ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-TUPM083

OPTIMIZATION OF SPIN COHERENCE TIME FOR ELECTRIC DIPOLE MOMENT (EDM) MEAUREMENTS IN A STORAGE RING

R. Shankar[†,1], A. Piccoli[1], P. Lenisa[1], Istituto Nazionale di Fisica Nucleare (INFN), Ferrara, Italy
[2]Università degli studi di Ferrara, Ferrara, Italy
on behalf of the JEDI Collaboration

Abstract

The JEDI experiment is dedicated to the search for the electric dipole moment (EDM) of charged particles using storage rings, which can be a very sensitive probe of physics beyond the Standard Model. In order to reach the highest possible sensitivity, a fundamental parameter to be optimized is the Spin Coherence Time (SCT), i.e., the time interval within which the particles of the stored beam maintain a net polarization greater than 1/e. To identify the working conditions that maximize SCT, accurate spin-dynamics simulations with the code BMAD have been performed on the lattice of a "prototype" storage ring which uses a combination of electric and magnetic fields for bending. This contribution presents an analysis of the mechanisms behind the decoherence, and a technique to maximize SCT through the optimization of second-order optical parameters.

INTRODUCTION

Of all the observable matter antimatter asymmetry in the universe only a small fraction is accounted for by the currently accepted Standard Model (SM). Assuming the CPT theorem to hold true, it appears that this asymmetry can only be explained by additional CP violating processes than those accounted for in the SM [1]. A noticeable manifestation of CP violation is the presence of an Electric Dipole Moment in a proton, whose magnitude can indicate the existence of additional CP violation Beyond the Standard Model (BSM). While the SM predicts an EDM $\leq 10^{-31}\ e \cdot cm$, possible contributions from BSM theories could place it orders of magnitude higher. The current upper limit on the proton EDM is $7.9 \times 10^{-25}\ e \cdot cm$ [2].

The JEDI collaboration is currently working on performing this measurement on protons using storage rings. If the spin of the proton is maintained to remain aligned with the momentum at all times (in a condition called "frozen spin"), the presence of EDM will result in a torque on the particle in response to a radial electric field. The visible effect of this torque can be observed as a gradual build-up of vertical polarisation in a stored proton bunch. To achieve a precision higher than the current lower limit on the proton EDM, the construction of a dedicated electrostatic storage ring would be needed [3, 4]

THE PROTOTYPE EDM STORAGE RING

The Prototype Storage Ring aims to demonstrate the principle of frozen spin for protons in a "scaled-down" version of the proposed ring but using a combination of electric and magnetic fields for confinement.

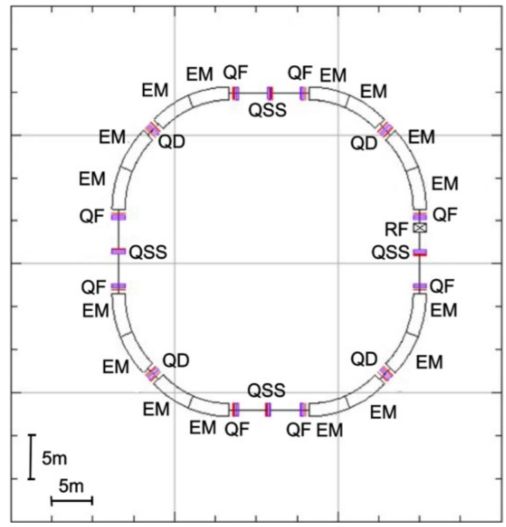

Figure 1: The software generated floor plan of the prototype EDM ring. Dipoles are labelled with 'EM', quadrupoles corresponding to their family with 'QF', 'QD' or 'QSS' and the cavity with 'RF'.

The proposed design [5], shown in Figure 1 consists of four unit-cells, each with two bending dipoles, 4 quadrupoles and 4 sextupoles to provide sufficient flexibility in beam optics.

The quadrupoles present on the ring are categorized into three families: QF (2 per unit cell, focussing), QD (1 per unit cell, defocussing) and QSS (1 per unit cell, in the straight section). The sextupoles are placed on the same locations as the quadrupoles and are categorized into similar families: SXF, SXD and SXSS. Each family of magnets have a common power supply for centralized control. During this study however, the QSS magnets were turned off. An RF-cavity is also placed at one of the straight sections for bunching (longitudinal focussing) of particles. The quadrupole and sextupole fields determine the first and second order optical properties of the lattice. Together, these optical properties form the parameter space within which the Spin Coherence Time must be optimized (see Figure 2).

SPIN COHERENCE TIME AND SPIN-TUNE ERROR

Assuming a bunch of n particles are maintained in the storage ring, let $\hat{s}_i(t)$ be the unit vector in the direction of the i^{th} particle's spin vector. The polarisation vector $\vec{P}(t)$ is [6]:

$$\vec{P}(t) = \frac{1}{n}\sum_{i=1}^{n} \hat{s}_i(t) \qquad (1)$$

ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-TUPM083

In a ring functioning in frozen-spin mode, if initially all particles distributed in phase space have their spins are aligned with their momenta ($|\vec{P}(0)| = 1$), the time τ taken for $|\vec{P}(\tau)| = \frac{1}{e}$ is defined as Spin Coherence Time (SCT).

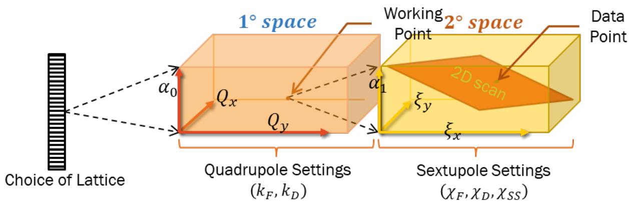

Figure 2: The organization of the parameter space explored in this study. The space formed by the betatron tunes Q_x, Q_y and the first-order momentum compaction factor α_0 is the first-order (1°) space, and the one formed by the chromaticities ξ_x, ξ_y and the second-order momentum compaction factor α_1 is the second-order (2°) space. A point in the first-order space is termed a working point, and one in the second-order space is termed a data point.

This quantity is ideal for evaluation of a storage ring for EDM measurements since very gradual polarization buildups would be noticeable only if the bunch remains spin-coherent for a long time. Therefore, longer SCT in a storage ring indicates a higher accuracy in potential EDM measurement. The spin tune spread $\Delta\theta_x$ measures the change in the direction of the polarisation vector from the reference particle in the plane of precession (here, assumed to be the ring plane):

$$\Delta\nu_s(t) = \frac{d}{dt}\big(\Delta\theta_x(t)\big) \approx \frac{d}{dt}\left(tan^{-1}\left(\frac{P_x(t)}{[\![\boldsymbol{s}_z(t)]\!]}\right)\right) \quad (2)$$

Here, P_x is the radial component of the polarization vector, and the hollow square brackets ⟦ ⟧ indicate properties of the reference particle. Also interesting is the spin tune error $\Delta\nu_x(t)$, which is the rate of change of spin tune spread.

RESULTS

demonstrates an instance of the time-development of the polarization as the particle bunch travels around the ring.

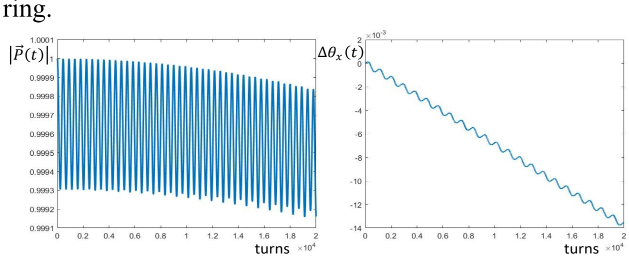

Figure 3: (left) A plot showing the decoherence of 1000 particles as a function of number of turns; (right) a plot showing the spin tune spread of the polarisation vector measured simultaneously.

From a detailed implementation of the Thomas-BMT equation on the prototype ring, it can be deduced that the change in the spin tune of an off-momentum particle stored in the ring would take the form (keeping terms up to the second order):

$$\Delta\nu_s = A\delta + B\delta^2 \quad (3)$$

, where $\delta = {}^{\Delta p}/_{p}$ is the momentum offset of the particle, and A and B are constants which depend on the Lorentz factor γ, and the G-factor of the proton. Further, from the analyses in [7] and [8], which was observed to be valid also for storage rings using electrostatic confinement (see Figure 4), the change in path length of the particle due to the contributions of both transverse and longitudinal motions is given by…

$$\frac{\Delta L}{L} = -\frac{\pi}{L}\left(\epsilon_x\xi_x + \epsilon_y\xi_y\right) + \alpha_0\delta + \alpha_1\delta^2 \\ = \alpha_1\delta^2 - \frac{\pi}{L}\epsilon_x\xi_x - \frac{\pi}{L}\epsilon_y\xi_y \quad (4)$$

…where the first-order longitudinal path-lengthening term is cancelled out in the long run by synchrotron oscillations. In general, path-lengthening effects manifest as an apparent speeding up of all particles with a non-zero emittance, which changes the effective Lorentz factor and thus the spin tune. This mechanism complements that of the momentum offset δ and can be observed in (right) showing the time development of the spin tune spread, where the local oscillations are due to the path-lengthening effect and the overall linear trend is due to the effective spin tune. Simulations at the origin ($\xi_x = 0$, $\xi_y = 0$, $\alpha_1 = 0$) show a downward trend without local oscillations due to the effect of the δ^2 term in eq. (3). Measurements of the error in the spin tune measured at different points in the vector-space $\vec{\xi} = (\xi_x, \xi_y, \alpha_1)$ have shown that $\Delta\nu_s$ can be modelled as a scalar potential with a constant gradient, and that the set of all points with $\Delta\nu_s = 0$ forms a plane in this space. The plane thus represents the second order optical configurations where the path-lengthening effect cancels out the original spin tune error.

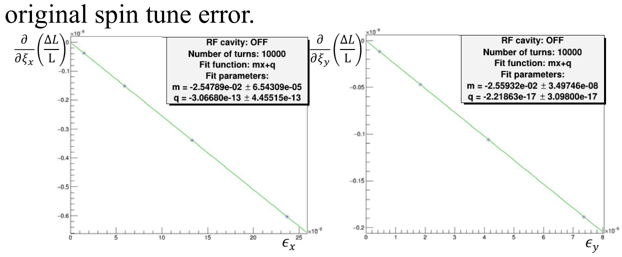

Figure 4: Plots of the rates of change of path length increase $\left({}^{\Delta L}/_{L}\right)$ with horizontal (left) and vertical (right) chromaticity, with their respective emittances, measured for the prototype lattice. The slopes of their linear fits exactly equal $-{}^{\pi}/_{L}$, confirming the validity of the formula for transverse path length variation with chromaticities as derived by [8] for the prototype ring.

The Spin Coherence Time (τ) was estimated by fitting the data in (left) with the decoherence model derived in [9]:

$$|\vec{P}(t)| \\ = |\vec{P}(0)|\left(\left[1 - \sqrt{\pi}\gamma_s(t)e^{-\gamma_s{}^2(t)}\,\mathrm{erfi}\big(\gamma_s(t)\big)\right]^2 \\ + \pi\gamma_s{}^2(t)e^{-2\gamma_s{}^2(t)}\right)^{\frac{1}{2}} \quad (5)$$

ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-TUPM083

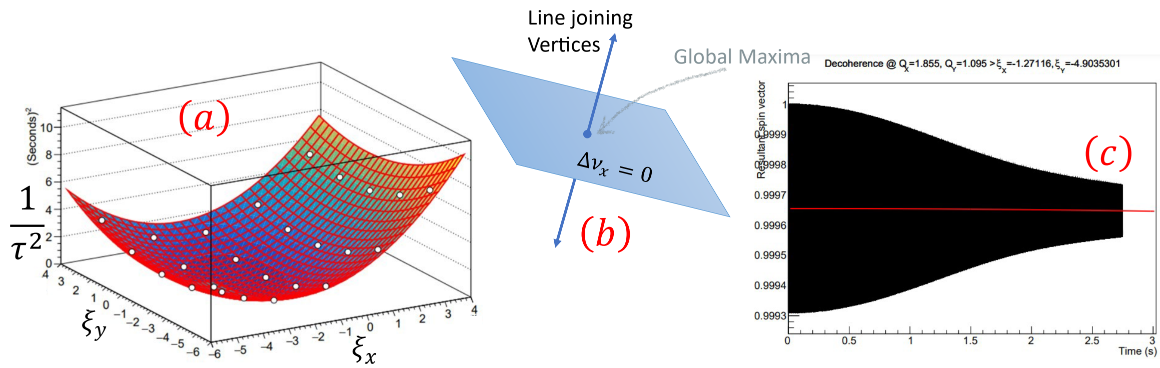

Figure 5: (a) Inverse-square of spin-coherence time almost exactly varies as a 2D paraboloid (or a family of concentric ellipses); The vertex of the paraboloid represents the optimized field setting in the chosen 2D slice. (b) A diagram showing a possible method to estimate the SCT maxima as the intersection point between the $\Delta\nu_s = 0$ plane and the line joining the vertices of many 2D paraboloid fits like (a). (c) A decoherence plot of the polarisation vector measured at an optimized point found at the intersection point depicted in (b), showing almost no decoherence.

Here, n is the turn number and $\gamma_s(t) = \sqrt{2}\pi\sigma t$ is termed the "damping parameter" where σ can be obtained from the fit. From this model, τ is obtained by solving $|\vec{P}(\tau)| = {}^1/_e$.

It was observed that the variation of ${}^1/_{\tau^2}$ across the 2° space follows the distribution of a three-dimensional paraboloid, specifically a family of ellipsoids…

$$\begin{aligned}\frac{1}{\tau^2} = \frac{1}{{\tau_0}^2} &+ L(\xi_x - \xi_x^o)^2 + M(\xi_y - \xi_y^o)^2 \\ &+ N(\alpha_1 - \alpha_1^o)^2 \\ &+ O(\xi_x - \xi_x^o)(\xi_y - \xi_y^o) \\ &+ P(\xi_y - \xi_y^o)(\alpha_1 - \alpha_1^o) \\ &+ Q(\alpha_1 - \alpha_1^o)(\xi_x - \xi_x^o)\end{aligned} \tag{6}$$

…where L, M, N, O, P, Q are constants representing the geometric properties of the paraboloid, and ξ_x^o, ξ_y^o, α_1^o are the coordinates of the optimized point where the spin coherence time reaches its maximum value (τ_0) in a given quadrupole setting. This was also confirmed using 2D paraboloid fits at different slices of the space as shown in Figure 5 (a). It was expected that the maximum spin coherence time would occur in a setting where the effective spin tune is zero, given that this is a frozen-spin lattice. The simulation results demonstrate that this is always true and can reliably be used as shown in Figure 5 (b), to narrow down the search during optimization.

Results of the optimization also show that the optimized settings always lie at negative chromaticities (like the example in Figure 5 (c)), which is contrary to the results at COSY [10], suggesting this may be an exclusive effect of the electric bending field.

Finally, optimization using these principles was performed at several quadrupole settings which exhibit optical properties within the recommended range in terms of beam lifetime [11] for this lattice, and spin coherence times of above 1000 s, which represents the target EDM sensitivity for the final lattice [3], was obtained at more than 10 points.

CONCLUSIONS

This paper presents the results of proton simulations at a storage ring in frozen-spin mode achieved using a combination of electric and magnetic bending fields. Investigations into the decoherence revealed the influence of path lengthening on the spin-tune error and subsequently the spin coherence time. The relevance of the transverse path-lengthening relation [8]…

$$\frac{\Delta L}{L} = -\frac{\pi}{L}\epsilon_x\xi_x - \frac{\pi}{L}\epsilon_y\xi_y \tag{7}$$

…for electrostatic and combined-field rings was also verified.

The study has also demonstrated the optimisation of spin coherence times of above 1000 seconds at several working points in the prototype lattice.

Also established is a robust method of optimisation which has demonstrated universality of working point, with successful optimisations at more than 90% of working points examined.

On the other hand, the study also highlights the limitations of this lattice in terms of optical flexibility, due to the placement of different sextupoles within the same straight section. Further studies shall explore new lattice configurations which could avoid these kinds of limitations.

REFERENCES

[1] A. Sakharov, "Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe," JETP Letters, vol. 5, pp. 24-27, 1967.

[2] V F Dmitriev and R A Sen'kov, "Schiff Moment of the Mercury Nucleus and the Proton Dipole Moment," Physical Review Letters, vol. 91, p. 212303, 2003.

[3] CPEDM Collaboration, "Storage ring to search for electric dipole moments for charged particles: Feasibility study," CERN Yellow Reports: Monographs, Geneva, 2021.

[4] P. Lenisa, "Search for Electric Dipole Moments of Charged Particles with Polarized Beams in Storage Rings," in The 18th International Workshop on Polarized Sources, Targets, and Polarimetry, Knoxville, 2019.

[5] A Lehrach, S Martin and R Talman, "Design of a Prototype EDM Storage Ring," in 23rd International Spin Physics Symposium, Ferrara, 2018.

[6] R. Shankar, M. Vitz and P. Lenisa, "Optimization of Spin Coherence Time at a Prototype Storage Ring for Electric

ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-TUPM083

Dipole Moment Measurements," in 24th International Spin Symposium (SPIN2021), Matsue, 2022.

[7] Marcel Stephan Rosenthal, "Experimental Benchmarking of Spin Tracking Algorithms for Electric Dipole Moment Searches at the Cooler Synchrotron COSY," PhD Thesis, RWTH Aachen University, Aachen, 2016.

[8] Yoshihiko Shoji, "Dependence of average path length betatron motion in a storage ring," Physical Review Special Topics - Accelerators and Beams, vol. 8, p. 094001, 2005.

[9] Dennis Eversmann, "High Precision Spin Tune Determination at the Cooler Synchrotron in Jülich," PhD Thesis, RWTH Aachen University, Aachen, 2018.

[10] G Guidoboni, E J Stephenson, A Wrońska and et. al., "Connection between zero chromaticity and long in-plane polarization lifetime in a magnetic storage ring," Physical review accelerators and beams, vol. 21, p. 024201, 2018.

[11] Saad Siddique, Andreas Lehrach and Jörg Pretz, "Simulations of Beam Dynamics and Beam Lifetime for a Prototype Electric Dipole Moment Ring," in 24th International Spin Symposium (SPIN2021), Matsue, 2022.