inference remain difficult? A textbook analysis for the phases of inference <i>Rüya Gökhan Koçer and Fulya Kula</i>	1000
Young children s informal statistical inference experiences through constructing a pictograph <i>Gamze Kurt Birel</i>	1002
What is taught and learnt on confidence interval? A case study <i>Kyeong-Hwa, Lee, Chang-Geun, Song and Yeong Ok, Chong</i>	1010
Prospective high school teachers' interpretation of hypothesis tests and confidence intervals <i>M^a del Mar López-Martín, Carmen Batanero and María M. Gea</i>	1018
Covariational reasoning patterns exhibited by high school students during the implementation of a hypothetical learning trajectory <i>Miguel Medina, Ernesto Sánchez and Eleazar Silvestre</i>	1026
A Case Study of Teacher Professional Development on Game- Enhanced Statistics Learning in the Early Years of Schooling <i>Maria Meletiou-Mavrotheris, Efi Papparistodemou, Loucas Tsouccas</i>	1034
The influence of the context of conditional probability problems on probabilistic thinking: A case study with teacher candidates <i>Beyza Olgun and Mine Isiksal-Bostan</i>	1042
In-service teachers' design, teaching and reflection on probability <i>Efi Papparistodemou and Maria Meletiou-Mavrotheris</i>	1050
Connecting context, statistics and software for understanding a randomization test: a case study <i>Susanne Podworny</i>	1057
Measuring probabilistic reasoning: the development of a brief version of the Probabilistic Reasoning Scale (PRS-B) <i>Caterina Primi, Maria Anna Donati, Sara Massino</i>	1065

An alternative method to compute confidence intervals for proportion <i>Antonio Francisco Roldán López de Hierro, María del Mar López-Martín and Carmen Batanero</i>	1073
Primary school students reasoning about and with the median when comparing distributions <i>Susanne Schnell and Daniel Frischemeier</i>	1075
A case study on design and application of creative tasks for teaching percentage bars and pie charts <i>Balgeum Song, Kyeong-Hwa Lee and Yeong Ok,Chong</i>	1083
Algorithms as a discovery process in frequentist approach to prediction interval <i>Jannick Trunkenwald and Dominique Laval</i>	1085
The investigative cycle: Developing a model to interpret written statistical reports of pre-service primary school teachers <i>Francisca Ubilla, Núria Gorgorió and Montserrat Prat</i>	1093
Introduction of inferential statistics in high school in Hungary <i>Ödön Vancsó, Peter Fejes-Tóth, and Zsuzsanna Jánvári</i>	1101
TWG06: Applications and modelling	1103
Introduction to the papers of TWG6: Applications and modelling <i>Berta Barquero, Susana Carreira, Jonas Bergman Ärlebäck, Britta Jessen, Katrin Vorhölter, Geoff Wake</i>	1104
The configuration of mathematical modelling activities: a reflection on perspective alignment <i>Lourdes Almeida and Susana Carreira</i>	1112
Enhancing future teachers' situation-specific modelling competencies by using staged videos <i>Alina Alwast and Katrin Vorhölter</i>	1120
An extension of the MAD framework and its possible implication for research <i>Jonas Bergman Ärlebäck and Lluís Albarracín</i>	1128
Simulations and prototypes in mathematical modelling tasks <i>Ana Margarida Baioa and Susana Carreira</i>	1136
Modelling praxeologies in teacher education: the cake box <i>Berta Barquero, Marianna Bosch, Floriane Wozniak</i>	1144
Analysis of textbook modelling tasks, in light of a modelling cycle <i>Ingeborg Katrin Lid Berget</i>	1152
Assessing Teaching Competencies for Mathematical Modelling <i>Rita Borromeo Ferri</i>	1154
On the positive influence of product-orientation in mathematical modeling: A research proposal <i>Wolfgang Bock, Martin Bracke, Patrick Capraro and Jean-Marie Lantau</i>	1162

Comprehending mathematical problem texts Fostering subjectspecific reading strategies for creating mental text representations <i>Jennifer Dröse</i>	1164
Mathematical modeling and the role of language proficiency <i>Sevinç Göksen-Zayim, Derk Pik, Rijkje Dekker and Carla van Boxtel</i>	1172
Pre-service mathematics teachers' learning through designing modelling tasks <i>Carolina Guerrero-Ortiz</i>	1174
Teachers learning to design and implement mathematical modelling activities through collaboration <i>Britta Eyrich Jessen</i>	1182
Development of group creativity in mathematical modeling <i>Hye-Yun Jung and Kyeong-Hwa Lee</i>	1190
Creating contact points between empirical modelling and theoretical modelling in teacher education: The case of pendulum problem <i>Masafumi Kaneko, Akihiko Saeki and Takashi Kawakami</i>	1192
Measuring an aspect of adaptive intervention competence in mathematical modelling processes <i>Heiner Klock and Hans-Stefan Siller</i>	1200
Research into students' evaluations of given modelling solutions and their suggestions for improved modelling - Results from two empirical studies <i>Sebastian Kuntze and Julian Eppler</i>	1208
Effective factors of a teacher training concerning the implementation of interdisciplinary STEM-modelling projects <i>Jean-Marie Lantau and Martin Bracke</i>	1216
Instrumental genesis and heuristic strategies as frameworks to geometric modeling in connecting physical and digital environments <i>Diego Lieban and Zsolt Lavicza</i>	1224
On the notion of mathematical model in educational research: Insights from a new proposal <i>Jesús Montejo-Gámez and Elvira Fernández-Ahumada</i>	1232
The extended theoretical framework of Mathematical Working Space (extended MWS): Potentialities in physics <i>Laurent Moutet</i>	1240
Downscaling and upscaling Fermi problems <i>Marta Pla-Castells and Irene Ferrando</i>	1248
Students of Development Studies learning about modelling and simulations as a research approach in their discipline <i>Amrit B. Poudel, Pauline Vos and F. LeRon Shults</i>	1256
On the role of multicriteria decision support in mathematical modelling <i>Stefan Ruzika and Lisa Schneider</i>	1264
Exploring pre-service teachers' flexibility in solving Fermi problems <i>Carlos Segura and Irene Ferrando</i>	1272

Features of modelling processes of group with visual and analytic mathematical thinking styles <i>Juhaina Awawdeh Shahbari and Rania Salameh</i>	1274
Professional competencies for teaching mathematical modelling supporting the modelling-specific task competency of prospective teachers in the teaching laboratory <i>Raphael Wess and Gilbert Greefrath</i>	1276
Horizontal mathematization: a potential lever to overcome obstacles to the teaching of modelling <i>Sonia Yvain-Prébiski and Aurélie Chesnais</i>	1284
TWG07: Adult Mathematics Education	1292
Introduction to TWG07 Adult Mathematics Education <i>Kees Hoogland, Beth Kelly and Javier Díez-Palomar</i>	1293
Doing the time in the mathematics class <i>Catherine Byrne, Brian Bowe and Michael Carr</i>	1301
Numeracy in adult education: discussing related concepts to enrich the Numeracy Assessment Framework <i>Javier Díez-Palomar, Kees Hoogland and Vincent Geiger</i>	1309
Mathematics and Physics Dialogic Gatherings: Fostering Critical Thinking Among Adult Learners <i>Javier Díez-Palomar and Evgenia Anagnostopoulou</i>	1317
Using dialogue scenes with adult mathematics learners: Research questions and methods <i>Graham Griffiths</i>	1325
Reproducing social difference in the concept of numeracy skills <i>Lisanne Heilmann</i>	1333
Numeracy and mathematics education in vocational education: a literature review, preliminary results <i>Kees Hoogland, Karen Heinsman and Paul Drijvers</i>	1341
Initiating a Common European Numeracy Framework <i>Kees Hoogland, Marlies Auer, Javier Díez-Palomar, Niamh O'Meara and Mieke van Groenestijn</i>	1349
What motivates adults to learn mathematics through trade unions in the workplace: social factors and personal feelings <i>Beth Kelly</i>	1356
Adult learners know more about maths than they think: helping learners to embrace their knowing <i>John J. Keogh, Theresa Maguire and John O'Donoghue</i>	1364
Numeracy practices in older age <i>Alina Redmer and Anke Grotlüschen</i>	1372

How theories of language-responsive mathematics can inform teaching designs for vocational mathematics <i>Lena Wessel</i>	1380
TWG08: Affect and the teaching and learning of mathematics	1388
Introduction to the work of TWG 8: Affect and the teaching and learning of mathematics <i>Stanislaw Schukajlow, Inés Ma Gómez-Chacón, Çiğdem Haser, Peter Liljedahl, Karen Skilling and Hanna Viitala</i>	1389
Verbal expression of emotions as entry points to examine failure experiences in secondary mathematics: A preliminary study <i>Lucy Alcântara, Nélia Amado and Susana Carreira</i>	1393
Learning geometry in self-made tutorials: the impact of producing mathematical videos on emotions, motivation and achievement in mathematical learning <i>Daniel Barton</i>	1401
Adolescents' Endorsement of Narratives Regarding the Importance of Mathematics: A Dialogic Perspective <i>Naama Ben-Dor and Einat Heyd-Metzuyanin</i>	1403
Learning styles in mathematics - the strength of the motivational factors at 10th grade Portuguese students <i>Miguel Figueiredo and Henrique Manuel Guimarães</i>	1411
Undergraduate students' mindsets in a computer programming mathematical learning environment <i>Wendy Ann Forbes, Joyce Mgombelo, Sarah Gannon and Chantal Buteau</i>	1419
Situation Specific-Skills Working Backward Reasoning: The Student's Perceptions and Affect Dimension <i>Inés M. Gómez-Chacón, Marta Barbero</i>	1427
Teacher-student eye contacts during whole-class instructions and small-group scaffolding: A case study with multiple mobile gaze trackers <i>Eeva Haataja, Visajaani Salonen, Anu Laine, Miika Toivanen and Markku S. Hannula</i>	1435
Collaborative processes when reasoning creatively about functions <i>Ellen Kristine Solbrekke Hansen</i>	1443
What can metaphor tasks offer for exploring preservice mathematics teachers beliefs? <i>Çiğdem Haser</i>	1445
"I don't like Maths as a subject but I like doing it": A methodology for understanding mathematical identity <i>Fionnán Howard, Siún Nic Mhuirí and Maurice O'Reilly</i>	1453
Students' Engagement in Inquiry-based Learning: Cognition, Behavior and Affect <i>Styliani K. Kourti</i>	1461
Investigating emotional intensity in mathematics classrooms: an enhanced methodology or affective gimmickry? <i>Elizabeth Lake</i>	1469

Pupils' perception of their understanding in mathematics and its connection to private supplementary tutoring <i>Gabriela Novotná</i>	1477
Reflections on the development and application of an instrument to access mathematical identity <i>Patricia Eaton, Christine Horn, Elizabeth Oldham and Maurice O'Reilly</i>	1485
Upper Secondary Mathematics Teachers' epistemological beliefs concerning the nature of mathematics <i>Marilena Pantziara, Alexandra Petridou and Marianna Karamanou</i>	1493
Engagement in mathematics through digital interactive storytelling <i>Giovannina Albano, Anna Pierri and Maria Polo</i>	1501
Self-concept in university mathematics courses <i>Stefanie Rach, Stefan Ufer and Timo Kosiol</i>	1509
First-person vicarious experiences as a mechanism for belief change <i>Annette Rouleau, Natalia Ruiz, Cristián Reyes and Peter Liljedahl</i>	1517
I am scared to make a drawing. Students' anxiety and its relation to the use of drawings, modelling, and gender <i>Stanislaw Schukajlow, Judith Blomberg, Johanna Rellensmann</i>	1525
Distinguishing engagement from achievement: understanding influential factors for engaged and disengaged low achieving mathematics students <i>Karen Skilling</i>	1533
Do students perceive mathematics and the mathematical subdomain of functions differently with regard to their self-concept and interest? <i>Ute Sproesser, Markus Vogel, Tobias Doerfler and Andreas Eichler</i>	1541
How can teachers influence their students' (mathematical) mindset? <i>Marloes van Hoeve, Michiel Doorman and Michiel Veldhuis</i>	1549
Low achiever's mathematical thinking: The case study of Maya <i>Hanna Viitala</i>	1557
The impact of a mathematics game programming project on student motivation in grade 8 <i>Eline Westerhout, Paul Drijvers and Arthur Bakker</i>	1565
"I want a high-educated job that pays well and is fun": Secondary students' relevance beliefs for taking advanced mathematics <i>Anders Wiik and Pauline Vos</i>	1573
TWG09: Mathematics and Language	1581
Introduction to TWG09: Transforming language-sensitive mathematics education research into papers and posters <i>Núria Planas, Marie Therese Farrugia, Jenni Ingram and Marcus Schütte</i>	1582
Taking advantage of the different types of mathematical languages to promote students' meaningful learning <i>Helen Alfaro Viquez</i>	1590

Learning mathematics in French at the undergraduate level in Algeria <i>Nadia Azrou</i>	1598
Textual configurations as an approach to evaluate textual difficulties of mathematical tasks <i>David Bednorz and Michael Kleine</i>	1606
Relations between school achievement and language abilities in mathematics classrooms <i>Aurélie Chesnais</i>	1614
Construction of mathematical discourses on generalization during group interaction <i>Judit Chico</i>	1622
Metacognitive and discursive activities - an intellectual kernel of classroom discussions in learning mathematics <i>Elmar Cohors-Fresenborg</i>	1630
Explorative study on language means for talking about enlarging figures in group work <i>Kirstin Erath</i>	1632
Young children's mathematising during free play with 'loose parts' <i>Marie Therese Farrugia</i>	1640
High school teachers' evaluation of argumentative texts in mathematics <i>Giovannina Albano, Cristina Coppola and Pier Luigi Ferrari</i>	1648
The role of writing in the process of learning to speak mathematically <i>Marei Fetzer</i>	1656
Solving problems collaboratively in multi-age classes - a possibility for learning? <i>Rachel-Ann Friesen, Marcus Schütte, Judith Jung</i>	1664
Exploration of patterns in different contexts <i>Anne Birgitte Fyhn and Lisbet Hansen</i>	1672
Centripetal and centrifugal forces in teacher-class dialogues in inquiry-based mathematics <i>Ingólfur Gíslason</i>	1680
The importance of a meaning-related language for understanding multiplication <i>Daniela Götze</i>	1688
Claims and demonstrations of understanding in whole class interactions <i>Jenni Ingram and Nick Andrews</i>	1696
Talking about standardized units in preschool supporting language and mathematical learning <i>Sarah Keuch and Birgit Brandt</i>	1704
Mathematical reasoning as a classroom discourse <i>Farran Mackay, Marieke Thurlings, Alexander Schueler-Meyer, and Birgit Pepin</i>	1712
Researching translanguaging: Functions of first and second languages in Maltese mathematics classrooms <i>Angel Mizzi</i>	1714

Adopting a discursive lens to examine functions learning and language use by bilingual undergraduate students <i>Máire Ní Riordáin and Éilis Flanagan</i>	1722
Specialized language support in mathematics education through the use of radio resources <i>Franziska Peters</i>	1730
Classroom communication: Defining and characterizing perpendicular lines in high school algebra <i>Valentina Postelnicu</i>	1738
Disentangling students' personal repertoires for meaning-making: The case of newly arrived emergent multilingual students <i>Susanne Prediger, Ángela Uribe, and Taha Kuzu</i>	1746
Preservice teachers' reflections on language diversity <i>Toril Eskeland Rangnes and Andrea Synnøve Blomsø Eikset</i>	1754
Sense and reference of signifiers for elements of polygons <i>Frode Rønning and Heidi Strømskag</i>	1762
Abstract thinking and bilingualism: Impact on learning mathematics <i>Leila Salekhova and Nail Tuktamishov</i>	1770
Teaching mathematics in an international class: Designing a path towards productive disposition <i>Marc Sauerwein</i>	1778
Which factors coincide with mathematical learning gains in bilingual classrooms? German language proficiency and mixed language use <i>Alexander Schüler-Meyer, Susanne Prediger, and Henrike Weinert</i>	1786
Publishing as an English non dominant language author. First results from a survey of support offered by mathematics education journals <i>Rudolf Sträßer</i>	1794
Variations in students' reading process when working on mathematics tasks with high demand of reading ability <i>Frithjof Theens</i>	1802
Generalizing distributive structures in primary school <i>Annika Umierski and Kerstin Tiedemann</i>	1810
TWG10: Diversity and Mathematics Education: Social, Cultural and Political Challenges	1818
Introduction to the work of TWG10: Diversity and Mathematics Education: Social, Cultural and Political Challenges <i>Hauke Straehler-Pohl, Anette Bagger, Laura Black, Anna Chronaki and David Kollosche</i>	1819
Construction of Critical Thinking Skills by the infusion approach in Probability and Statistics in Daily Life <i>Einav Aizikovitsh-Udi</i>	

The politics of early assessment in mathematics education <i>Anette, Helena Vennberg and Lisa Björklund Boistrup</i>	1831
Critical uses of knowledge and identity: Embedded mathematics as a site for/of class struggle in educational praxis <i>Laura Black, Julian Williams, David Swanson, Sophina Choudry and Emilia Howker</i>	1839
Mathematics education s solidarity assimilation methodology <i>Tânia Cabral, Alexandre Pais and Roberto Baldino</i>	1846
Assessing students' perceptions of democratic practices in the mathematics classroom <i>Wajeeh Daher</i>	1854
Diversity, inclusion and the question of mathematics teacher education - How do student teachers reflect a potential-related view? <i>Timo Dexel, Ralf Benölken and Marcel Veber</i>	1862
A farewell to mathematics: A personal choice or social exclusion? <i>Oğuzhan Doğan</i>	1870
A call for nuancing the debate on gender, education and mathematics in Norway <i>Trine Foyen</i>	1878
The relevance of mathematics and students' identities <i>Anduaem Tamiru Gebremichael</i>	1886
Social inequalities in mathematics from a socialization theoretical point of view - Analysis of problem-solving processes of students <i>Belgüzar Kara, Bärbel Barzel</i>	1894
Towards guidelines for the analysis of teaching materials in linguistically and culturally diverse mathematics classrooms <i>David Kolloosche</i>	1896
How do we teach mathematics to refugee students? A qualitative study of the teaching and learning of mathematics in International Preparatory Classes <i>Maike Lüssenhop and Gabriele Kaiser</i>	1904
Using photo-elicitation in early years mathematics research <i>Dorota Lembrér</i>	1912
Referring and proffering: An unusual take on what school mathematics is about <i>Jean-François Maheux, Jérôme Proulx, Rox-Anne L'Italien-Bruneau and Marie-Line Lavallée-Lamarche</i>	1920
The contextual power dynamics in defining and utilising problem solving tasks: A case study at an Egyptian private school <i>Mariam Makramalla and Andreas J. Stylianides</i>	1928
Discourse of otherness in a Universally Designed undergraduate mathematics course <i>Juuso Henrik Nieminen</i>	1936
Lávvu and mathematics <i>Siv Ingrid Nordkild and Ole Einar Hætta</i>	1944

Interdisciplinarity, culturally sustaining pedagogies, and the problem of pandisability as culture: Co-creating diverse mathematics learning contexts <i>Alexis Padilla and Paulo Tan</i>	1946
The teaching of higher education mathematics by pre-service mathematics teacher educators: How might this contribute to social justice? A consideration of a possible approach <i>Hilary Povey</i>	1954
The student's perspective on school mathematics - a case study <i>Ann-Sofi Røj-Lindberg and Kirsti Hemmi</i>	1962
Agency and identity of female Arab students entering a technological university <i>Soryna Sabbah and Einat Heyd-Metzuyanin</i>	1970
Students' experiences of learner autonomy in mathematics classes <i>Shipra Sachdeva</i>	1978
Locally disrupting institutional racism by enacting mathematics in a U. S. laboratory classroom <i>Sabrina Bobsin Salazar</i>	1986
Visible pedagogy and challenging inequity in school mathematics <i>Pete Wright</i>	1994
Real-life mathematics: Politicization of natural life and rethinking the sovereign <i>Ayşe Yolcu</i>	2002
TWG11: Comparative studies in mathematics education	2010
Thematic working group 11: Comparative studies in mathematics education <i>Paul Andrews, Eva Jablonka, and Constantinos Xenofontos</i>	2011
Order of factors in multiplying decimal numbers and gender differences: a comparison of tasks <i>Cascella Clelia and Giberti Chiara</i>	2014
The centrality of the unit of analysis in comparative research in mathematics education: comparing analytical accounts of student collaborative activity in different social groups <i>David Clarke and Man Ching Esther Chan</i>	2022
Impact of access requirements on the mathematical knowledge of students admitted to Primary Teaching programs: a microcomparative study <i>Núria Gorgorió, Lluís Albarracín, and Anu</i>	2031
Introduction to early algebra in Estonia, Finland and Sweden - some distinctive features identified in textbooks for Grades 1-3 <i>Kirsti Hemmi, Madis Lepik, Lars Madej, Kajsa Bråting and Joakim Smedlund</i>	2039
A preliminary comparison of Chinese and German state mandated curricula for mathematics education (years 1 to 6) <i>Eva Jablonka</i>	2047
Reform on a shaky ground? A comparison of algebra tasks from TIMSS and Swedish textbooks <i>Kristina Palm Kaplan</i>	2055

Time series analysis: Moving averages as an approach to analysing textbooks <i>Jöran Petersson, Judy Sayers and Paul Andrews</i>	2057
A cross-cultural study of curriculum systems: mathematics curriculum reform in the U.S., Finland, Sweden, and Flanders <i>Hendrik Van Steenbrugge, Heidi Krzywacki, Janine Remillard, Tuula Koljonen, Rowan Machalow and Kirsti Hemmi</i>	2062
The use of problem in upper-primary and lower-secondary textbooks of the Republic of Cyprus <i>Constantinos Xenofontos</i>	2073
A cross-cultural comparative study into teachers questioning patterns in lower secondary mathematics lessons in the UK and China <i>Wenping Zhang and David Wray</i>	2081
TWG12: History in Mathematics Education	2089
Introduction to the papers of TWG12: History in Mathematics Education <i>Renaud Chorlay, Aline Bernardes, Tanja Hamann and Antonio Oler-Marcén</i>	2090
Generalizing a 16th century arithmetic problem with prospective Secondary education teachers <i>Álvaro Barreras and Antonio M. Oller-Marcén</i>	2093
On the didactical function of some items from the history of calculus <i>Stephan Berendonk</i>	2101
Justifying a calculation technique in years 3 and 6 <i>Renaud Chorlay</i>	2109
New Math at primary schools in West Germany a theoretical framework for the description of educational reforms <i>Tanja Hamann</i>	2117
Relevance of mathematics journals for Dutch teachers in the 18th and 19th century <i>Jenneke Krüger</i>	2125
Semiotic potential of a tractional machine: a first analysis <i>Michela Maschietto, Pietro Milici and Dominique Tournès</i>	2133
On the historical development of algorithms - hidden in technical devices? <i>Regina Möller and Peter Collignon</i>	2141
Historical tasks to foster problematization <i>Bruna Moustapha-Corrêa Aline Bernardes and Victor Giraldo</i>	2149
The development of Thales theorem throughout history <i>Slim MRABET</i>	2158
Original sources, ICT and mathemacy <i>Marianne Thomsen and Inge Marie Olsen</i>	2160
Rizanesander's Recknekonsten or "The art of arithmetic" - the oldest known textbook of mathematics in Swedish <i>Reza Russell Hatami and Johanna Pejlare</i>	2162

I Spy with my Little Eye – Teachers’ linkages about historical snippets in textbooks <i>Sebastian Schorcht</i>	2170
Exams in calculations/mathematics in Norway 1946 2017 content and form <i>Bjørn Smestad and Aina Fossum</i>	2170
History of mathematics in Dutch teacher training <i>Desiree van den Bogaart-Agterberg</i>	2180
History of mathematics and current developments in education <i>Ysette Weiss and Rainer Kaenders</i>	2188
Lucas Bunt and the rise of statistics education in the Netherlands <i>Bert Zwaneveld and Dirk De Bock</i>	2196
TWG13: Early Years Mathematics	2204
Early Years Mathematics: Introduction to TWG 13 <i>Ingvald Erfjord, Bozena Maj-Tatsis, Esther Levenson, Priska Sprenger and Marianna Tzekaki</i>	2205
Pips (times) Pods: Dancing towards multiplicative thinking <i>Sandy Bakos and Nathalie Sinclair</i>	2209
Framework for analysing children s ways of experiencing numbers <i>Camilla Björklund and Ulla Runesson Kempe</i>	2217
Pre-schoolers ability to synchronise multiple representations of numerosity in embodiment of a counting-on-strategy <i>Morten Bjørnebye</i>	2225
Development of strategies for a combinatorial task by a 5 year old child <i>Kerstin Bräuning</i>	2233
Kindergarten teachers’ orchestration of mathematical learning activities: the balance between freedom and structure <i>Svanhild Breive</i>	2241
Creative processes of one primary school child working on an open-ended task <i>Svenja Bruhn</i>	2249
Characterising the mathematical discourse in a kindergarten <i>Ingvald Erfjord, Martin Carlsen, and Per Sigurd Hundeland</i>	2257
Paper plate patterns: teachers developing patterning in pre-school <i>Sue Gifford and Helen Thouless</i>	2265
Investigating mediation strategies used by early years mathematics teachers in Malawi <i>Fraser Gobede</i>	2267
Developing mathematical literacy in an inquiry-based setting working with play-coins in a second-grade classroom <i>Benedikte Grimeland and Svein Arne Sikko</i>	2269
Student teachers' definitions of the concept teaching mathematics in preschool <i>Maria Johansson, Timo Tossavainen, Ewa-Charlotte Faarinen, and Anne Tossavainen</i>	2277

Using finger patterns - the case of communicating age <i>Miriam M. Lüken</i>	2285
Evidence of relational thinking at kindergarten level <i>Andrea Maffia and Simona Mancarella</i>	2293
Task characteristics that promote mathematical reasoning among young students: An exploratory case study <i>Bozena Maj-Tatsis and Konstantinos Tatsis</i>	2301
Exploring how primary teacher education prepares pre-service teachers to teach early years Mathematics <i>Justina Longwe Mandala</i>	2309
Preschool children's understanding of length and area measurement in Japan <i>Nanae Matsuo and Nagisa Nakawa</i>	2311
The use of structure as a matter of language <i>Lara Müller and Kerstin Tiedemann</i>	2319
Children's use of mathematical language <i>Beate Nergård</i>	2321
Fostering children's repeating pattern competencies by physical activity <i>Kathrin Nordemann and Thomas Rottmann</i>	2329
Toddlers exploring structural elements in play <i>Hanna Palmér and Camilla Björklund</i>	2331
What can five-, six- and seven-year-olds tell us about the transition from mathematics in kindergarten to that in school in Norway? <i>Geir Olaf Pettersen and Monica Volden</i>	2339
Inquiry-based implementation of a mathematical activity in a kindergarten classroom <i>Chrysanthi Skoumpourdi</i>	2347
Perceiving and using structures in sets the contribution of eye-tracking in a three- level evaluation process <i>Priska Sprenger and Christiane Benz</i>	2355
Construction of scientific basis for pre-school teacher education <i>Christina Svensson</i>	2363
Shedding light on preschool teachers' self-efficacy for teaching patterning <i>Pessia Tsamir, Dina Tirosh, Esther Levenson, Ruthi Barkai</i>	2365
Generalization in early arithmetic <i>Marianna Tzekaki and Evaggelia Papadopoulou</i>	2373
Characterisation of the learning trajectory of children aged six to eight years old when acquiring the notion of length measurement <i>Yuly Vanegas, Montserrat Prat and Alba Rubio</i>	2381
Chances and obstacles of 'indirect' learning processes in situations with preschool teachers <i>Anna-Marietha Vogler</i>	2389

Developing a social training of spatial perception and spatial cognition <i>Martina Wernicke, Jasmin Kizilirmak, Barbara Schmidt-Thieme and Kristian Folta-Schoofs</i>	2397
TWG14: University Mathematics Education	2399
Introduction to the papers of TWG14: University Mathematics Education <i>Alejandro S. González-Martín, Irene Biza, Jason Cooper, Imène Ghedamsi, Ghislaine Gueudet, Vilma Mesa, Alon Pinto and Olov Viirman</i>	2400
How Lisa's mathematical reasoning evolved at undergraduate level on the role of metacognition and mathematical foundation <i>Aaltje Berendina Aaten, Gerrit Roorda, Johan Deprez and Martin Goedhart</i>	2408
Students' conceptions of the definite integral in the first year of studying science at university <i>Inen Akrouti</i>	2416
The transition from high school to university mathematics: the effect of institutional issues on students' initiation into a new practice of studying mathematics <i>Amalia-Christina Bampili, Theodossios Zachariades and Charalampos Sakonidis</i>	2418
Students' understanding of the interplay between geometry and algebra in multidimensional analysis: representations of curves and surfaces <i>Matija Bašić and Željka Milin Šipuš</i>	2426
Understanding the secondary-tertiary transition in mathematics education: contribution of theories to interpreting empirical data <i>Christer Bergsten and Eva Jablonka</i>	2434
External Didactic Transposition in Undergraduate Mathematics <i>Marianna Bosch, Thomas Hausberger, Reinhard Hochmuth and Carl Winsløw</i>	2442
Students' difficulties to learn derivatives in the Tunisian context <i>Rahma Bouguerra</i>	2450
The integration of digital resources into teaching and learning practices of the derivative concept <i>Danilo Christo and Sonia Iglioni</i>	2452
Restored narratives on the agency and disputes of mathematicians and mathematics educators in the education of mathematics teachers <i>Cleber Dias da Costa Neto, Victor Giraldo and Elena Nardi</i>	2454
Evaluation of a connecting teaching format in teacher education <i>Andreas Datzmann and Matthias Brandl</i>	2462
Students difficulties at the secondary-tertiary transition: the case of random variables <i>Camille Doukhan and Ghislaine Gueudet</i>	2464
Participation of female students in undergraduate Mathematics at the University of Malawi <i>Levis Eneya, Lisnet Mwadzaangati and Mercy Kazima</i>	2472
Supporting the use of study skills in large mathematics service courses to enhance students' success - one example <i>Frank Feudel and Hans M. Dietz</i>	2474

Digital learning materials in traditional lectures and their evaluation at the example of a voluntary pre-university bridging course <i>Yael Fleischmann, Tobias Mai, Rolf Biehler and Alexander Gold</i>	2482
Study and research paths at university level: managing, analysing and institutionalizing knowledge <i>Ignasi Florensa, Berta Barquero, Marianna Bosch and Josep Gascón</i>	2484
Two situations for working key properties of \mathbb{R} <i>Macarena Flores González, Alain Kuzniak, Elizabeth Montoya Delgadillo and Laurent Vivier</i>	2492
To whom do we speak when we teach proofs? <i>Mika Gabel and Tommy Dreyfus</i>	2494
Towards an interplay between TDS and ATD in a Design-Based Research project at the entrance to the university <i>Imène Ghedamsi and Thomas Lecorre</i>	2502
The graph of a function and its antiderivative: a praxeological analysis in the context of Mechanics of Solids for engineering <i>Alejandro S. González-Martín and Gisela Hernandez-Gomes</i>	2510
Calculus students' difficulties with logical reasoning <i>Gabriel Herrera Alva, Antonio Rivera Figueroa and Kinrha Aguirre-De la Luz</i>	2518
The VEMINT-Test: Underlying Design Principles and Empirical Validation <i>Reinhard Hochmuth, Marcel Schaub, Andreas Seifert, Regina Bruder and Rolf Biehler</i>	2526
An exploration of the relationship between continuous assessment and resource use in a service mathematics module <i>Emma Howard, Maria Meehan and Andrew Parnell</i>	2534
How to assess students learning in mathematics literacy education: An attempt to use students' comments for assessment <i>Mitsuru Kawazoe</i>	2542
On tasks that lead to praxeologies formation: a case in vector calculus <i>Margo Kondratieva</i>	2544
What can be 'annoying' about mathematical conventions? Analysing post-exchanges of mathematically competent discursants <i>Igor' Kontorovich</i>	2552
Beliefs about learning attributed to recognized college mathematics instructors <i>Valentin A. B. Küchle and Shiv S. Karunakaran</i>	2560
Features of innovative lectures that distinguish them from traditional lectures and their evaluation by attending students <i>Christiane Kuklinski, Michael Liebendörfer, Reinhard Hochmuth, Rolf Biehler, Niclas Schaper, Elisa Lankeit, Elena Leis and Mirko Schürmann</i>	2562
Students' work with a task about logical relations between various concepts of multidimensional differentiability <i>Elisa Lankeit and Rolf Biehler</i>	2570

Teaching of discontinuous functions of one or two variables: A didactic experience using problem posing and levels of cognitive demand <i>Uldarico Malaspina and Carlos Torres</i>	2578
Identifying sense-making in algebra instruction at U.S. post-secondary colleges <i>Angeliki Mali, Anne Cawley, Irene Duranczyk, Vilma Mesa, April Ström and Laura Watkins</i>	2586
From university mathematics students to postsecondary teachers <i>Sarah Mathieu-Soucy, Claudia Corriveau and Nadia Hardy</i>	2594
The structure of EQIPM, a video coding protocol to assess the quality of community college algebra instruction <i>Vilma Mesa, Irene Duranczyk, Laura Watkins and the AI@CC Research group</i>	2596
University Student Use of Dynamic Textbooks: An Exploratory Analysis <i>Vilma Mesa, Angeliki Mali and Elena Castro-Rodríguez</i>	2604
An exploratory study of calculus students' understanding of multivariable optimization problems <i>Thembinkosi P. Mkhathshwa</i>	2606
Towards a better understanding of engineering students use and orchestration of resources: Actual Student Study Paths <i>Birgit Pepin and Zeger-Jan Kock</i>	2614
Conceptualising knowledge of mathematical concepts or procedures for diagnostic and supporting measures at university entry level <i>Guido Pinkernell</i>	2622
Formative assessment of proof comprehension in undergraduate mathematics: Affordances of iterative lecturer feedback <i>Alon Pinto and Jason Cooper</i>	2630
Relations between academic knowledge and knowledge taught in secondary education: Klein's second discontinuity in the case of the integral <i>Gaëtan Planchon</i>	2638
Theorizing coordination and the role of course coordinators <i>Chris Rasmussen, Jessica E. Hagman and Naneh Apkarian</i>	2640
OPTES+ A Mathematical Bridging Course for Engineers <i>Anna-Katharina Roos, Gerhard Götz, Hans-Georg Weigand and Jan Franz Wörler</i>	2642
The connectivity in resources for student-engineers: the case of resources for teaching sequences <i>Hussein Sabra</i>	2644
TWG15: Teaching Mathematics with Technology and Other Resources	2652
Introduction to the papers of TWG15: Teaching Mathematics with Technology and Other Resources <i>Alison Clark-Wilson, Ornella Robutti, Melih Turgut and Iveta Kohanová</i>	2653
Teachers' strategic choices when implementing technology <i>Henrik P. Bang, Niels Grønbaek and Claus R. Larsen</i>	2661

Turning dilate from point tool into part of an instrument: an example of a preservice mathematics teacher working on a dynamic geometry system <i>Gulay Bozkurt, Candas Uygan and Melih Turgut</i>	2663
Towards linking teaching, technology and textbooks <i>Maike Braukmüller, Angelika Bikner-Ahsbabs and Dirk F. Wenderoth</i>	2671
Project in preparation - Connected classroom technology (CCT) to enhance formative assessment in mathematics education <i>Mats Brunström and Maria Fahlgren</i>	2679
Key Factors for Successfully Embedding a Programming Approach to the Primary Maths Curriculum at Scale <i>Alison Clark-Wilson, Richard Noss, Celia Hoyles, Piers Saunders and Laura Benton</i>	2681
The co-design of a c-book by students and teachers as a process of meaning generation: The case of co-variation <i>Dimitris Diamantidis, Chronis Kynigos, Ioannis Papadopoulos</i>	2689
Addressing the problem of digital tools with digital methods <i>Jonas Dreyøe</i>	2697
The forgotten technology. Teachers' use of mini white-boards to engage students <i>Thomas F. Eidissen, Karen Bjerkeli, and Ove Gunnar Drageset</i>	2699
Investigating similar triangles using student-produced videos <i>Anders Støle Fidje and Ingvald Erfjord</i>	2701
Silent video tasks: Towards a definition <i>Bjarnheiður (Bea) Kristinsdóttir, Freyja Hreinsdóttir and Zsolt Lavicza</i>	2709
A new era of manipulatives: making your own resources with 3D printing and other technologies <i>Diego Lieban, Eva Ulbrich, Marina Menna Barreto and Zsolt Lavicza</i>	2711
Surveying teachers' conception of programming as a mathematics topic following the implementation of a new mathematics curriculum <i>Morten Misfeldt, Attila Szabo and Ola Helenius</i>	2713
Designing an problem for learning mathematics with programming <i>Morten Munthe</i>	2721
Developing MAP for integrating mathematical applets in teaching sequence <i>Gal Nakash-Stern and Anat Cohen</i>	2723
The impact of technology on the teachers' use of different representations <i>Helena Rocha</i>	2731
An overview of gamification and gamified educational platforms for mathematics teaching <i>Cecilia Russo and Zsolt Lavicza</i>	2739
Professional development through a web-based portal: The progress of mathematics teachers teaching algebra based on hypothetical learning trajectories <i>Dilek Tanışlı, Nilüfer Köse and Melih Turgut</i>	2741

Self-efficacy the final obstacle on our way to teaching mathematics with technology? <i>Daniel Thurm and Bärbel Barzel</i>	2749
How to distinguish simulations? Development of a classification scheme for digital simulations for teaching and learning mathematics <i>Jan Franz Wörler</i>	2757
Mathematical practices of teachers in technology-enhanced classrooms: A case of teaching slope concept <i>Seçil Yemen Karpuzcu and Mine Işıksal Bostan</i>	2765
TWG16: Learning Mathematics with Technology and Other Resources	2773
Introduction to the papers of TWG16: Learning Mathematics <i>Hans-Georg Weigand, Nathalie Sinclair, Ana Donevska-Todorova, Eleonora Faggiano, Eirini Geraniou, Osama Swidan</i>	2774
Desirable difficulties while learning mathematics: Interleaved practice based on e-learning <i>Maria Afrooz and Rita Borromeo Ferri</i>	2778
Types of graphic interface design and their role in learning via mathematical applets at the elementary school <i>Eitan Ben-Haim, Anat Cohen and Michal Tabach</i>	2780
Meanings in Mathematics: using Internet Memes and Augmented Reality to promote mathematical discourse <i>Giulia Bini, Ornella Robutti</i>	2788
Students Process and Strategies as They Program for Mathematical Investigations and Applications <i>Chantal Buteau, Eric Muller, Kirstin Dreise, Joyce Mgombelo, and Ana Isabel Sacristán</i>	2796
Big events in Mathematics using math trails <i>Amélia Caldeira, Ana Moura and Christian Mercat</i>	2804
Multiplicative reasoning through two-handed gestures <i>Sean Chorney, Canan Gunes and Nathalie Sinclair</i>	2806
Physical and virtual classroom in the learning of mathematics: analysis of two episodes <i>Agnese Del Zozzo and George Santi</i>	2814
Design of a multi-dimensional instrument for reviewing the quality of apps for elementary geometry <i>Ana Donevska-Todorova and Katja Eilerts</i>	2816
Student engagement in mobile learning activities: breakdowns and breakthroughs <i>Khristin Fabian</i>	2818
A picture is worth a thousand words: visualizing collaboration through gaze synchrony graphs <i>Enrique Garcia Moreno-Esteva, Jessica F. A. Salminen-Saari, Miika Toivanen & Markku S. Hannula</i>	2826

University students' engagement with an asynchronous online course on digital technologies for mathematical learning <i>Eirini Geraniou and Cosette Crisan</i>	2828
Dynamic vs. static! Different visualisations to conceptualize parameters of quadratic functions <i>Lisa Göbel and Bärbel Barzel</i>	2836
The effect of digital tools on visual attention during problem solving: Variance of gaze fixations when working with GeoGebra or on paper <i>Markku S. Hannula and Miika Toivanen</i>	2838
An Exploration of the effect of Bray's Activity Design Heuristics on Students' Learning of Transformation Geometry <i>Aoife Harrison, Aibhín Bray and Brendan Tangney</i>	2846
Didactical resource purposes as an aspect of students' decision making regarding resources used to learn mathematics <i>Eivind Hillesund</i>	2848
Guidelines for design of didactic sequences utilizing dynamic geometry affordances related to mathematical reasoning competency <i>Ingi Heinesen Højsted</i>	2850
Issues in modelling terms involving subtraction in a manipulative environment for linear equations and a possible solution <i>Thomas Janßen, David Reid & Angelika Bikner-Ahsbals</i>	2852
Exploring with digital media to understand trigonometric functions through periodicity <i>Myrto Karavakou, Chronis Kynigos</i>	2860
Moving fingers and the density of rational numbers: An inclusive materialist approach to digital technology in the classroom <i>Doyen Kim and Oh Nam Kwon</i>	2868
Investigating students use of dynamic materials addressing conceptions related to functional thinking <i>Edith Lindenbauer</i>	2876
A virtual environment dedicated to spatial geometry to help students to see better in space <i>Xavier Nicolas and Jana Trgalova</i>	2884
Students perceptions in a situation regarding eigenvalues and eigenvectors <i>José Orozco-Santiago, Carlos-Armando Cuevas and Luc Trouche</i>	2892
Why digital tools may (not) help by learning about graphs in dynamics events? <i>Sonia Palha</i>	2900
Learning environments applying digital learning tools to support argumentation skills in primary school: first insights into the project <i>Melanie Platz</i>	2908
Digital media support functional thinking: How a digital selfassessment tool can help learners to grasp the concept of function <i>Hana Ruchniewicz and Bärbel Barzel</i>	2916

Exploring pre-calculus with augmented reality. A design-based-research approach <i>Florian Schacht and Osama Swidan</i>	2925
Exploring critical aspects of students' mathematics learning in technology-enhanced and student-led flipped learning environments <i>Stefanie Schallert and Robert Weinhandl</i>	2933
Practicing place value: How children interpret and use virtual representations and features <i>Axel Schulz and Daniel Walter</i>	2941
Exploring non-prototypical configurations of equivalent areas through inquiring-game activities within DGE <i>Carlotta Soldano and Cristina Sabena</i>	2949
The graphing calculator as an instrument of semiotic mediation in the construction of the function concept <i>Manuela Subtil and António Domingos</i>	2957
Drawing topology using Ariadne <i>Moritz L. Sümmermann</i>	2959
Semiotic analysis of modelling activities in a rich-digital environment <i>Osama Swidan and Eleonora Faggiano</i>	2961
Automated feedback on the structure of hypothesis tests <i>Sietske Tacoma, Bastiaan Heeren, Johan Jeuring and Paul Drijvers</i>	2969
Problem-solving techniques in the context of an educational video game: the Mudwall puzzle in Zoombinis <i>Georgios Thoma and Irene Biza</i>	2977
Repeated sampling in a digital environment: A remix of data and chance <i>Marianne van Dijke-Droogers, Paul Drijvers and Arthur Bakker</i>	2985
Reflections on item characteristics of non-routine items in diagnostic digital assessment <i>Irene van Stiphout and Madelon Groenheiden</i>	2993
Role of tablets in teaching and learning mathematics <i>Fabrice Vandebrouck and Barbara Jaworski</i>	3001
The long-term effects of MathCityMap on the performance of German 15 year old students concerning cylindrical tasks <i>Joerg Zender and Matthias Ludwig</i>	3003
Exploring the role of context in students meaning making for algebraic generalization <i>Angela Zoupa and Giorgos Psycharis</i>	3011

TWG17: Theoretical Perspectives and Approaches in Mathematics

Education Research 3019

Introduction to the Thematic Working Group 17 on Theoretical Perspectives and Approaches in Mathematics Education Research of CERME11 <i>Angelika Bikner-Ahsbabs, Arthur Bakker, Heather Lynn Johnson, Man Ching Esther Chan</i>	3020
--	------

Table of Contents: Thematic Working Group 17	3028
Comparing a priori analyses <i>Ivy Kidron</i>	3029
Rethinking the connection between theory and methodology: a question of mutual affordances <i>Man Ching Esther Chan, David Clarke</i>	3038
Boundary crossing by design(ing): a design principle for linking mathematics and mathematics education in preservice teacher training <i>Erik Hanke, Angelika Bikner-Ahsbabs</i>	3046
How concepts turn into objects: an investigation of the process of objectification in early numerical discourse <i>Felix Lensing</i>	3054
On the epistemology of the Theory of Objectification <i>Luis Radford</i>	3062
An epistemological and philosophical perspective on the question of mathematical work in the Mathematical Working Space theory <i>Alain Kuzniak, Laurent Vivier</i>	3070
A phenomenological methodology based on Husserl's work in the service of mathematics education research <i>Andonis Zagorianakos</i>	3078
Creating art laboratory settings and experiencing mathematical thinking: towards a philosophical dimension in mathematics education research <i>Cláudia Regina Flores</i>	3086
Networking theories in design research: an embodied instrumentation case study in trigonometry <i>Rosa Alberto, Arthur Bakker, Oia Walker-van Aalst, Peter Boon, Paul Drijvers</i>	3088
Generativity in design research: the case of developing a genre of action-based mathematics learning activities <i>Arthur Bakker, Anna Shvarts, Dor Abrahamson</i>	3096
Theory, methodology and design as an insightful bundle: a case of dual eye-tracking student-tutor collaboration on an embodied mathematical task <i>Anna Shvarts</i>	3104
Assembling mathematical concepts through trans-individual coordinated movements: the role of affect and sympathy <i>Elizabeth de Freitas, Francesca Ferrara, Giulia Ferrari</i>	3106
Locally integrating theories to investigate students' transfer of mathematical reasoning <i>Heather Lynn Johnson, Evan McClintock, Amber Gardner</i>	3114
Epistemological and methodological foundations of creating a learning trajectory of children's mathematics <i>Nicole L. Fonger, Amy Ellis, Muhammed F. Dogan</i>	3122

Conceiving teaching trajectories in the form of series of problems: a step for the theoretical reconstruction of the Hungarian Guided Discovery approach <i>Katalin Gosztonyi</i>	3130
Students' learning paths about ratio and proportion in geometry: an analysis using Peirce's theory of signs <i>Georgia Bampatsikou, Triandafillos A. Triandafillidis, Stefanos Asimopoulos, Kostas Hatzikiriakou</i>	3138
Connected Working Spaces: the case of computer programming in mathematics education <i>Jean-baptiste Lagrange, Dominique Laval</i>	3146
Multi-theoretical approach when researching mathematics teachers' professional development in selforganized online groups <i>Yvonne Liljekvist, Jorryt van Bommel, Ann-Christin Randahl, Christina Olin-Scheller</i>	3154
Extending Yackel and Cobb's socio-mathematical norms to ill-structured problems in an inquiry-based classroom <i>Katie Makar, Jill Fielding-Wells</i>	3162
Understanding powers – individual concepts and common misconceptions <i>Christos Itsios, Bärbel Barzel</i>	3170
Questioning the paradidactic ecology: internationally shared constraints on lesson study? <i>Koji Otaki, Yukiko Asami-Johansson, Jacob Bahn</i>	3178
TWG18: Mathematics teacher education and professional development	3186
International perspectives on mathematics teacher professional development <i>Stefan Zehetmeier, João Pedro da Ponte, Laurinda Brown, Àngela Buforn, Janne Fauskanger, Marita Friesen, Maria Mellone, Libuse Samkova</i>	3187
Working with example sets: A productive focus in Lesson Study <i>Jill Adler and Jehad Alshwaikh</i>	3191
Reflections upon a research project seen as a means for teachers' professional development <i>Mette Andresen</i>	3199
Portfolios as a way of documenting and reflecting learning processes in a mathematics teachers professional development program <i>Jennifer Bertram</i>	3201
The story of Maia: I will try to survive! <i>Annette Hessen Bjerke</i>	3203
Connecting mathematical knowledge with engagement in mathematics teaching practices <i>Timothy Boerst and Meghan Shaughnessy</i>	3211
High school teacher training challenges in the Italian interdisciplinary project Liceo Matematico <i>Laura Branchetti, Roberto Capone, Francesco Saverio Tortoriello</i>	3219

Researching as a mathematics teacher educator: analysing mathematics teachers' detailed descriptions of first lessons <i>Laurinda Brown</i>	3221
Using mixed-assessments to evaluate opportunities to learn in mathematics teacher education <i>Nils Buchholtz, Katrin Vorhölter, Anna Orschulik and Nadine Krosanke</i>	3229
Mathematical knowledge for teaching of a prospective teacher having a progressive incorporation perspective (PIP) <i>Esra Bukova Güzel, Gülseren Karagöz Akar, Aytuğ Özaltun Çelik, Semiha Kula Ünver, Nurdan Turan</i>	3231
Teachers' Collaboration in a Mathematics Lesson Study <i>João Pedro da Ponte and Marisa Quaresma</i>	3239
How to visualize classroom norms through social interaction - A pilot study of two frameworks <i>Jonas Dahl, Christina Svensson and Richard Wester</i>	3247
Understanding student teachers' professional development by looking beyond mathematics teacher education <i>Andreas Ebbelind</i>	3249
Retrospective reflections on 'Missions' as pedagogies of practice <i>Elisabeta Eriksen, Annette Hessen Bjerke, Camilla Rodal and Ida Heiberg Solem</i>	3257
Learning to represent students' mathematical ideas through teacher time-outs in rehearsals <i>Janne Fauskanger and Kjersti Wæge</i>	3265
How does the professional vocabulary change when pre-service teachers learn to analyse classroom situations? <i>Marita Friesen, Carmel Mesiti and Sebastian Kuntze</i>	3273
Prospective mathematics teachers' interpretative knowledge: focus on the provided feedback <i>Jeannette Galleguillos and Miguel Ribeiro</i>	3281
Using participant generated influence maps to gain insights into the influences on early career primary teachers' teaching of mathematics <i>Alison Godfrey</i>	3289
A zone theory analysis of identity formation in mathematics teacher educators <i>Merrilyn Goos and Anne Bennison</i>	3297
Analyzing attitude towards learning and teaching mathematics in members of professional learning communities: A case study <i>Birgit Griese</i>	3305
Math MOOC UniTo & MathCityMap - Exploring the potentials of a review system in a MOOC environment <i>Iwan Gurjanow, Eugenia Taranto, Matthias Ludwig, Virginia Alberti, Roberta Ferro</i>	3313
Cross-linking maths: Using keynotes to structure a curriculum for future teachers <i>Tanja Hamann and Barbara Schmidt-Thieme</i>	3321

Teacher education: Developing the individual within the collaborative <i>Fiona Haniak-Cockerham</i>	3323
Teacher Professional Development and Collegial Learning: A literature review through the lens of Activity System <i>Frida Harvey and Anna Teledahl</i>	3331
Between natural language and mathematical symbols (<, >, =): the comprehension of pre-service and preschool teachers' perspective of Numbers and Quantity <i>Dina Hassidov and Bat-Sheva Ilany</i>	3339
Similarities and differences in problem solving: case of "exhibition grounds" <i>Radka Havlíčková, David Janda, Derek Pilous and Veronika Tůmová</i>	3347
Formulating our formulations: the emergence of conviction as becoming mathematics teacher educators <i>Tracy Helliwell and Julian Brown</i>	3349
Tasks Designed for Training Secondary Mathematics Teachers Using Technology <i>Alexánder Hernández, Josefa Perdomo-Díaz and Matías Camacho-Machín</i>	3357
Opportunities for Adopting a Discourse of Explorations in a Professional Development Setting <i>Einat Heyd-Metzuyanin and Talli Nachlieli</i>	3362
On the efficiency of a professional development program for mathematics teachers in upper-secondary schools in Iceland <i>Freyja Hreinsdóttir</i>	3373
Algebraic solutions of German out-of-field elementary school teachers <i>Lara Huethorst</i>	3375
Mathematics teacher educators' critical collegiality <i>Suela Kacerja and Rune Herheim</i>	3384
Prospective Secondary Mathematics Teachers Development of Core-Practices During Methods Courses: Affordances of Quantitative Reasoning <i>Gulseren Karagöz Akar, Esra Bukova Güzel, Serkan Özel</i>	3392
From judgmental evaluations to productive conversations: Mathematics teachers' shifts in communication within a video club <i>Ronnie Karsenty, Yochai Peretz and Einat Heyd-Metzuyanin</i>	3400
Construction of teachers' roles in collegial discussions <i>Odd Tore Kaufmann and Andreas Ryve</i>	3408
From instrumental to relational Malawi mathematics teacher educators' research lessons <i>Mercy Kazima, Janne Fauskanger and Arne Jakobsen</i>	3416
How can we help teachers using guided discovery method, who have not used it before <i>Eszter Kovács-Kószó, Katalin Gosztonyi and József Kosztolányi</i>	3424
Diagnostic competence of future primary school teachers hypothesizing about causes of students' errors <i>Macarena Larrain</i>	3426

Thinking about Mathematics classroom culture through spontaneous videos <i>Cristina Loureiro, Cristina Morais and Helena Gil Guerreiro</i>	3435
Preservice teachers learning about critical mathematics education <i>Camilla Meidell, Diana Paola Piedra Moreno and Georgia Kasari</i>	3443
Retrospective competence assessment in a PD course on teaching statistics with digital tools in upper secondary schools <i>Ralf Nieszporek and Rolf Biehler</i>	3445
Learning through/about Culturally Relevant Pedagogy in Mathematics Teacher Education <i>Kathleen Nolan and Lindsay Keazer</i>	3453
Using teachers' research to elicit professional development among pre- and in-service mathematics teachers: a qualitative meta-analysis of mathematics education in graduate programs <i>Tikva Ovadiya</i>	3455
A quasi-experimental impact study of a professional development course for secondary mathematics teachers in South Africa <i>Craig Pournara and Patrick Barmby</i>	3463
Orchestrating collective mathematical discussions: practices and challenges <i>Ana Maria Roque Boavida, Ana Paula Silvestre and Cátia Prata</i>	3471
Pre-service mathematics teachers interpret observed teachers' responses to students' statements <i>Sigal Rotem, Michal Ayalon and Shula Weissman</i>	3473
Using Concept Cartoons in future primary school teacher training: the case of problem posing and open approach <i>Libuše Samková</i>	3481
Generation and generation tasks in mathematics didactics <i>Marianne Schäfer and Rita Borromeo Ferri</i>	3489
Interweaving Mathematical-News-Snapshots as a facilitator for the development of Mathematical Knowledge for Teaching <i>Ruti Segal, Atara Shriki, Nitsa Movshovitz-Hadar and Boaz Silverman</i>	3497
Examining pedagogical and classroom discourse through the lens of figured worlds: The case of an elementary school teacher <i>Galit Shabtay and Esther Levenson</i>	3505
Impacts of a mathematical mistake on preservice teachers' eliciting of student thinking <i>Meghan Shaughnessy, Rosalie DeFino, Erin Pfaff, Merrie Blunk and Timothy Boerst</i>	3513
Beliefs and expectations at the beginning of the bachelor teacher training program in mathematics <i>Evelyn Süss-Stepancik and Stefan Götz</i>	3521
Pre-service teachers' geometrical discourses when leading classroom discussions about defining and classifying quadrilaterals <i>Gry Anette Tuset</i>	3523

Developing professional development programmes with gamification for mathematics teachers in Uruguay <i>Fabián Vitabar, Zsolt Lavicza and Markus Hohenwarter</i>	3531
Action research as a potent methodology for improving teaching and learning in mathematics <i>Eda Vula</i>	3539
TWG19: Mathematics Teaching and Teacher Practice(s)	3547
Introduction to the papers of TWG19: Mathematics Teaching and Teacher Practice(s) <i>Charalampos Sakonidis, Reidar Mosvold, Ove Gunnar Drageset, Siún Nic Mhuirí, Rukiye Didem Taylan</i>	3548
Comparison of students and researchers' choice of significant events of math lessons <i>Christoph Ableitinger, Astrid Anger and Christian Dorner</i>	3556
Between-desk-teaching as a deliberate act of making content available: The case of Bernie teaching ratio <i>Nick Andrews</i>	3564
Teachers' learning goals for a mathematics lesson <i>Kristin Krogh Arnesen and Yvonne Grimeland</i>	3572
A math teacher's participation in a classroom design research: teaching of ratio and proportion <i>Rukiye Ayan, Mine Isiksal-Bostan and Michelle Stephan</i>	3580
A comparison of the treatment of mathematical errors arising from teacher-initiated and student-initiated interactions <i>Fay Baldry</i>	3588
Teacher responses to public apparent student error: A critical confluence of mathematics and equitable teaching practice <i>Hyman Bass and Reidar Mosvold</i>	3596
Conceptual learning opportunities in teachers' differentiated task designs for inclusive mathematics education <i>Christian Büscher</i>	3604
Effects of a Scaffolding Model for small groups in mathematics <i>Sharon Calor, Rijkje Dekker, Jannet van Drie, Bonne Zijlstra and Monique Volman</i>	3612
How to improve teacher students' awareness of critical aspects in a lesson plan <i>Jonas Dahl, Anna Wernberg and Cecilia Winström</i>	3620
How teachers use interactions to craft different types of student participation during whole-class mathematical work <i>Ove Gunnar Drageset</i>	3622
Beyond the immediate - illuminating the complexity of planning in mathematics teaching <i>Helena Grundén</i>	3630
The work of equitable mathematics teaching: Leading a discussion of student solutions <i>Mark Hoover and Imani Goffney</i>	3638

Learning from Lessons: A study on structure and construction of mathematics teacher knowledge - First results of case study <i>Judith Huget and Andrea Peter-Koop</i>	3646
Balancing the live use of resources towards the introduction of the Iterative Numerical method <i>Lina Kayali and Irene Biza</i>	3648
The King's birthday, potentials for developing mathematics teaching <i>Bodil Kleve, Ida Heiberg Solem and Gerd Ånestad</i>	3656
Learning trajectories and fractions: primary teachers meaning attributions <i>Anna Klothou, Charalampos Sakonidis and Vagia Arsenidou</i>	3664
Good mathematics teaching at lower primary school level <i>Marianne Maugesten</i>	3672
Analysis of differences between teachers' activity during their regular and constructivist lessons <i>Janka Medová, Kristína Bulková and Soňa Čeretková</i>	3680
Noticing mathematical potential - A proposal for guiding teachers <i>Elisabet Mellroth and Jesper Boesen</i>	3688
The work of positioning students and content in mathematics teaching <i>Reidar Mosvold and Raymond Bjuland</i>	3696
Evolving discourse of practices for quality teaching in secondary school mathematics <i>Talli Nachlieli, Yishay Mor, Einat Gil and Yael Kashtan</i>	3704
A lens on two classrooms: Implications for research on teaching <i>Siún Nic Mhuirí</i>	3712
Assessing how teachers promote students' metacognition when teaching mathematical concepts and methods <i>Edyta Nowińska</i>	3720
Teachers probing questions in mathematical classrooms connected to their practice of encouraging students to explain their thinking <i>Anna Östman</i>	3728
Linking the micro and macro context: A sociocultural perspective <i>Maria Pericleous</i>	3730
Teachers attempts to address both mathematical challenge and differentiation in whole class discussion <i>Giorgos Psycharis, Despina Potari, Chrissavgi Triantafillou and Theodossios Zachariades</i>	3738
Investigating the relation between teachers' actions and students' meaning making of mathematics <i>Karin Rudsberg, Marcus Sundhäll and Per Nilsson</i>	3746
Mathematics teaching and teachers' practice: tracing shifts in meaning and identifying potential theoretical lenses <i>Charalampos Sakonidis</i>	3754

Pre-service teachers' experiences in selecting and proposing challenging tasks in secondary classrooms <i>Leonor Santos, Hélia Oliveira, João Pedro da Ponte, Ana Henriques</i>	3762
Teachers actions in classroom and the development of quantitative reasoning <i>Lurdes Serrazina, Margarida Rodrigues and Ana Caseiro</i>	3770
Contradictions in prospective mathematics teachers' initial classroom teaching as sources for professional learning <i>Konstantinos Stouraitis and Despina Potari</i>	3778
Analyzing a novice teacher's instructional actions in response to unexpected moments in teaching <i>Rukiye Didem Taylan and Merve Esmer</i>	3786
Feedback for creative reasoning <i>Anna Teledahl and Jan Olsson</i>	3794
Revisiting teacher decision making in the mathematics classroom: a multidisciplinary approach <i>Steven Watson</i>	3802
TWG20: Mathematics teacher knowledge, beliefs, and identity	3810
Introduction to the papers of TWG20: mathematics teacher knowledge, beliefs, and identity <i>Miguel Ribeiro, Francesca Martignone, Fatma Aslan-Tutak, Kirsti Rø, Miguel Montes, Sebastian Kuntze</i>	3811
Mathematical and didactical knowledge about patterns and regularities mobilized by teachers in a professional learning task <i>Marcia Aguiar, Alessandro Jacques Ribeiro and João Pedro da Ponte</i>	3815
Mathematics Teachers Specialized Knowledge Model as a Metacognitive Tool for Initial Teacher Education <i>Álvaro Aguilar-González and Luis J. Rodríguez-Muñiz</i>	3823
Knowledge of a mathematician to teach divisibility to prospective secondary school teachers <i>Marieli Vanessa Rediske de Almeida, Miguel Ribeiro, Dario Fiorentini</i>	3831
Emergent model for teachers' conceptions of argumentation for mathematics teaching <i>Michal Ayalon & Samaher Naama</i>	3839
A student teacher's responses to contingent moment and task development process <i>Emine Aytakin and Mine Işıksal Bostan</i>	3847
Using the Knowledge Quartet to analyse interviews with teachers manipulating dynamic geometry software <i>Nicola Bretscher</i>	3855
An analysis of the nature of the knowledge disseminated by a mathematics teacher training policy: The PROFMAT case <i>Marlova Estela Caldatto and Miguel Ribeiro</i>	3863

Prospective primary teachers' knowledge about the mathematical practice of defining <i>Myriam Codes, Nuria Climent and Isabel Oliveros</i>	3871
Empowering teachers conceptually and pedagogically through supporting them in seeing connections between school mathematics and relevant advanced mathematics knowledge <i>Cosette Crisan</i>	3879
Teacher educators understanding of mathematical knowledge for teaching <i>Matthew Dahlgren, Reidar Mosvold and Mark Hoover</i>	3887
Exemplifying mathematics teacher's specialised knowledge in university teaching practices <i>Rosa Delgado-Rebolledo and Diana Zakaryan</i>	3895
The probability subjective view: developing teachers' knowledge to give sense to students' productions <i>Rosa Di Bernardo, Maria Mellone, Ciro Minichini and Miguel Ribeiro</i>	3903
"Sometimes it goes wrong!" Teachers' beliefs concerning experiments in mathematics <i>Sebastian Geisler and Sarah Beumann</i>	3905
Chinese and Dutch mathematics teachers' beliefs about inquiry-based learning <i>Luhuan Huang, Michiel Doorman and Wouter van Joolingen</i>	3913
Preservice teachers' mathematical knowledge for teaching combinatorial thinking <i>Veronika Hubeňáková and Ingrid Semanišinová</i>	3921
Malawian preservice teachers' perceptions of knowledge at the mathematical horizon <i>Everton Lacerda Jacinto and Arne Jakobsen</i>	3929
Applying the Knowledge Quartet to student teachers lesson plans: An intervention <i>Lena Karlsson</i>	3937
Preservice teachers noticing of mathematical opportunities <i>Hulya Kilic, Oguzhan Dogan, Nil Arabaci and Sena Simay Tun</i>	3939
Design of repertory grids for research on mathematics teacher conceptions of process-related mathematical thinking <i>Peter M. Klöpping and Ana Kuzle</i>	3947
Constructing tasks for primary teacher education from the perspective of Mathematics Teachers Specialised Knowledge <i>Montes, Miguel, Climent, Nuria, Carrillo, José, Contreras, Luis Carlos</i>	3955
A Look into Turkish Preservice Teachers Translation Skills: Case for Model Representations <i>Zeynep Pehlivan and Fatma Aslan-Tutak</i>	3963
Prospective primary teachers' knowledge of problem solving process <i>Juan Luis Piñeiro, Elena Castro-Rodríguez and Enrique Castro</i>	3970
Conceptualising tasks for teacher education: from a research methodology to teachers' knowledge development <i>Milena Policastro, Maria Mellone, Miguel Ribeiro and Dario Fiorentini</i>	3978

Kindergarten teachers' knowledge in and for interpreting students' productions on measurement <i>Milena Policastro, Miguel Ribeiro and Alessandra Rodrigues de Almeida</i>	3986
Developing an identity as a secondary school mathematics teacher: Identification and negotiability in communities of practice <i>Kirsti Rø</i>	3988
Students abilities on the relationship between beliefs and practices <i>Safrudiannur, and Benjamin Rott</i>	3996
Secondary school preservice teachers' references to the promotion of creativity in their master's degree final projects <i>Alicia Sánchez, Vicenç Font and Adriana Breda</i>	4004
The problem of 0.999 &: Teachers school-related content knowledge and their reactions to misconceptions <i>Verena Spratte, Laura Euhus and Judith Kalinowski</i>	4012
"In school you notice the performance gap and how different it is between the students" - Student teachers' collective orientations about the learners' heterogeneity in mathematics <i>Ann-Kristin Tewes, Elisa Bitterlich, Judith Jung</i>	4020
Teachers noticing of language in mathematics classrooms <i>Carina Zindel</i>	4028
TWG21: Assessment in mathematics education	4036
Introduction to the papers of TWG21: Assessment in mathematics education <i>Paola Iannone, Michal Ayalon, Johannes Beck, Jeremy Hodgen and Francesca Morselli</i>	4037
Strategies of formative assessment enacted through automatic assessment in blended modality <i>Alice Barana and Marina Marchisio</i>	4041
National standardized tests database implemented as a research methodology in mathematics education. The case of algebraic powers. <i>Giorgio Bolondi, Federica Ferretti, George Santi</i>	4049
Students' attitudes and responses to pair-work testing in mathematics <i>Eszter Bóra and Péter Juhász</i>	4057
Classroom assessment tasks and learning trajectories <i>Eleni Demosthenous, Constantinos Christou and Demetra Pitta-Pantazi</i>	4059
Diagnosis of basic mathematical competencies in years 8 and 9 <i>Christina Drüke-Noe and Hans-Stefan Siller</i>	4067
Assessment and argumentation: an analysis of mathematics standardized items <i>Rossella Garuti and Francesca Martignone</i>	4075
Evaluating students' self-assessment in large classes <i>Jokke Häsä, Johanna Rämö and Viivi Virtanen</i>	4083

Large scale analysis of teachers' assessment practices in mathematics <i>Julie Horoks, Julia Pilet, Sylvie Coppé, Marina De Simone and Brigitte Grugeon-Allys</i>	4091
Preservice Middle School Mathematics Teachers Development in Formative Assessment <i>Gözde Kaplan Can and Çiğdem Haser</i>	4099
An assessment of non-standardized tests of mathematical competence for Norwegian secondary school using Rasch analysis <i>Morten Riise Klegseth, Eivind Kaspersen and Trygve Solstad</i>	4107
Mathematics described proficiency levels: connecting psychometric and teaching analyses <i>Cristina Lasorsa, Rossella Garuti, Marta Desimoni, Donatella Papa, Antonella Costanzo and Rosalba Ceravolo</i>	4115
Teachers' Perceptions of Using Incentives in State Examinations to Increase the uptake of Higher Level Mathematics <i>Mark Prendergast, Páraic Treacy and Niamh O'Meara</i>	4117
Teachers use of Descriptive Assessment methods in primary school mathematics education in Iran <i>Ali Akbar Sangari, Marja van den Heuvel-Panhuizen, Michiel Veldhuis and Zahra Gooya</i>	4119
Developing and evaluating an online linear algebra examination for university mathematics <i>Christopher Sangwin</i>	4121
The role of formative assessment in fostering individualized teaching at university level <i>Annalisa Cusi and Agnese Ilaria Telloni</i>	4129
Teaching, learning and assessing in grade 10: an experimental pathway to the culture of theorems <i>Fiorenza Turiano, Paolo Boero and Francesca Morselli</i>	4137
Using student and instructor characteristics to predict student success in Algebra courses <i>Laura Watkins, Rik Lamm, Nidhi Kohli, Patrick Kimani, and the AI@CC Research group</i>	4145
TWG22: Curriculum resources and task design in mathematics education	4153
Introduction to the papers of TWG22: Curriculum resources and task design in mathematics education <i>Birgit Pepin, Sean Delaney, Andreas Eckert, Nataly Essonnier, and Andreas J. Stylianides</i>	4154
Exploring teachers' assignment of mathematical competencies to planned lessons using Epistemic Network Analysis <i>Benjamin Brink Allsopp, Jonas Dreyøe, Andreas Lindenskov Tamborg, Brendan Eagan, Sara Tabatabai, David Williamson Shaffer and Morten Misfeldt</i>	4162

Supporting the reinvention of the slope of a curve in a point: A smooth slope to slide is a smooth slide to slope <i>Rogier Bos, Michiel Doorman, Kristijan Cafuta, Selena Praprotnik, Sanja Antoliš, Matija Bašić</i>	4170
Conjecturing tasks for undergraduate calculus students <i>Sinéad Breen, Caitríona Ní Shé, and Ann O'Shea</i>	4178
Developing the task progressions framework <i>Scott A. Courtney and Dubravka Glasnović Gracin</i>	4186
Combining Differentiation and Challenge in Mathematics Instruction: A Case from Practice <i>Seán Delaney and Ann Marie Gurhy</i>	4194
Designing tasks with self-explanation prompts <i>Anneli Dyrvold & Ida Bergvall</i>	4202
Designing for digitally enriched Math Talks The case of pattern generalization <i>Andreas Eckert and Per Nilsson</i>	4210
Social creativity in the collaborative design of a digital resource embedding mathematics into a story <i>Nataly Essonnier, Chronis Kynigos and Jana Trgalova</i>	4218
Building bridges between school mathematics and workplace mathematics <i>Rune Herheim and Suela Kacerja</i>	4226
Task design with a focus on conceptual and creative challenges <i>Jonas Jäder</i>	4234
The Influence of Teacher Guides on Teachers Practice: A Longitudinal Case Study <i>Ljerka Jukić Matić and Dubravka Glasnović Gracin</i>	4242
Community documentation targeting the integration of inquiry-based learning and workplace into mathematics teaching <i>Elissavet Kalogeria and Giorgos Psycharis</i>	4244
The use of variation theory in a problem-based task design study <i>Berie Getie Kassa and Liping Ding</i>	4252
Web of problem threads (WPT) a theoretical frame and task design tool for inquiry-based learning mathematics <i>Dániel Katona</i>	4260
Secondary school mathematics teachers' selection and use of resources <i>Zeger-Jan Kock and Birgit Pepin</i>	4262
Multiplicative reasoning task design with student teachers in Scottish schools: valuing diversity, developing flexibility and making connections <i>Helen Martin</i>	4264
Comparison of Japanese and Turkish textbooks: Giving opportunities for creative reasoning in terms of proportion <i>Yasin Memiş</i>	4266

Making Mathematics fun: The Fear Room game <i>Christina Misailidou and Ronald Keijzer</i>	4274
Examination of mathematical opportunities afforded to learners in grade 1 Malawian primary mathematics textbooks <i>Lisnet Mwadzaangati</i>	4276
Two primary school teachers pedagogical design capacity of using mathematics textbooks in Delhi, India <i>Meghna Nag Chowdhuri</i>	4284
Ability maps in the context of curriculum research <i>Lisbeth Liv Nøhr, Mia Onsvig Gregersen</i>	4292
Task design fostering construction of limit confirming examples as means of argumentation <i>Annalisa Cusi and Shai Olsher</i>	4294
Towards improving teaching and learning of algorithmics by means of resources design: a case of primary school education in France <i>Rafalska Maryna</i>	4302
Length measurement in the textbooks of German and Taiwanese primary students <i>Silke Ruwisch and Hsin-Mei E. Huang</i>	4310
A Hypothetical Learning Trajectory for the learning of the rules for manipulating integers <i>Jan Schumacher and Sebastian Rezat</i>	4318
"First you have to know it exists." Cultivating teachers' thinking about resource options <i>Helen Siedel</i>	4326
Alignment of mathematics curriculum to standards at high schools in Colombia <i>Silvia Solano, Pedro Gómez and María José González</i>	4334
The concept of function in secondary school textbooks over time: An analysis made with the Theory of Conceptual Fields <i>Patricia Sureda and Laura Rossi</i>	4336
The potential of Problem Graphs as a representational tool with focus on the Hungarian Mathematics Education tradition <i>Eszter Varga</i>	4344
From teacher s naming system of resources to teacher's resource system: Contrasting a Chinese and a Mexican case <i>Chongyang Wang, Ulises Salinas-Hernández and Luc Trouche</i>	4346
TWG 23: Implementation of research findings in mathematics education	4354
Introduction to the papers of TWG23: Implementation of research findings in mathematics education <i>Mario Sánchez Aguilar, Ana Kuzle, Kjersti Wæge and Morten Misfeldt</i>	4355
Identification and diagnosis of students' mathematical misconceptions in a dynamic online environment <i>Morten Elkjær</i>	4363

Implementation research in primary education: Design and evaluation of a problem-solving innovation <i>Inga Gebel and Ana Kuzle</i>	4365
Operationalizing implementation theory in mathematics education research - identifying enablers and barriers in the Swedish “Boost for Mathematics” <i>Rikke Maagaard Gregersen, Sine Duedahl Lauridsen and Uffe Thomas Jankvist</i>	4373
A learning path for rational numbers through different representations <i>Helena Gil Guerreiro, Cristina Morais, Lurdes Serrazina and João Pedro da Ponte</i>	4381
Coherence through inquiry based mathematics education <i>Per Øystein Haavold and Morten Blomhøj</i>	4389
Competencies and curricula: Danish experiences with a two-dimensional approach <i>Tomas Højgaard and Jan Sølberg</i>	4397
Adapting implementation research frameworks for mathematics education <i>Uffe Thomas Jankvist, Mario Sánchez Aguilar, Jonas Dreyøe and Morten Misfeldt</i>	4405
Implementation through participation: Theoretical considerations and an illustrative case <i>Boris Koichu and Alon Pinto</i>	4413
Designing inquiry-based teaching at scale: Central factors for implementation <i>Dorte Moeskær Larsen, Mette Hjelmberg, Bent Lindhart, Jonas Dreyøe, Claus Michelsen and Morten Misfeldt</i>	4421
Towards a common understanding of implementation research in mathematics education research <i>Andreas Lindenskov Tamborg</i>	4429
Capturing Problem Posing landscape in a grade-4 classroom: A pilot study <i>Ioannis Papadopoulos and Nafsika Patsiala</i>	4431
Models of school governance and research implementation. A comparative study of two Swedish cases, 1960-2018 <i>Johan Prytz</i>	4439
Implementing alternative models for introducing multiplication <i>Anna Ida Säfström, Ola Helenius and Linda Marie Ahl</i>	4447
Implementing theories for preschool teaching with play-based pedagogies <i>Nina Ullsten Granlund</i>	4455
TWG24: Representations in mathematics teaching and learning	4462
Introduction to the work of TWG24: Representations in mathematics teaching and learning <i>Anna Baccaglini-Frank, Carla Finesilver, Samet Okumus and Michal Tabach</i>	4463
From concrete to abstract and back: Metaphor and Representation <i>Pierre Arnoux and Jorge Soto-Andrade</i>	4467
“He’s so fast at drawing” – Children’s use of drawings as a tool to solve word problems in multiplication and division <i>Heidi Dahl</i>	4475

Functional representations produced and used by students during their introduction to the concept of derivative: a window on their understanding processes <i>Sarah Dufour</i>	4483
Moving towards understanding graphical representations of motion <i>Carolien Duijzer, Marja Van den Heuvel-Panhuizen, Michiel Veldhuis, and Michiel Doorman</i>	4491
A training in visualizing statistical data with a unit square <i>Andreas Eichler, Carolin Gehrke, Katharina Böcherer-Linder and Markus Vogel</i>	4499
Learning to 'deal': A microgenetic case study of a struggling student's representational strategies for partitive division <i>Carla Finesilver</i>	4507
Utilizing dynamic representations to foster functional thinking <i>Stephan Michael Günster</i>	4515
Connecting the everyday with the formal: the role of bar models in developing low attainers' mathematical understanding <i>Sue Hough, Steve Gough and Yvette Solomon</i>	4523
Visualization of fractions - a challenge for pre-service teachers? <i>Ellen Konstanse Hovik and Camilla Markhus Rodal</i>	4531
Leveraging difference to promote students' conceptions of graphs as representing relationships between quantities <i>Heather Lynn Johnson, Evan McClintock, and Amber Gardner</i>	4539
Diagrammatic representations for mathematical problem solving <i>Vinay Kathotia and Elizabeth Kimber</i>	4547
Capturing 'time': characteristics of students' written discourse on dynagraphs <i>Giulia Lisarelli, Samuele Antonini and Anna Baccaglini-Frank</i>	4549
Effective choices of representations in problem solving <i>Jasmina Milinkovic, Aleksandra Mihajlovic and Mirko Dejjic</i>	4557
Geometric prediction: proposing a theoretical construct to analyze students' thinking in geometrical problem-solving <i>Elisa Miragliotta</i>	4565
Fibre mathematics: exploring topological forms through material practices <i>Kate O'Brien and Elizabeth de Freitas</i>	4573
Prospective mathematics teachers extrapolative reasoning about misleading bar graphs <i>Samet Okumus and Ercan Dede</i>	4581
The role of students' drawings in understanding the situation when solving an area word problem <i>Manuel Ponce de León Palacios and Jose Antonio Juárez López</i>	4589
Exploring Strategies Used to Solve a Non-Routine Problem by Chilean Students; an Example of "Sharing Chocolates" <i>Farzaneh Saadati and Mayra Cerda</i>	4591

Who is right? Theoretical analysis of representational activities <i>Michal Tabach and Boris Koichu</i>	4593
Teaching practice regarding grade 3 pupils use of representations <i>Isabel Velez, João Pedro da Ponte and Lurdes Serrazina</i>	4601
Explaining geometrical concepts in sign language and in spoken language a comparison <i>Annika M. Wille and Christof Schreiber</i>	4609
Characterizing fraction addition competence of preservice teachers using Rasch analysis <i>Marius Lie Winger, Julie Gausen, Eivind Kaspersen and Trygve Solstad</i>	4617
TWG25: Inclusive Mathematics Education – challenges for students with special needs	4619
Introduction to the work of TWG25: Inclusive Mathematics Education challenges for students with special needs <i>Petra Scherer, Evelyn Kroesbergen, Hana Moraová, Helena Roos</i>	4620
Strategies that promote the mathematical activity of students with language disorders: an analysis of language interactions <i>Raquel Isabel Barrera-Curin, Laurie Bergeron and Audrey Perreault</i>	4628
Low entrance or reaching the goals? Mathematics teachers' categories for differentiating with open-ended tasks in inclusive classrooms <i>Sarah Buró and Susanne Prediger</i>	4636
Teaching mathematics to students with intellectual disability: What support do teachers need? <i>Chun-ip Fung and Dichen Wang</i>	4644
K-12 Namibian teachers' beliefs on learning difficulties in mathematics: Reflections on teachers' practice <i>Shemunyenge Taleiko Hamukwaya</i>	4652
An in-service training to support teachers of different professions in the implementation of inclusive education in the mathematics classroom <i>Laura Korten, Marcus Nührenbörger, Christoph Selter, Franz Wember and Tobias Wollenweber</i>	4660
Designing mathematical computer games for migrant students <i>Janka Medová, Kristína Bulková and Soňa Čeretková</i>	4662
Diagnosis tools of dyscalculia contribution of didactics of mathematics to numerical cognition <i>Florence Peteers and Cécile Ouvrier-Bufferet</i>	4664
I just don't like math, or I think it is interesting, but difficult... Mathematics classroom setting influencing inclusion <i>Helena Roos</i>	4672
The potential of substantial learning environments for inclusive mathematics student teachers' explorations with special needs students <i>Petra Scherer</i>	4680

Mathematical discourses of a teacher and a visually impaired pupil on number sequences: Divergence, convergence or both? <i>Angeliki Stylianidou and Elena Nardi</i>	4688
Do hearing-impaired students learn mathematics in a different way than their hearing peers? A review <i>Kinga Szűcs</i>	4696
Supporting braille readers in reading and comprehending mathematical expressions and equations <i>Annemiek van Leendert, Michiel Doorman, Paul Drijvers, Johan Pel and Johannes van der Steen</i>	4704
TWG26: Mathematics in the context of STEM education	4712
Introduction to TWG26: Mathematics in the context of STEM education <i>Behiye Ubuz, Koeno Gravemeijer, Michelle Stephan and Patrick Capraro</i>	4713
Enhancing mathematics and science learning through the use of a Human Orrery <i>Maha Abboud, Philippe Hoppenot and Emmanuel Rollinde</i>	4721
Integrating the methods of mathematical modelling and engineering design in projects <i>Imad Abou-Hayt, Bettina Dahl, and Camilla Østerberg Rump</i>	4729
Mathematics and art in primary education textbooks <i>Teresa F. Blanco, Valeria González-Roel and Jose Manuel Diego-Mantecón</i>	4737
Mathematical modeling of musical fountains and light organs - Where is the M in interdisciplinary STEM projects? <i>Wolfgang Bock, Martin Bracke and Patrick Capraro</i>	4739
STEM analysis of a module on Artificial Intelligence for high school students designed within the I SEE Erasmus+ Project <i>Laura Branchetti, Olivia Levrini, Eleonora Barelli, Michael Lodi, Giovanni Ravaioli, Laura Rigotti, Sara Satanassi and Giulia Tasquier</i>	4747
Promoting mathematics teaching in the framework of STEM integration Maria Cristina Costa and António Domingos	4749
Reflecting on the value of mathematics in an interdisciplinary STEM course <i>Nelleke den Braber, Jenneke Krüger, Marco Mazereeuw and Wilmad Kuiper</i>	4757
Complex modeling: Does climate change really exist? Perspectives of a project day with high school students <i>Maren Hattebuhr and Martin Frank</i>	4765
Inter TeTra Interdisciplinary teacher training with mathematics and physics <i>Eduard Krause, Nguyen Van Bien, Tran Ngoc Chat, Nguyen Phuong Chi, Frederik Dilling, Jochen Geppert, Kathrin Holten, Tuan Anh Le, Simon Kraus and Chu Cam Tho</i>	4767
Student mathematical preparedness for learning science and engineering at university <i>Ciara Lane and Gráinne Walshe</i>	4775
Pre-service teachers' perspectives on the role of statistics in a learning scenario for promoting STEM integration <i>Hélia Oliveira, Ana Henriques and Mónica Baptista</i>	4783

Student conception of angles and parallel lines in engineering context <i>Premkumar Pugalenti, Michelle Stephan and David Pugalee</i>	4791
Opportunities to engage in STEM practices: Technology and design course <i>Behiye Ubuz</i>	4799
Gamification with Moodle in higher education <i>Ana Júlia Viamonte and Isabel Perdigão Figueiredo</i>	4807
Complex Modeling: Insights into our body through computer tomography perspectives of a project day on inverse problems <i>Kirsten Wohak and Martin Frank</i>	4815

ADDENDUM

TWG11: Comparative studies in mathematics education	4823
Cultural effects on mathematics lessons: through the international collaborative development of a lesson in two countries <i>Stéphane Clivaz and Takeshi Miyakawa</i>	4824
TWG14: University Mathematics Education	4832
Relevant aspects of proficiency in secondary school arithmetic for a successful start in STEM subjects <i>David Schönwälder, Guido Pinkernell and Gerhard Götz</i>	4833
A mathematician's deliberation in reaching the formal world and students' world views of the eigentheory <i>Sepideh Stewart, Jonathan Epstein, Jonathan Troup and David McKnight</i>	4835
A case study on mathematical routines in undergraduate biology students' group-work <i>Floridona Tetaj and Olov Viirman</i>	4843
Recall and substantiation routines in exam scripts: injective and surjective functions <i>Athina Thoma and Elena Nardi</i>	4851
Students as Partners in Complex Number Task Design <i>Stephanie Treffert-Thomas, Barbara Jaworski, Dave Hewitt, Nikolaos Vlaseros and Marinos Anastasakis</i>	4859
Post-secondary students' enactment of identity in a programming and mathematics learning environment <i>Amanjot Toor, Joyce Mgombelo and Chantal Buteau</i>	4867
Using history of mathematics to inform the transition from school to university: Affective and mathematical dimensions <i>Seyda Uysal and Kathleen M. Clark</i>	4875
Epistemological characteristics influencing didactic choices in course planning – the cases of Basic Topology and Differential Geometry <i>Olov Viirman and Magnus Jacobsson</i>	4883
Calculus variations as figured worlds for math identity development <i>Matthew Voigt, Chris Rasmussen and Antonio Martinez</i>	4891

National standardized tests database implemented as a research methodology in mathematics education. The case of algebraic powers.

Giorgio Bolondi¹, Federica Ferretti², George Santi³

¹University of Bolzano, Faculty of Education, Bolzano, Italy; giorgio.bolondi@unibz.it

²University of Bolzano, Faculty of Education, Bolzano, Italy; federica.ferretti@unibz.it

³University of Bolzano, Faculty of Education, Bolzano, Italy; gesanti@unibz.it

In this work we show the use of the INVALSI database as an instrument to collect organized data from national standardized tests in order to study students' mathematical thinking on a large scale. It allows us to create appropriate correlations to single out the behavior of the students relative to a specific mathematical topic or a cognitive issue. In this study we have used the INVALSI database to study how grade 10 students work with powers in the domain of algebra. We analyze the quantitative data within a semiotic theoretical framework. Analyzing items taken from the INVALSI tests we are able to single out a general behavior that involves a population of Italian students and that persists in time. We interpret the results within our theoretical framework, thereby giving a quantitative validation to a theoretical perspective that has been proven consistent by qualitative investigations.

Keywords: Standardized assessments, qualitative and quantitative methodologies, algebra, semiotics, powers.

Introduction.

It is more than 10 years that the Italian Ministry of Education (MIUR) has established national standardized tests (INVALSI) in mathematics. In detail, every year since 2008 INVALSI administers Italian and Mathematics (and since 2018, English) large-scale assessment tests to Italian students from Primary School to High Secondary School. The tests are devised according to a robust theoretical framework that has been developed according to mathematics education research. As we will see, a key tool that made it possible to focus our investigation was a database containing data about the INVALSI assessment.

The aim of this paper is to show how the INVALSI database can be effectively used also for research purposes. We study how high school students deal with powers analyzing their answers to items that have been administered throughout several school years. The items have been collected using the INVALSI database GESTINV (Gestinv, 2018) in order to single out consistent data regarding powers.

Our research interest focused on syntactic aspects because INVALSI tests show that Italian high school students have severe difficulties in handling the meaning of algebraic formalism when dealing with powers. This is particularly interesting if we take into account the efforts and attention that both students and teachers devote to algebraic calculations in the first two years of high school.

Our study wants to shed some light on the following issues:

What information can standardized assessments provide about student's learning difficulties regarding powers? What research tools can we use?

What information can we acquire from standardized assessments about mathematical practices in Italian high schools?

Theoretical framework.

Gestinv is a database which includes all the administered INVALSI tests in Italy. There are now 13,000 users amongst in service and preservice teachers and researchers. As regards mathematics there are almost 1700 items available.

Gestinv contains all the items of the INVALSI tests indexed according to the National Curricula, the results from the statistical point of view, the content, the key words, the percentage of correct, wrong and invalid answers and other characteristics. It is possible to carry out searches correlating these indexes. It has been used as a research tool in several researches to single out difficulties in specific mathematics topics or problems, for example in Ferretti and Gambini (2018), or in Ferretti, Giberti and Lemmo (2018). Gestinv allows both a fine grain and a coarse grain analysis of mathematics didactical phenomena and the data can be organized according to the basic contents and the indications of the Italian national curriculum.

To create a consistent framework for our study, it is necessary to combine the potentialities of Gestinv with a theoretical lens that allows us to look into the complexity of the didactical phenomenon that we want to investigate with the database. In the case of this paper our aim is to use Gestinv to observe the behavior of students in algebra, in particular when working with powers.

The learning of algebra is a key research issue in mathematics education, that involves a leap from procedural to relational and general cognitive functioning. Such a change in cognitive perspective is underpinned by the use of semiotic representations that in algebra implies the use of symbolic language intertwined with other semiotic systems.

Duval's (1995, 2008) semiotic approach highlights a specific cognitive functioning in mathematics, due to the special epistemological nature of its objects that do not allow ostensive references. Thinking and learning in mathematics is identified with the coordination of semiotic registers via treatment and conversion. Treatment is a semiotic transformation from a representation into another within the same register and conversion is a semiotic transformation from a representation in one register into another representation in another register.

The inaccessible nature of mathematical objects leads the student in a cognitive paradox that obliges him to identify the mathematical object with its semiotic representations (Duval, 1993). The cognitive paradox can hinder the student's meaning making processes since he is unable to establish the correct relationship between the mathematical object (the signified) and several representations (its signifiers). Conversion is particularly difficult to carry out because there is no syntactic rule that binds the first representation in one register with the second in the other register. In conversion, the pupil ends up with a series of unrelated representations that he is unable to refer to the same mathematical concept. According to Duval, conversion is the key cognitive function that guarantees the conceptual acquisition of mathematical objects.

Treatments instead rest on the structure of the semiotic register that provides the pupil with the syntactic rules to go from one representation to the other via the semiotic transformation. Therefore, we expect that students are able recognize that different representations involved in treatment transformations refer to the same mathematical object. Within this framework our research questions are: Q1: What precise information, regarding powers, can we acquire from a research that implements Gestinv? Q2: Is it possible to collect information that is coherent with solid research findings? Q3: What methodological tools does Gestinv provide?

Methodology.

Our research methodology intertwines qualitative and quantitative methodologies according to Johnson and Onwuegbuzie (2004) and Iori (2018). In our research design we mixed quantitative

and qualitative methodologies according to the following scheme: QUAL -----> QUAN -----> QUAN+QUAL. From a qualitative point of view our aim was to ascertain higher school students' difficulties in algebra. In particular our focus was on the students' ability to give meaning to the syntactic structure of algebra. Gestinv allowed us to carry out a quantitative analysis based on the INVALSI grade ten mathematics items, selecting the ones with lower scores. Among these we noticed that the management of powers yielded the worst results, in particular the tasks involving treatment operations. At this point we carry out a Quan/Qual analysis that combines our semiotic lens (Qual) with the study of the characteristic curves and the distractor plots (Quan).

In Figure 3 we see an example of our basic tool. On the x-axis are the students' level of competence, measured on the basis of the whole test and on the y-axis are the probabilities of answers to the distractors in function of the level of competence. The solid line represents the curve predicted by the model (characteristic curve) whereas the dotted lines are the empirical results of the test, plotted by deciles. Characteristic curves and distractor plots are an effective quantitative tool to highlight correlations between the students' answers to a specific item according to the basic contents and the indications of the Italian national curriculum, which informs the structure of the INVALSI test.

We point out the fact that we selected items close to the kind of algebra activity usually performed in Italian high schools during the first two years of secondary school. Moreover the items were selected according to the INVALSI theoretical framework, which considers the ability to handle semiotic transformations as one of the basic cognitive processes. We used the functions of Gestinv to choose items that matched our research needs in terms of cognitive processes, mathematical content and learning objectives. We turned out with items that highlight students' difficulties in giving sense to algebraic representations as they undergo treatment transformations. It is important to keep in mind that in Italy students, when learning algebra, are mainly exposed to complicated, heavy, and long algebraic calculations, basically treatments. This is also true for the teaching and learning of powers.

Results.

The following task was administered in 2017 Mathematics Grade 10 INVALSI test.

You can write the expression $\left(a - \frac{1}{a}\right)^2$ like:

A. $a^2 + \frac{1}{a^2} - 2$

B. $\frac{a^4 + 1}{a^2}$

C. $a^2 - \left(\frac{1}{a}\right)^2$

D. $\frac{a^2 - 2a + 1}{a^2}$

Figure 1: Task in Mathematics Grade 10 INVALSI Test 2017

This INVALSI test was administered to 540.000 grade 10 students; the results refer to a sample of about 48.000 students representative of the whole population. The task requires to perform a

treatment operation in the algebraic register, applying a well-known identity. As we can see in the following figure (Figure 2), only 34% of the students gave the correct answer; the majority of students who did not solve the task correctly chose option C. This item disproves Duval's claim that conversion is the key semiotic transformation and that treatment supports students in the construction of the correct meaning of mathematical concepts, in this study, powers. All the incorrect options reveal difficulties in the treatment operations and the mathematical meaning is completely lost. For example, option C formally resembles the original algebraic expression but there is no link to the correct mathematical meaning of powers. It is interesting how the students do not rely on the algebraic transformation rules to give sense to their mathematical activity. This item provides a quantitative confirm of qualitative researches that show how treatment transformations can result in a change or loss of meaning (D'Amore, 2007; Santi, 2011). Such researches show that meaning cannot be reduced to the structure of semiotic systems, but it is rooted in the range of social activity, at a personal and cultural level. In absence of a meaningful personal activity, the student, according to the cognitive paradox, interprets the semiotic representations as if they were unrelated mathematical objects, thereby losing the reference to the common mathematical object, even if he can rely on precise transformation rules. Rules and procedures that Italian students have trained over and over again in their algebra classes.

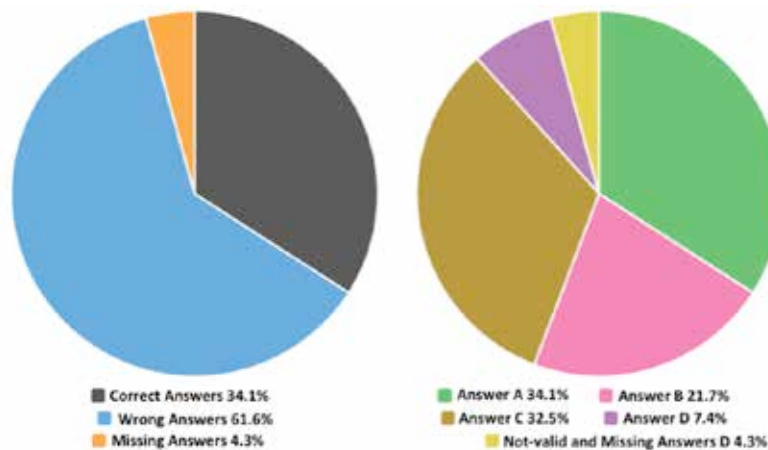


Figure 2: Results referred to the task in Mathematics Grade 10 INVALSI Test 2017

As we can see in the following graph (Figure 3), among the various options of incorrect answers, option C is the most frequently chosen at all levels of competence, up to medium-high skill levels. It testifies that this loss of meaning roots in convictions and beliefs that affect even the most competent students.

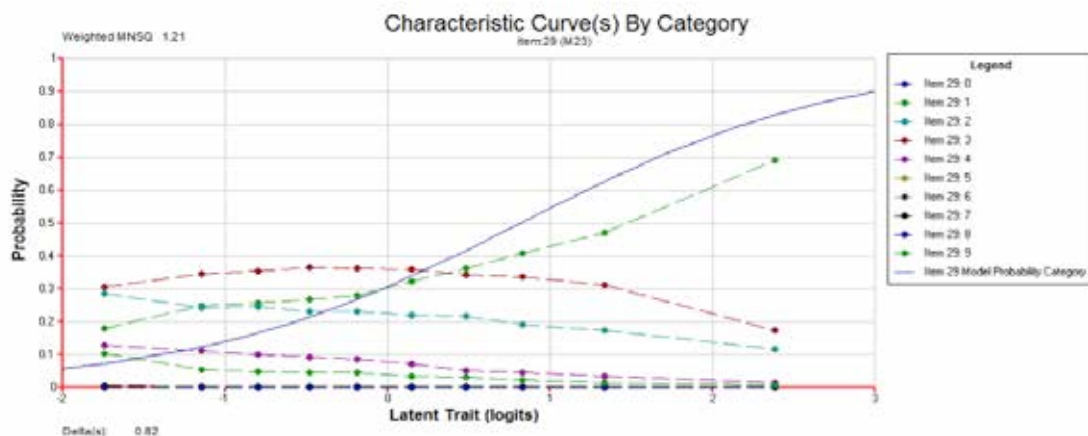


Figure 3: Characteristic Curve referred to the task in Mathematics Grade 10 INVALSI Test 2017

The following task was administered in Mathematics Grade 10 INVALSI Test in the year 2015.

The expression $a^{43} + a^{44}$ is equal to:

- A. a^{44-43}
 B. $a^{43} \cdot (a+1)$
 C. a^{87}
 D. $2a^{87}$

Figure 4: Task in Mathematics Grade 10 INVALSI Test 2015

In 2015 almost 550000 Grade 10 students performed this INVALSI test, and the national results referred to a sample of 48440 students. As we can see in the following graphs, only a third of Italian students provided the correct answer; among the incorrect options, the most chosen option is C. In option C the exponents of the two powers in the text are summed. Again, this protocol shows a loss of meaning, due to a treatment, when the student goes from the original $a^{43} + a^{44}$ to a^{87} . The expression $a^n + a^m$ puzzles the student who is not able to frame it appropriately in the context of powers, thereby he resorts to the well-known identity $a^n \cdot a^m = a^{(n+m)}$ which leads to a loss of the original meaning. This result suggests that factoring out the GCF is meaningless to most students, despite the thorough practice in terms of treatment transformations they are exposed to. Meaningless in the sense that they confuse the algebraic representations (signifiers) with the mathematical object (signified) and they are not able to establish the correct semiotic reference to the mathematical object.

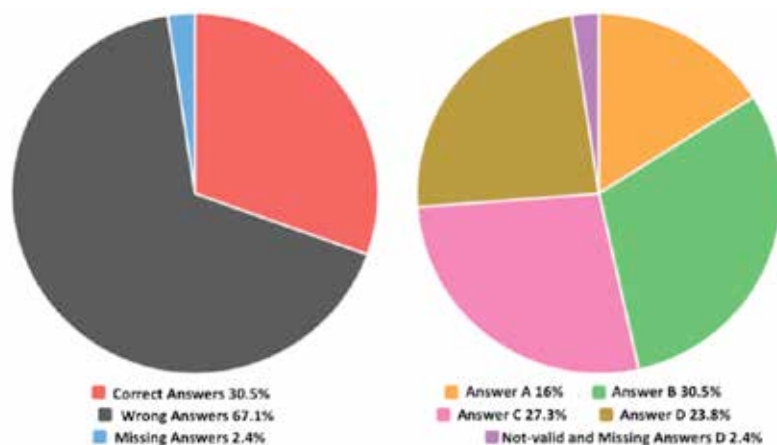


Figure 5: Results referred to the task in Mathematics Grade 10 INVALSI Test 2015

The Characteristic Curve (Figure 6), shows that, among the incorrect options, Option C is the most chosen at all levels of competence.

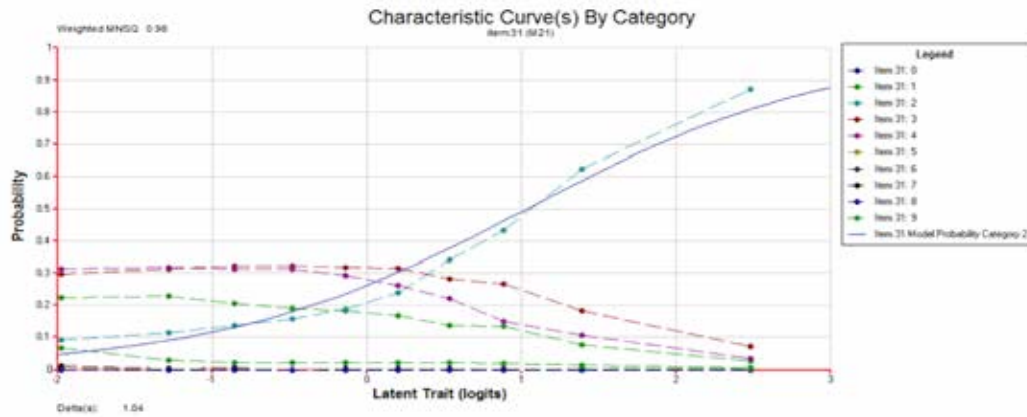


Figure 6: Characteristic Curve referred to the task in Mathematics Grade 10 INVALSI Test 2015

The following question was in the Mathematics INVALSI test that was administered in Italy in 2012 to all Grade 10 students. Almost 533.000 students performed the test and the results referred to a sample of almost 50.000 Grade 10 Italian students. To answer the task correctly, it is necessary to manipulate the exponents of an algebraic expression with two terms.

The expression $a^{37} + a^{38}$ is equal to:

- A. $2a^{75}$
- B. a^{75}
- C. $a^{37}(a+1)$
- D. $a^{37 \cdot 38}$

Figure 7: Task in Mathematics Grade 10 INVALSI Test 2012

As we can see in the following graphs (Figure 8), only 35% of the students answered correctly. More than 25% of students chose option B, in which the exponent of the power is the sum of the exponents of the power in the text. Also, in this situation, option B is the most chosen incorrect option at all levels of competences (Figure 9). If we compare this item we the former we can see that the behavior of the students is exactly the same both in terms of scores and of its characteristic curves. The combination of the last two items shows how Gestinv provides quantitative and qualitative information that we can compare across time to gain further insight on a teaching-learning phenomenon

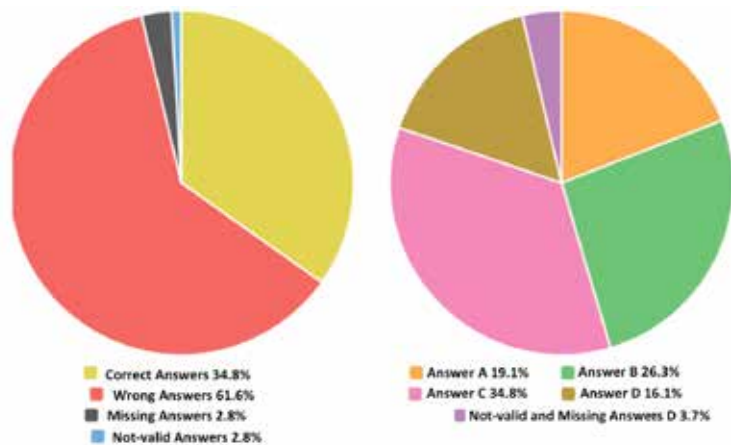
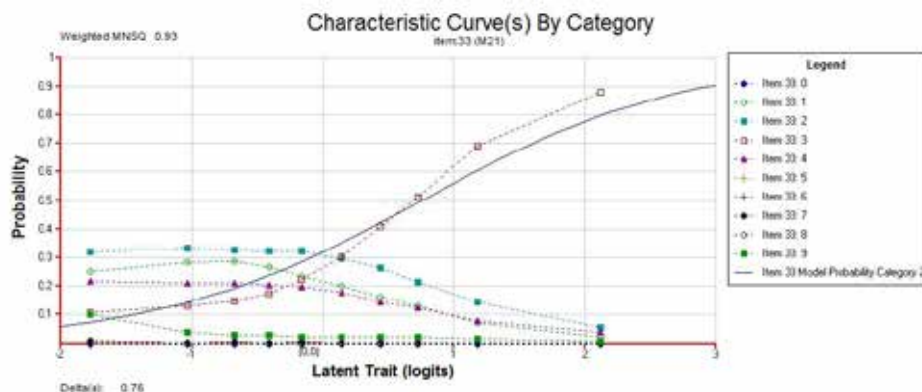


Figure 8: Results referred to the task in Mathematics Grade 10 INVALSI Test 2012**Figure 9: Characteristic Curve referred to the task in Mathematics Grade 10 INVALSI Test 2012****Answer to the research questions.**

A1) At a coarse-grained level, Gestinv highlights rooted difficulties in students facing treatments regarding powers. This is a quantitatively relevant macro phenomenon that persists with the same features across time.

A2) At a fine-grained level, Gestinv provides data that are coherent with solid research findings. In particular, it unveils at a quantitative level the phenomenon of change/loss of meaning due to treatment semiotic transformations. Several studies (D'Amore, 2007; D'Amore & Fandiño Pinilla, 2007; Santi, 2011) have shown that at all school levels, including prospective teachers, also treatment bewilders students who experience a loss or a change of meaning in treatment transformations. The loss and/or change of meaning due to treatment transformations implies that mathematical cognition in general and in particular the algebraic one cannot be reduced to a complex transformation of signs. Meaning is beyond the mere relation sign-object and it is necessary to take into account other basic features that characterize sense-making processes in mathematics. Moreover, data easily available in Gestinv show that this phenomenon also affects students with medium-high levels of competences. Thus teacher's didactical awareness is not only aimed at helping weak students but also the so-called stronger ones, devising an effective didactical transposition and didactical engineering that encompass the complexity of mathematical thinking and learning.

A3) Gestinv is an effective tool that entangles quantitative and qualitative research methodologies. As regards the quantitative approaches, they are based on a statistically significant population. It allows us to provide a quantitative validation of theoretical results, confirmed at a qualitative level. Furthermore, the characteristic curves are a powerful tool to intertwine quantitative and qualitative analyses.

Conclusions.

Gestinv implements a new research method that combines quantitative and qualitative approaches. In this study, it allowed us to highlight a macro didactical phenomenon that is quantitatively relevant. Thanks to the combination of the quantitative and qualitative approaches we were able both to frame the students' behavior within an appropriate theoretical framework and provide a statistically based quantitative validation of such a theoretical result.

From an educational point of view, Gestinv shows how the school mathematical practices confirm the research results of mathematics education.

Further research is required to understand if Gestinv, implemented as a research method, is able to single out new didactical phenomena that cannot be framed in the current perspectives and therefore prompts new theoretical research.

References

- D'Amore, B. (2007). Mathematical objects and sense: how semiotic transformations change the sense of mathematical objects. *Acta Didactica Universitatis Comenianae*, 7, 23-45.
- D'Amore, B. & Fandiño Pinilla, M.I. (2007). Change of the meaning of mathematical objects due to the passage between their different representations. How other disciplines can be useful to the analysis of this phenomenon. *Rome, Symposium on the occasion of the 100th anniversary of ICMI, March 2008. WG5: The evolution of theoretical framework in mathematics education*, organizers: Gilah Leder and Luis Radford. www.unige.ch/math/EnsMath/Rome2008.
- Duval, R. (1993). *Registres de représentations sémiotique et fonctionnement cognitif de la pensée. Annales de Didactique et de Sciences Cognitives*, ULP, IREM Strasbourg, 5, 37-65.
- Duval, R. (1995). *Sémiosis et pensée humaine: Registres sémiotiques et apprentissages intellectuels*. Berne: Peter Lang.
- Duval, R. (2008). Eight problems for a semiotic approach in mathematics education. In L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in mathematics education: Epistemology, history, classroom, and culture* (pp. 39–61). Rotterdam: Sense Publishers.
- Ferretti, F., & Gambini, A. (2017). A vertical analysis of difficulties in mathematics from secondary school to university level; some evidence stems from standardized assessment. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education*. Dublin, Ireland: DCU Institute of Education & ERME.
- Ferretti, F., Giberti, C., & Lemmo, A. (2018). The Didactic Contract to Interpret Some Statistical Evidence in Mathematics Standardized Assessment Tests. *EURASIA Journal of Mathematics, Science and Technology Education*, 14(7), 2895-2906. <https://doi.org/10.29333/ejmste/90988>
- Gestinv 2.0. (07.01.2018). *Archivio interattivo delle prove Invalsi*. Retrieved from <http://www.gestinv.it>.
- Iori, M. (2018). Teachers' awareness of the semio-cognitive dimension of learning mathematics. *Educational Studies in Mathematics*, 98(1), 95-113.
- Johnson, R. B., & Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher*, 33(7), 14-26.
- Santi, G. (2011). Objectification and semiotic function. *Educational Studies in Mathematics*, 66(77), 285–311.