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**Amplitude analysis of
 $B^0 \rightarrow \eta_c(1S)K^+\pi^-$ decays at LHCb and
development and characterisation of a
novel photodetector for the future
upgrade of RICH detectors**

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Abstract

An amplitude analysis of the $B^0 \rightarrow \eta_c(1S)K^+\pi^-$ decay channel, with $\eta_c(1S) \rightarrow p\bar{p}$, is performed using a sample corresponding to an integrated luminosity of 9 fb^{-1} of pp collision data collected by the LHCb detector at centre-of-mass energies of $\sqrt{s} = 7, 8$, and 13 TeV . The motivation for this study lies in the exploration of the resonant structures in the $\eta_c(1S)\pi^-$ system, searching for the possible existence of exotic hadronic states beyond the conventional quark model. The data are described with a model including only intermediate contributions from known K^* resonances, and no evidence is found for an exotic resonance in the $\eta_c(1S)\pi^-$ system, previously reported in an earlier analysis of this decay channel. The inclusive branching fraction of the $B^0 \rightarrow \eta_c(1S)K^+\pi^-$ decays is measured to be

$$\mathcal{B}(B^0 \rightarrow \eta_c(1S)K^+\pi^-) = (5.82 \pm 0.20 \pm 0.23 \pm 0.55) \times 10^{-4},$$

where the first uncertainty is statistical, the second systematic, and the third arises from the limited knowledge of external branching fractions.

The LHCb Upgrade I, designed to operate at an instantaneous luminosity five times higher than in previous runs, introduces major improvements in hadron identification through upgraded Ring Imaging Cherenkov detectors. To maximise this performance, a dedicated software framework is developed to monitor and evaluate online particle identification during data taking, ensuring stable calibration and optimal operations under high-luminosity conditions.

A major upgrade planned for 2034–2036 will further increase the instantaneous luminosity in LHCb beyond $\mathcal{L} = 1.0 \cdot 10^{34} \text{ cm}^{-2}\text{s}^{-1}$, requiring a complete redesign of the Ring Imaging Cherenkov detectors. Within this framework, the 4DPHOTON project is developing next-generation photon detectors capable of single-photon counting with superior spatial and temporal resolutions, based on a vacuum tube containing a photocathode, a microchannel plate (MCP), and a Timepix4 CMOS application-specific integrated circuit as its readout anode. The first prototype is characterised through laboratory measurements, showing dark count rates between $\sim 1 \text{ Hz/cm}^2$ and $\sim 30 \text{ Hz/cm}^2$, depending on the MCP bias, with gain values ranging from $\mathcal{O}(10^3)$ to $\mathcal{O}(10^5)$. A non-uniform dark count distribution is

observed across the matrix, and further studies are underway to investigate its origin. The timing resolution yields a single-pixel resolution of 103 ± 3 ps and a cluster resolution of 88 ± 9 ps when operating the MCP at 2400 V. The results of a testbeam campaign carried out at the end of 2024 confirm the laboratory measurements, showing agreement with the Geant4 simulation, and yielding a time resolution of 283 ± 7 ps when operating the MCP at 2100 V.

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Chapter 1

Introduction

Hadrons beyond conventional $q\bar{q}$ mesons and qqq baryons have been proposed since the introduction of the Quark Model. Quantum Chromodynamics allows for the existence of nonstandard states composed of quarks and gluons, such as glueballs, hadronic molecules, and tightly bound multiquark states like tetraquarks ($qq\bar{q}\bar{q}$) and pentaquarks ($qqqq\bar{q}$) [1, 2]. The first of these so-called exotic hadron candidates, the $\chi_{c1}(3872)$ meson, was discovered in 2003 [3], followed by many charmonium- and bottomonium-like states [4, 5, 6, 7]. Some of their properties challenge the conventional $q\bar{q}$ picture. Competing models attribute these exotics to different binding mechanisms and internal structures [8, 9, 10, 11, 12, 13]. Progress in their understanding requires exploring new production and decay channels, as well as searching for additional candidates. Ch. 3 outline a theoretical overview of exotic hadrons, together with the current experimental evidences of non-conventional candidates.

During my PhD, I worked on this topic by performing an amplitude analysis of the $B^0 \rightarrow \eta_c(1S)K^+\pi^-$ decays, with the $\eta_c(1S)$ reconstructed via its $p\bar{p}$ decay mode. Alongside the amplitude analysis, the branching ratios of these decays were measured. The full available dataset was used, corresponding to Run 1 (2011–2012) at $\sqrt{s} = 7$ and 8 TeV, and Run 2 (2015–2018) at $\sqrt{s} = 13$ TeV, for a total integrated luminosity of 9 fb^{-1} . This analysis, presented in Ch. 4, contributes to the understanding of exotic hadron spectroscopy by probing new decay channels of charmonium-like states. The LHCb detector during Run 1 and Run 2 is described in Chapter 2.

A key tool enabling such analyses these decays provide powerful and complementaryes at LHCb is high-performance hadron identification (PID) over a wide range of momenta, provided by two Ring Imaging Cherenkov (RICH) detectors. Following the

LHC second long shutdown (LS2), the RICH1 optical system was fully redesigned, and the opto-electronics chains of both RICH1 and RICH2 were upgraded to support continuous 40 MHz readout and single-photon counting at hit densities up to approximately 100 MHz/cm². These improvements restore the excellent Run 1 and Run 2 performances, allowing precise separation of long-lived charged hadrons with momenta ranging from 2.6 GeV to beyond 100 GeV, even in events with up to seven visible interactions. To support these advances, I developed a tool for quasi-online evaluation of the PID performance of the LHCb RICH detectors, as discussed in Ch. 5.

Looking further ahead, the LHCb Upgrade II, described in Chapter 6, planned between 2034 and 2036 aims to operate at instantaneous luminosities above $\mathcal{L} = 1.0 \cdot 10^{34} \text{ cm}^{-2}\text{s}^{-1}$, more than five times higher than the current $\mathcal{L} = 2 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$. As the RICH detectors are central for flavour-physics measurements, they will be fully redesigned and equipped with advanced photon detectors and upgraded front-end electronics. In this context, the 4DPHOTON project is developing a novel photon detector based on a vacuum tube housing a photocathode, a microchannel plate, and a Timepix4 CMOS integrated circuit as the readout anode. This technology aims to detect up to one billion photons per second over a 7 cm² active area, achieving spatial resolution of 5–10 μm and timing resolution better than 50 ps, as detailed in Ch. 7. I contributed to this project through software development, laboratory measurements, a testbeam campaign, and the simulation of the testbeam setup.⁸

Chapter 2

The LHCb experiment at CERN

The Large Hadron Collider (LHC) is currently the world’s highest-energy particle accelerator. Located at CERN, it collides protons and ions within a 27 km tunnel beneath the Franco-Swiss border, the same tunnel that previously housed the Large Electron-Positron Collider (LEP). The LHC achieved its first proton-proton collisions in 2009, observed by its four major experiments: ALICE, ATLAS, CMS, and LHCb.

The primary objective of the LHC experiments is to explore physics beyond the Standard Model (SM), commonly referred to as new physics (NP). In particular, the LHCb experiment is optimised to search for indirect signatures of NP by conducting high-precision measurements of processes involving beauty (b) and charm (c) quarks. Deviations from Standard Model predictions, such as unexpected decay patterns or enhanced production rates caused by hypothetical particles appearing in quantum loops, could signal contributions from NP. This makes LHCb especially sensitive to phenomena that might elude direct detection at current LHC energy scales. Some anomalies that hint at potential departures from SM predictions have already been observed.

An introduction to the LHC accelerator complex and the four large experiments is given in Sec. 2.1. The LHCb experiment is described in Sec. 2.2.

2.1 The Large Hadron Collider

The Large Hadron Collider (LHC) [14] is a pp and ion collider. The choice of hadronic collisions is driven by two main advantages with respect to an electron-positron collider: the significantly higher event rates resulting from larger interaction cross-sections, and the ability to reach higher energies, thanks to the lower

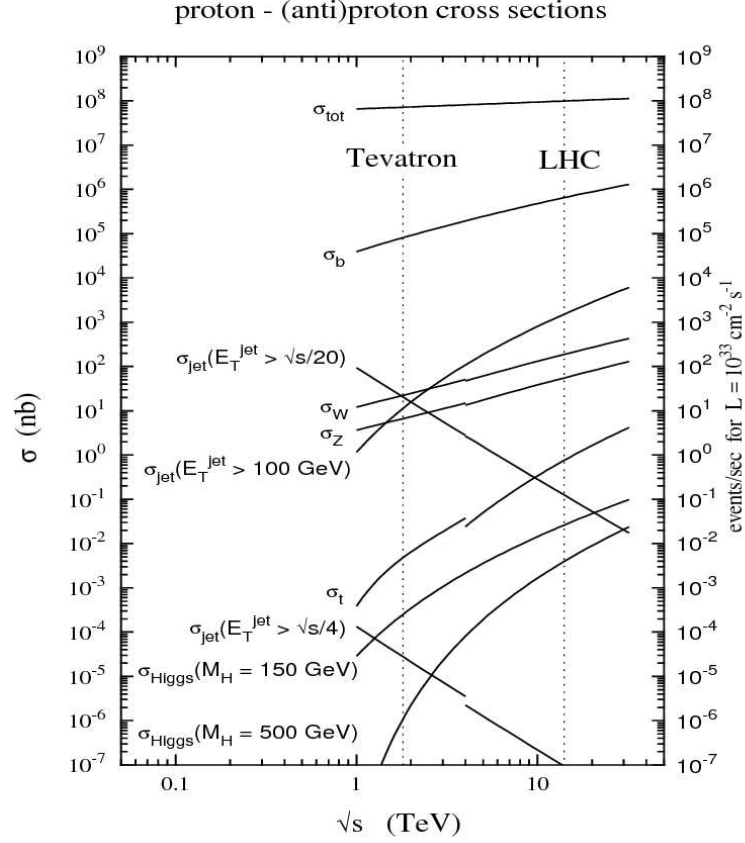


Figure 2.1: The cross-sections for various processes from pp or $p\bar{p}$ collisions as a function of the centre-of-mass energy. The vertical dotted lines indicate the centre-of-mass energy of Tevatron and of the LHC.

synchrotron radiation emitted by massive particles ($\propto 1/m^4$). The cross-sections of several processes as a function of the centre-of-mass energy are shown in Figure 2.1. This comes at the cost of a large QCD background for electroweak and rare SM processes posing challenges in the design of the experiments. The two beams are bent and focused by a repeating structure of dipole magnets and higher-order multipole magnets, respectively. The 1323 NbTi dipole magnets generate magnetic fields up to 8.3 T. LHC uses eight radiofrequency (RF) cavities for each beam to accelerate the particles, each delivering 2 MV and operating at 400 MHz. Protons in the LHC are accelerated to an energy of 6.8 TeV per beam, resulting in a center-of-mass energy of $\sqrt{s} = 13.6$ TeV for pp collisions.

The CERN accelerator complex consists of a series of machines that successively

increase the energy of a particle beam before transferring it to the next stage in the chain. A schematic representation of the CERN accelerator complex is shown in Figure 2.2.

The proton source is hydrogen gas. An electric field is applied to separate electrons from protons. The first stage of acceleration is carried out by LINAC 4, which boosts the protons to an energy of 160 MeV before injecting them into the Proton Synchrotron Booster that accelerates the protons to 2.0 GeV, followed by the Proton Synchrotron (PS), which increases their energy to 25 GeV. Next, the Super Proton Synchrotron (SPS) accelerates the beam to 450 GeV. Finally, the protons are injected into the LHC, where they reach their maximum energy of 6.8 TeV per beam. The LHC is also capable of accelerating ions, extracted from different sources. In this case, ions are first accelerated by LINAC 3 and then injected into the Low Energy Ion Ring (LEIR). From there, they follow the same acceleration chain as protons—passing through the PS, SPS, and finally the LHC, reaching a center-of-mass energy of 5.36 TeV per nucleon pair for lead-lead collisions.

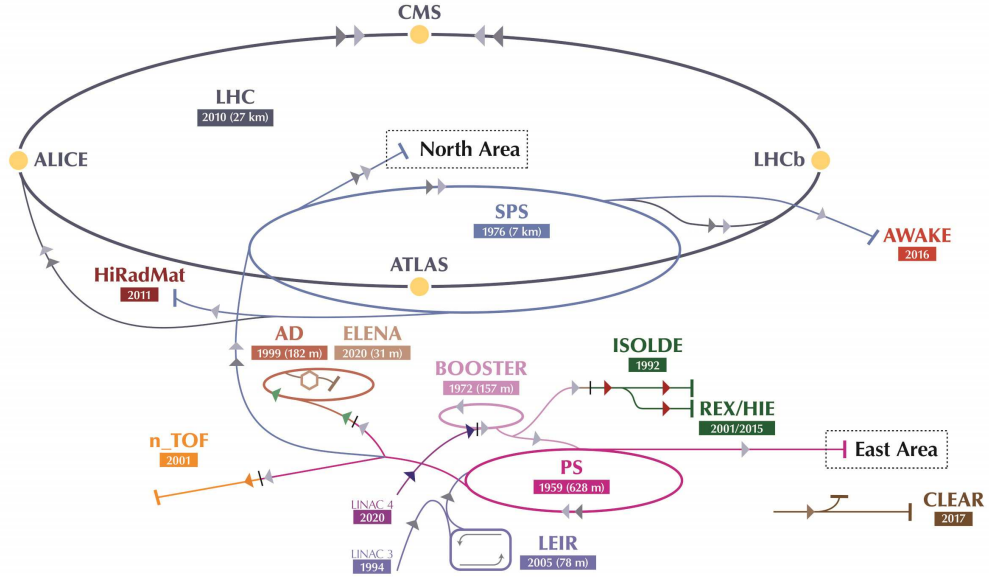


Figure 2.2: Schematic view of the CERN accelerator complex.

There are four large experiments designed to exploit the collisions delivered by the LHC: ALICE, ATLAS, CMS and LHCb.

- ALICE (A Large Ion Collider Experiment) [15]: A specialised detector focused on studying the quark-gluon plasma and the early stages of matter through heavy-ion collisions. By colliding lead ions at a center-of-mass energy per nucleon pair of $\sqrt{s_{NN}} = 2.76$ TeV and a peak luminosity of

$\mathcal{L} = 10^{27} \text{ cm}^{-2}\text{s}^{-1}$, ALICE explores the conditions of extreme energy density thought to have existed shortly after the Big Bang.

- ATLAS (A Toroidal LHC ApparatuS) [16]: A general-purpose detector designed to perform precision measurements within the SM, search for and study the Higgs boson, and investigate potential new physics phenomena. With a length of 44 m, a diameter of 25 m, and a weight of approximately 7,000 tonnes, ATLAS is the largest experiment at the LHC in terms of physical size.
- CMS (Compact Muon Solenoid) [17]: Another general-purpose detector similar in scope to ATLAS. It features a powerful internal magnetic field of 3.8 T generated by a superconducting solenoid. CMS is characterised by its compactness relative to its mass: it weighs about 14,000 tonnes, with a diameter of 15 m and a length of 28.7 m.
- LHCb (Large Hadron Collider beauty experiment) [18]: Dedicated to precision flavour physics, the primary goal of LHCb is to perform indirect searches for physics beyond the Standard Model by studying decays of b - and c -hadrons. Its detector is a single-arm forward spectrometer, optimised based on the production cross-section properties of $b\bar{b}$ pairs in high-energy pp collisions. A detailed description of the LHCb experiment follows in the next section.

2.2 The LHCb detector

The primary objective of the LHCb experiment is to search for signatures of new physics through the study of CP-violating processes and rare decays of beauty and charm hadrons. Some of these phenomena are precisely predicted within the SM, and potential contributions from new physics may manifest via loop-level processes, leading to measurable deviations, in angular distributions, decay rates or production cross-sections. Rare decays mediated by flavour-changing neutral currents play a central role in this field, offering an exceptionally sensitive probe of physics beyond the SM. Since these transitions are forbidden at tree level, they proceed only through higher-order quantum loops, which makes their rates and angular distributions both strongly suppressed and sharply predictable within the SM. This suppression enhances their susceptibility to potential contributions from new heavy particles that could modify the loops or effective couplings. As a result, precision measurements of branching ratios, angular observables, and CP asymmetries in these decays provide powerful and complementary tests alongside direct high-energy searches. Figure 2.3 shows the Feynman diagram corresponding

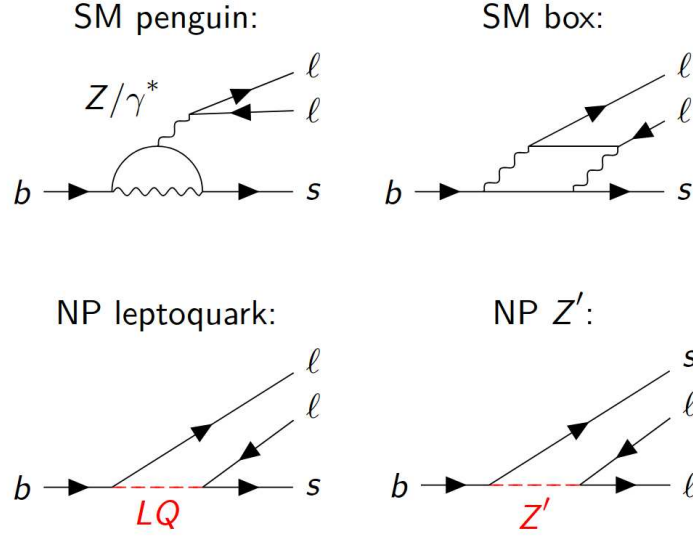


Figure 2.3: Feynman diagram for (top) SM contribution and (bottom) NP contribution to flavour changing neutral currents decays.

to SM and NP contribution to these decays. By exploiting the copious production of beauty and charm hadrons at the LHC, LHCb achieves world-leading precision on key observables such as the Cabibbo-Kobayashi-Maskawa (CKM) angle γ , the mixing phase ϕ_s in the B_s system, and the ratios $|V_{ub}|/|V_{cb}|$ from semileptonic decays. These quantities are predicted with minimal theoretical ambiguity, making them especially clean probes of the unitarity of the CKM matrix. Any deviation from the Standard Model expectations would signal new sources of CP violation or flavour-changing dynamics, with profound implications for our understanding of matter–antimatter asymmetry. The current constraints on the CKM triangle are shown in Figure 2.4.

To achieve its physics goals, the LHCb detector must be able to separate the b -hadrons produced by the LHC from the background, since the charm ($2369 \pm 3 \pm 152 \pm 118 \mu\text{b}$ [20]) and beauty ($144 \pm 1 \pm 21 \mu\text{b}$ [21]) cross-sections are orders of magnitude smaller than the total pp cross-section ($75.4 \pm 3.0 \pm 4.5 \text{ mb}$ [22]). The LHCb detector [18] is designed as a single-arm forward spectrometer, optimised for the kinematics of heavy-flavour production at the LHC, dominated by the gluon-gluon fusion, given the acceptance in pseudorapidity, the corresponding Bjorken variable and the proton probability density functions. It covers a polar angular range from 15 mrad to 300 mrad (250 mrad) in the bending (non-bending) plane of the dipole magnet, corresponding to a pseudorapidity range of $2 < \eta < 5$. The pseudorapidity η is related to the polar angle θ (measured with respect to the

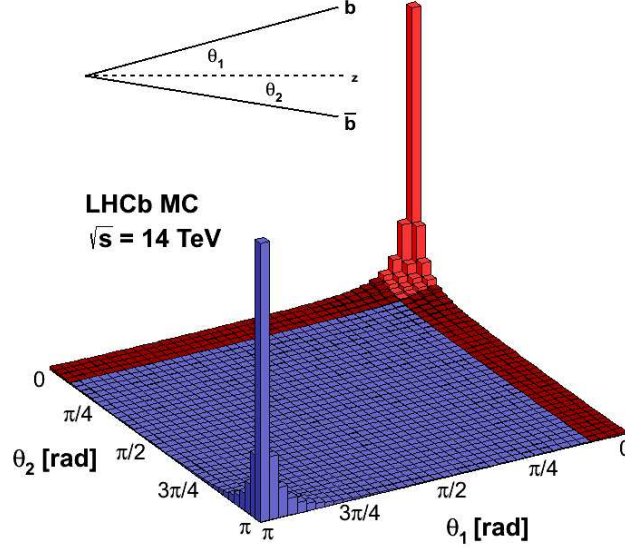


Figure 2.5: Distribution of the polar angle of the $b\bar{b}$ quark pairs at a collision energy of $\sqrt{s} = 14 \text{ TeV}$. The red part of the distribution represents the geometrical acceptance of the LHCb detector.

beam axis) by:

$$\eta = -\log \left(\tan \left(\frac{\theta}{2} \right) \right) \quad (2.1)$$

This angular coverage was specifically chosen to maximise the acceptance for b -hadrons, as $b\bar{b}$ cross-section at high energies is large at small polar angles relative to the beam direction, as illustrated in Figure 2.5.

The layout of the LHCb detector is shown in Figure 2.6.

‘ The coordinate system adopted by the LHCb experiment is defined with its origin at the pp interaction point. The z -axis is aligned with the beam direction and points downstream, toward the end of the detector. The y -axis points vertically upward, while the x -axis points horizontally outward from the LHC center. The plane in which the magnetic field bends charged particles is the x - z plane. Due to their relatively long lifetimes ($\mathcal{O}(1 \text{ ps})$), b - and c -hadrons typically decay at measurable distances from the production vertex, namely the primary vertex (PV), resulting in displaced decay vertices, referred to as secondary vertices (SV). To identify these decays, excellent vertex resolution is essential, and precise proper-time resolution is required to resolve oscillations in time-dependent CP-violation

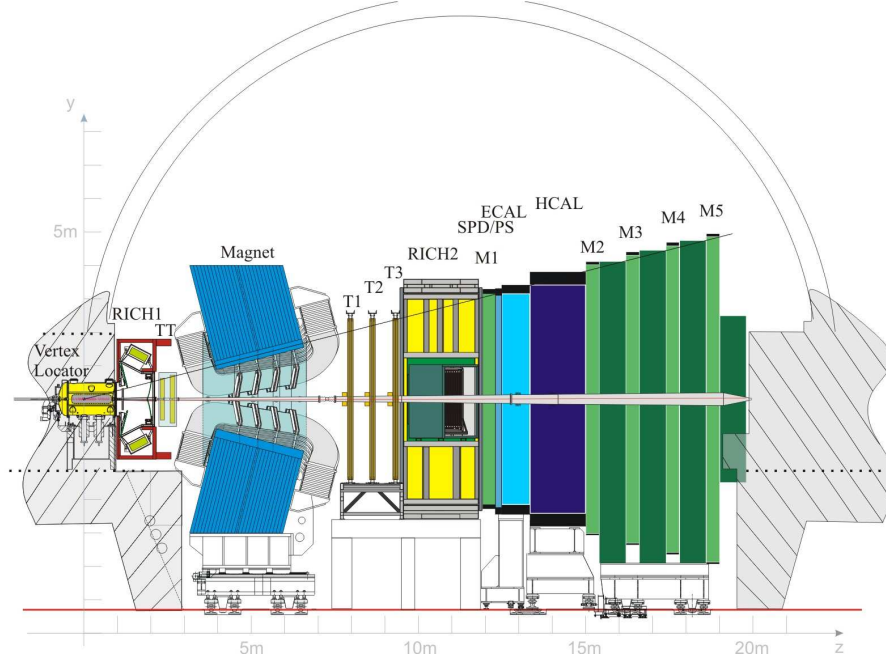


Figure 2.6: Schematic view of the LHCb detector in the non-bending vertical plane with the subdetectors labelled

measurements. To suppress combinatorial background, events arising from random combinations of tracks that imitate the signal, common in hadronic environments, a precise invariant-mass reconstruction is essential. This demands high momentum resolution. Furthermore, since many decay modes share similar or identical topologies, a robust particle identification (PID) system is crucial, especially for separating charged hadrons. In order to efficiently collect large samples of rare decays with small branching fractions, the experiment employs a flexible and high-performance trigger system. These stringent requirements are met by the various subdetectors that make up the LHCb apparatus. The LHCb experiment operates at a lower luminosity compared to the ATLAS and CMS experiments. It maintains an approximately constant instantaneous luminosity to minimise pile-up, i.e. the number of primary vertices per bunch crossing. This is particularly important for flavour physics, which requires precise and stable performances, something that becomes significantly more challenging in high pile-up conditions. To achieve a luminosity about two orders of magnitude below the LHC design value, LHCb employs a luminosity-levelling technique. This method introduces a controlled offset between the two colliding beams. An automated procedure continuously realigns the beams throughout the duration of an LHC fill to keep the luminosity stable, guided by online luminosity monitoring as displayed in Figure 2.7.

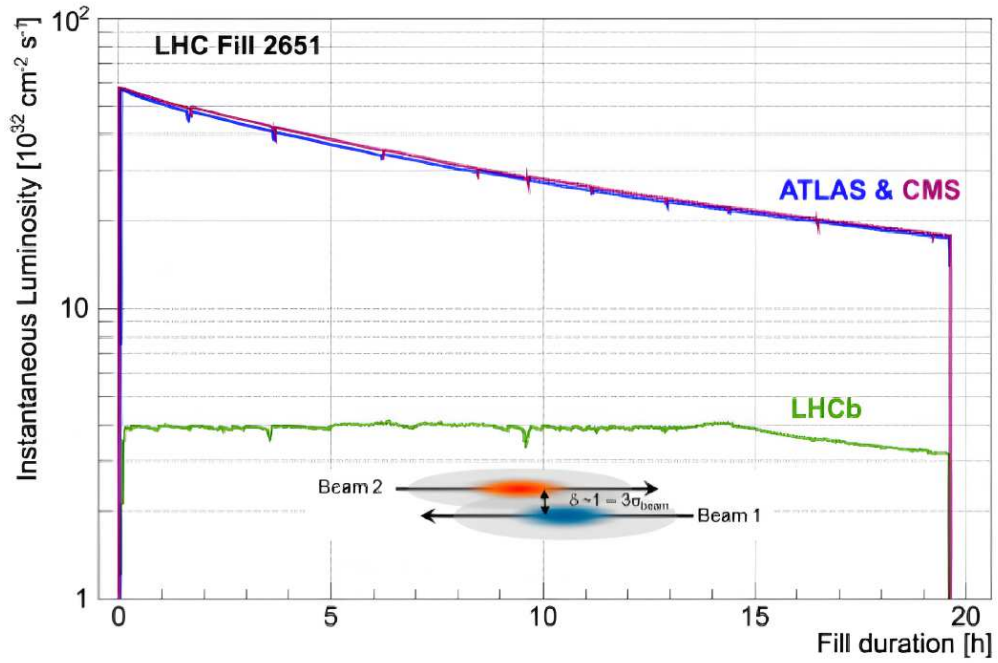


Figure 2.7: Time dependence of the instantaneous luminosity for ATLAS, CMS and LHCb during LHC fill 2651. After ramping to the desired luminosity value for LHCb, the luminosity is kept stable in a range of 5% by adjusting the transversal beam overlap. The difference in luminosity towards the end of the fill between ATLAS, CMS and LHCb is due to the difference in the final focusing at the collision points.

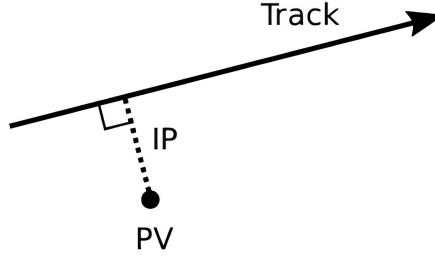


Figure 2.8: Impact parameter of a track in relation to the primary vertex. The figure is drawn in the plane containing the track and the PV.

The LHCb apparatus described below refers to the detector configuration used during Run 1 (2011–2012) and Run 2 (2015–2018). The upgraded apparatus for Run 3 will be presented in Sec. 5.1.

2.2.1 The Tracking system

The tracking system is designed to reconstruct charged particles trajectories by analysing the positions where particles interact with the tracking detectors, referred to as hits. A particle momentum (p) is determined from the angle of its deflection as it traverses the magnetic field produced by a warm dipole magnet. In order to achieve a momentum resolution of $\delta p/p = 4 \cdot 10^{-3}$ for 10 GeV particles, chosen to ensure the precise measurements of b -hadrons, an integrated field of 4 Tm is required. The first component of the system is the Vertex Locator (VELO), a silicon microstrip detector surrounding the interaction point. VELO provides precise measurements of the primary and secondary vertex positions and the track impact parameters (IP) a scheme of the IP definition is shown in Figure 2.8. Upstream of the magnet, a second silicon microstrip detector, the Tracker Turicensis (TT), improves the momentum resolution of reconstructed tracks and helps suppress fake, duplicate tracks originating from a single particle. Downstream of the magnet, the T stations further refine the determination of a particle momentum and flight direction. These stations integrate two detection technologies, with silicon microstrips placed near the beamline and strawtube detectors in the outer regions.

All tracking detectors are characterised by high spatial resolution (in one or two dimensions) and a minimal material budget to reduce scattering. Tracks reconstructed using the full LHCb tracking system achieve a momentum resolution between 0.5% and 1% for momenta in the range $5 < p < 200$ GeV.

The magnetic field polarity is periodically reversed during data-taking, so that LHCb collects approximately equal integrated luminosity for both magnet configurations. This procedure helps identify and correct for detection asymmetries, in order to reduce systematic uncertainties, particularly important in measurements of CP violation.

The LHCb reconstruction software defines five standard track categories, illustrated in Figure 2.9, each based on the specific sub-detectors involved in the reconstruction process:

- **VELO Tracks:** these tracks consist of hits in the VELO detector and are not associated with hits in other tracking stations. They are primarily used for primary vertex reconstruction.
- **Long Tracks:** comprising hits in both the VELO, T-stations and can additionally have hits in TT. Long tracks are the most critical for physics analyses due to their superior momentum resolution.
- **Downstream Tracks:** these originate from long-lived charged particles that decay significantly downstream of the interaction point. They produce hits only in the TT and T-stations, lacking VELO information. Consequently, the absence of a long lever arm results in reduced momentum resolution compared to long tracks.
- **Upstream Tracks:** created by low-momentum particles that are strongly deflected by the magnetic field, these tracks do not reach the T-stations. They are reconstructed using information from the VELO and TT, and are used to improve RICH1 reconstruction process.
- **T Tracks:** reconstructed solely from hits in the T-stations, these tracks typically originate from interactions with detector material or from decays of very long-lived particles.

The Vertex Locator

The Vertex Locator (VELO) [23] is a silicon-strip detector that provides high-precision measurements of particle trajectories near the interaction point, with an IP resolution varying from 10 to 80 μm . Its primary function is to determine the positions of the PVs and SVs with high accuracy, enabling precise reconstruction of the flight paths of heavy-flavour hadrons. The VELO consists of 42 silicon strip modules arranged in planes perpendicular to the beam axis. Each module measures the radial coordinate r and the azimuthal angle ϕ . To maximise spatial resolution, the VELO is positioned as close as possible to the interaction region,

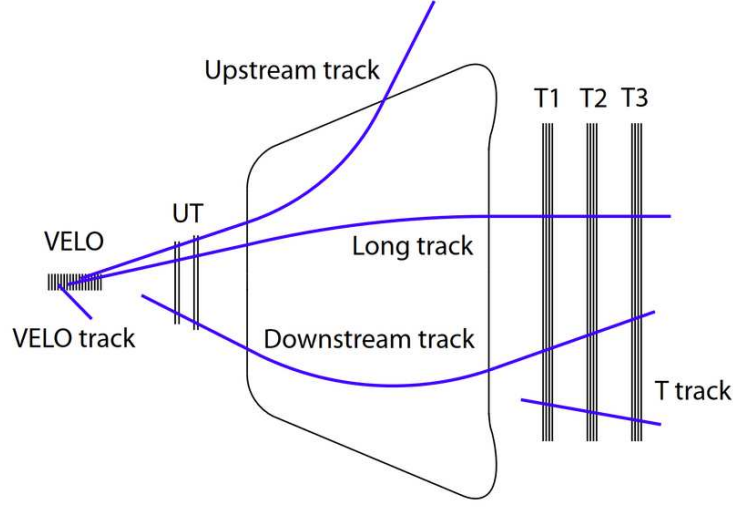


Figure 2.9: A schematic illustration of the various track types: long, upstream, downstream, VELO and T tracks

approximately 7 mm from the beam line, while also meeting strict radiation tolerance requirements due to the intense radiation environment ($\sim 10^{14}$ neq/cm²). During injection, the LHC beams have a larger aperture and can be susceptible to instabilities, therefore the VELO is designed to be retractable. It is divided vertically into two symmetric halves, each containing an equal number of modules on either side of the beam pipe. Before beam injection, the VELO halves are retracted by 30 mm to protect the sensors. Once stable beam conditions are achieved, they return to their nominal closed position, centered around the interaction region. Each VELO half is housed in a lightweight aluminium vacuum vessel. The inner faces of these vessels, known as RF-foils, isolate the VELO vacuum from the machine primary vacuum and shield the sensors from RF-induced backgrounds. The aluminium RF-foils are designed to minimise material in the path of traversing charged particles, reducing multiple scattering and preserving tracking performance. The average contribution of the VELO detector to the radiation length is $0.175 X_0$ [18]. Their geometry also ensures that the two VELO halves overlap slightly when in the closed position, maximising coverage and vertex resolution. A schematic of the VELO is shown in Figure 2.10.

The r -sensor strips of the VELO are concentric rings with a variable pitch, increasing linearly from 38 μm at the inner edge to 102 μm at the outer edge. The ϕ -sensor strips are wedge-shaped and segmented into two radial regions, separated at $r = 17.25$ mm, to control occupancy and avoid excessively large pitches that would degrade hit resolution. In the inner region, the pitch varies from 38 μm to

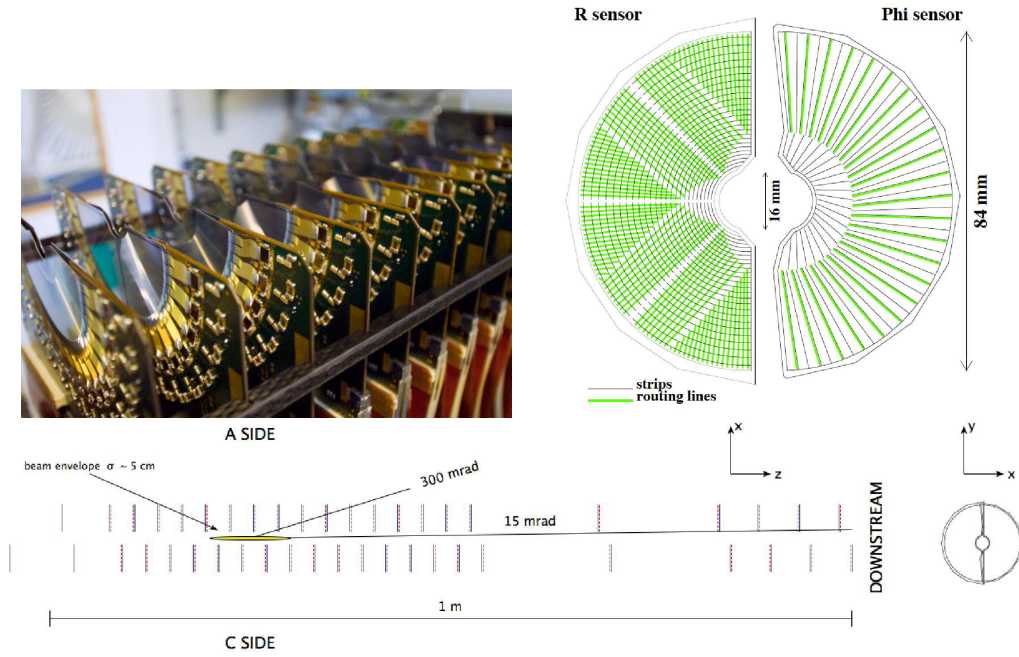


Figure 2.10: Top left: photograph of an half of the VELO detector. Top right: sketch illustrating the $r\phi$ geometry of the VELO sensors. Bottom: cross-section in the xz plane at $y = 0$ of the sensors and a view of the sensors in the xy plane. The detector is shown in its closed position. r (ϕ) sensors are shown with solid blue (dashed red) lines. The modules at positive (negative) x are known as the left or A-side (right or C-side).

78 μm , while in the outer region it ranges from 39 μm to 97 μm . To reduce pattern recognition ambiguities, the ϕ -strips are skewed with respect to the radial direction by 20° in the inner region and 10° in the outer region. The VELO covers the angular acceptance of the downstream LHCb detectors, detecting charged particles emerging from the PV within the longitudinal range $|z| < 10.6$ cm and pseudo-rapidity range $1.6 < \eta < 4.9$. Reconstructed tracks are required to pass through at least three VELO modules. The resolution on the PV position achieved by the VELO depends on the number of tracks used. For the transverse (x , y) coordinates, it varies between 9 μm and 35 μm , while for the longitudinal (z) coordinate, it ranges from 50 μm to 280 μm . For a typical event with 25 PV-associated tracks, the average transverse and longitudinal resolutions are approximately 15 μm and 71 μm , respectively. An additional key quantity measured by the VELO is the impact parameter, defined as the distance of closest approach between a particle extrapolated trajectory and the PV. The IP is a critical observable in LHCb physics analyses, allowing discrimination between signal tracks from displaced vertices, such as those originating from b -hadron decays, and background tracks produced directly at the PV. The VELO, being the detector whose sensitive material is the closest to the interaction region above all the LHC experiments, provides the highest IP resolution among all major LHC experiments, with values ranging from 10 μm to 80 μm , depending on the transverse momentum (p_T) of the track. The dominant source of uncertainty on the IP measurement is multiple scattering as the particles traverse the VELO material.

The Tracker Turicensis

The tracking stations located upstream of the magnet are known as the Tracker Turicensis (TT). Positioned immediately before the dipole magnet, they provide a reference for charged particle trajectories. This improves the precision of momentum measurements, reduces the rate of spurious track reconstructions from random hits, and enables the reconstruction of long-lived particles that decay outside the VELO detector [24], such as K_s^0 and Λ . By extrapolating reconstructed track segments from the VELO to the corresponding energy deposits in the TT detectors [25], a preliminary estimate of the particle momentum can be obtained. This allows a prediction of its path through the magnetic field, allowing to restrict the search for matching hits in the downstream tracking stations to narrow tolerance windows. As a result, the data rate from the reconstruction algorithm is reduced, and overall performance is improved. The TT detector consists of two stations separated by approximately 30 cm, each comprising two detection layers. All layers are equipped with 512 silicon strips featuring a pitch of 183 μm , covering the full LHCb acceptance. The first and last layers measure the track coordinate

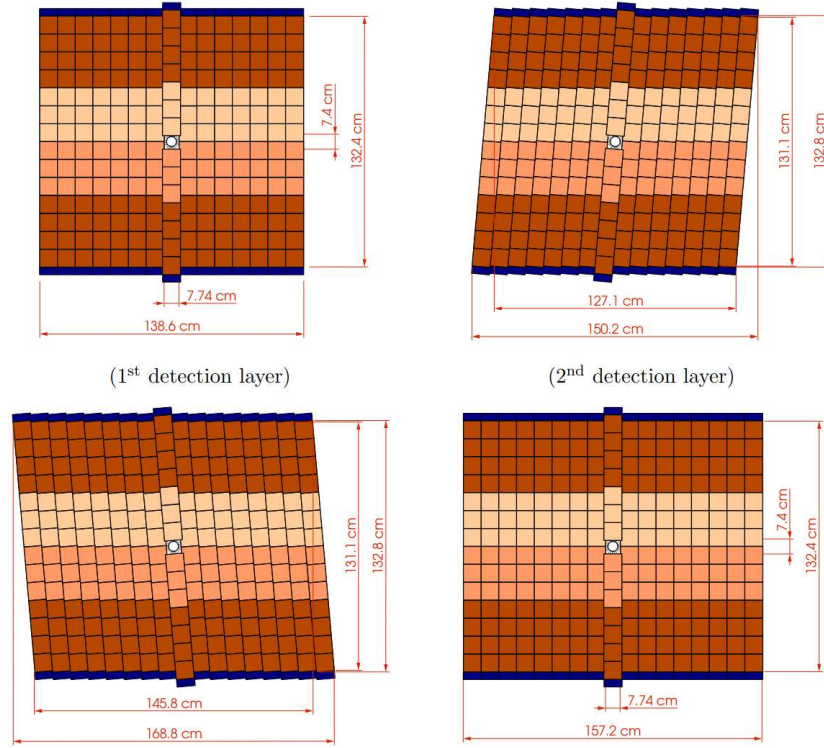


Figure 2.11: Layout of the four detection layers of the TT station. The different readout sectors along a silicon ladder are indicated by different shading.

along the x -axis, while the two intermediate layers are tilted by $\pm 5^\circ$ in the $x - y$ plane, defining the u and v axes. By combining the signals from all four layers, ambiguities in track reconstruction are effectively resolved. This structure allows a spatial resolution of $\sim 50 \mu\text{m}$ in the x coordinate. The layout for each of the four detection layers is shown in Figure 2.11.

The Inner and Outer Tracker

Downstream of the magnet, three additional tracking stations, each composed of four detection planes, are used to reconstruct the trajectories of particles after they are deflected by the magnetic field. Two distinct tracking technologies are implemented to address the varying detector environments, as illustrated in Figure 2.12. In the inner region, where particle flux and detector occupancy are higher, silicon strip detectors are employed due to their high radiation tolerance, fine granularity, and excellent spatial resolution. These strips are arranged in the same configuration and installed with the same pitch as those in the TT detector. Although the Inner Tracker (IT) [26] covers about 2% of the total LHCb acceptance, it handles approximately 20% of the total particle flux because of the increased track density

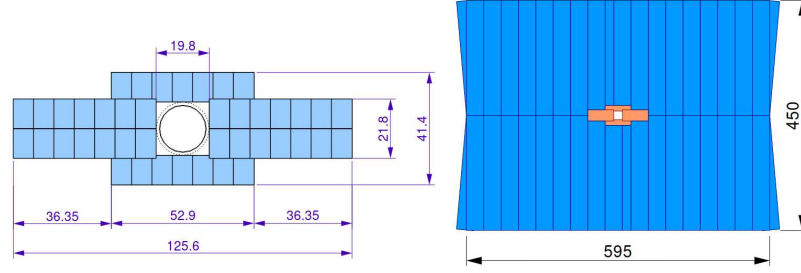


Figure 2.12: Front views of one station of the IT (left) and OT (right) detectors. Dimensions are indicated in centimetres.

at high pseudorapidity.

Beyond the radial region where occupancy drops to around 10% , the IT is replaced by the Outer Tracker (OT) [27, 28], which uses drift tubes. These tubes are 2.4 meters long, have an inner diameter of 4.9 mm, and are filled with a gas mixture of Ar/CO₂/O₂ (70% /28.5% /1.5%), ensuring a drift time below 50 ns. The achieved spatial resolutions are approximately 50 μm for the IT and 200 μm for the OT [29].

Tracking system performance

A Kalman filter model [30] is employed for track fitting, effectively accounting for multiple scattering effects. The resulting χ^2 per degree of freedom (χ^2/ndof) from the fit is used as a quality metric for the reconstructed tracks. To eliminate redundancies caused by overlapping reconstruction strategies, a clone killer algorithm [31] is applied to remove duplicated tracks. Additionally, a neural network classifier estimates the probability that a track was formed from hits belonging to multiple different particles. The output of this classifier is referred to as Ghost-Prob [32] and is used in subsequent selection algorithms.

The tracking efficiency for long tracks is evaluated using a tag-and-probe method based on $J/\psi \rightarrow \mu^+ \mu^-$ decays [33]. One muon is fully reconstructed and acts as the tag. The second muon, the probe, is only partially reconstructed, using a subset of the detector. After fitting J/ψ candidates via their invariant-mass, a search is conducted for a long track match corresponding to the probe muon. The tracking efficiency is then calculated as the ratio of matched candidates to the total number of probe candidates. As shown in Figure 2.13, the reconstruction of long tracks achieves an efficiency of over 94% across a wide range of momenta. The figure also displays the momentum resolution, which remains below 1% for a broad momentum interval.

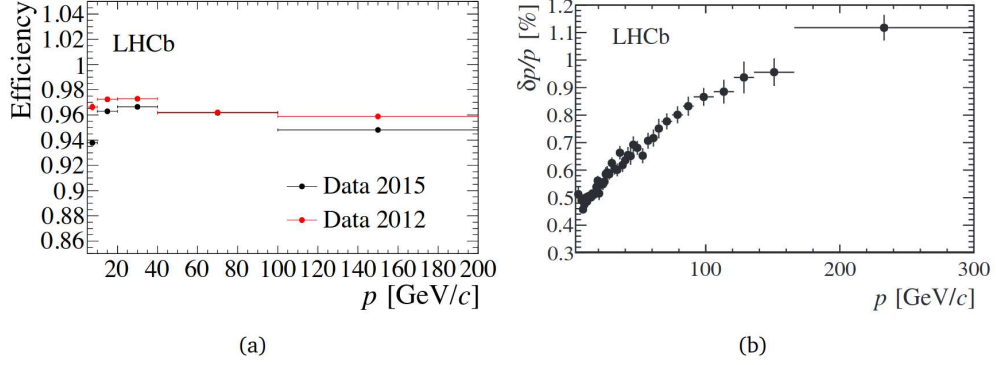


Figure 2.13: (a) Long track reconstruction efficiency as a function of momentum. (b) Momentum resolution as a function of momentum in Run 1 [34].

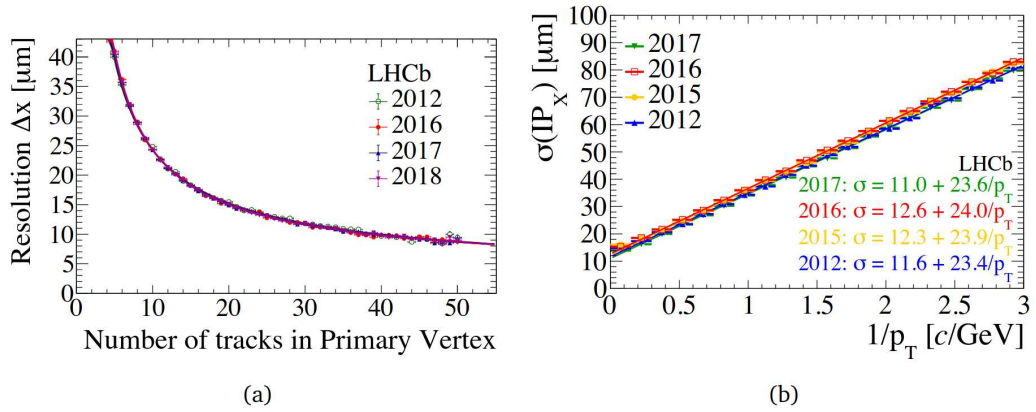


Figure 2.14: (a) The primary vertex resolution along the x-axis during the Run 1 and Run 2 data collection periods, as a function of the number of tracks comprising the vertex. (b) The distribution of impact parameter resolution along the x-axis in relation to the inverse transverse momentum, measured during the Run 1 and Run 2 data acquisition phase [35]

The PV reconstruction algorithm determines the event main interaction points by analysing the spatial distribution of tracks. The process involves two key steps: seeding and fitting. During seeding, candidate PVs are identified by locating clusters of track intersections. Then, these candidates are refined using a weighted least squares fit, applied in order of decreasing track multiplicity to avoid secondary vertices capturing tracks from true PVs. After fitting, the algorithm checks the separation between new and existing vertices to prevent false PV identifications. This cycle of seeding and fitting repeats until no new PVs are found. While higher track multiplicity generally simplifies PV reconstruction due to increased track coverage in the VELO detector, challenges arise when multiple PVs are present in the same event. Figure 2.14 shows the primary vertex and IP resolutions. The linear dependence of IP resolution on $1/p_T$ arises from multiple scattering and detector geometry. At high p_T , IP resolution is dominated by PV resolution rather than track precision, as tracking accuracy improves but PV resolution depends on the number of associated tracks.

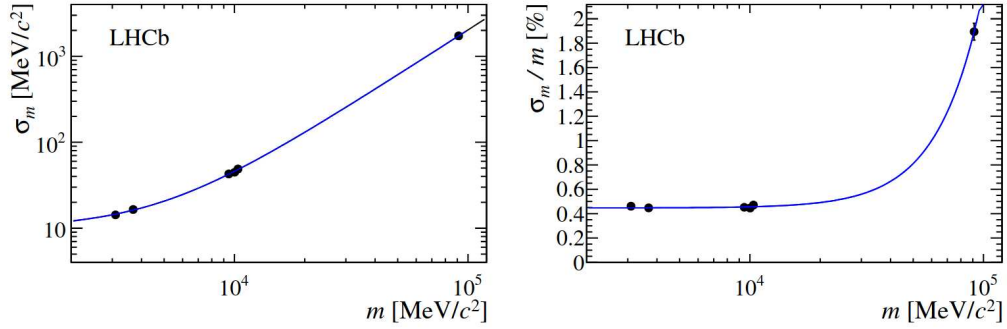


Figure 2.15: Mass resolution σ_m (left) and relative mass resolution (right) are shown as functions of the dimuon resonance mass m . The muon mass is negligible in the invariant-mass calculation for these resonances. The mass resolution is extracted from fits to the mass distributions, with the overlaid curve representing an extrapolation based on an empirical power-law fit.

For tracks originating from the same vertex, the invariant-mass is calculated and fitted. The resolution of this invariant-mass measurement for di-muon resonances is shown in Figure 2.15, ranging from approximately 0.5% to 2% for masses between 3 and 100 GeV. The momentum measurement is calibrated, referred to as the momentum scale calibration, by comparing the measured masses of well-known resonances to their world-average values [36]. This procedure utilises large samples of decays, such as $J/\psi \rightarrow \mu^+\mu^-$ and $B^+ \rightarrow J/\psi K^+$. A systematic uncertainty of 0.03% is assigned to the momentum scale.

2.2.2 The Particle Identification System

To precisely identify different particle species across the momentum range of interest (approximately 2–150 GeV), a combination of complementary experimental techniques is required. At the LHCb experiment, several dedicated sub-detectors are employed for Particle IDentification (PID). Two Ring Imaging CHerenkov (RICH) detectors are used to mainly discriminate between charged pions, charged kaons and protons. The Calorimeter system, designed to measure the energy of incoming particles by absorbing the showers produced when particles interact with the detector material. It consists of the Electromagnetic Calorimeter (ECAL), optimised for reconstructing the neutral pions, through the $\pi^0 \rightarrow \gamma\gamma$ decay, photons, electrons, and positrons, and the Hadronic Calorimeter (HCAL), which absorbs hadronic showers. Two additional scintillator planes, Scintillating Pad Detector (SPD) and PreShower (PS), are placed upstream of the ECAL. These assist in triggering by providing a fast estimate of detector occupancy, help initiate particle showers before the calorimeters and discriminating e^+/e^- from γ . Finally, a Muon system is employed to identify muons, by matching extrapolated track information with hits in these detectors.

The RICH system

The LHCb experiment employs two RICH detectors [37] which provide highly efficient charged hadron identification. As illustrated in Figure 2.16, Cherenkov radiation is emitted when a charged particle traverses a dielectric medium, referred to as the radiator, at a speed β exceeding the velocity of light in that medium, corresponding to c/n , where n is the refractive index of the material [38]. This phenomenon can be qualitatively understood as a result of asymmetric polarisation in the medium, induced by the passage of the charged particle. The medium becomes polarised ahead of and behind the particle, resulting in a time-varying net electric dipole moment. For particles fast enough, this polarisation exhibits axial symmetry, forming a region in which atoms or molecules behave like microscopic electric dipoles. As these dipoles return to equilibrium, they emit electromagnetic radiation. When $\beta > 1/n$, the emitted radiation from different points along the particle trajectory interfere constructively, leading to the formation of a coherent wavefront. This wavefront propagates in the form of a cone at a fixed angle relative to the particle path, known as the Cherenkov angle. The cosine of this angle is given by:

$$\cos(\theta_c) = \frac{1}{n\beta} \quad (2.2)$$

A RICH detector uses spherical mirrors to focus the emitted Cherenkov light cones into approximately circular rings on a photon detector plane, positioned

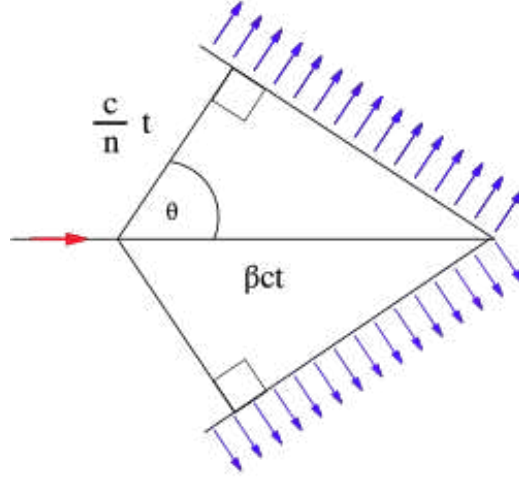


Figure 2.16: Diagram illustrating the process of Cherenkov radiation emission.

outside the acceptance employing a system of planar mirrors. The radius of each Cherenkov ring is proportional to the Cherenkov angle θ_c and the focal length (f) of the spherical mirror, defined as $f = R_c/2$ with R_c the radius of curvature of the mirror, according to the equation $R \sim \theta_c \cdot f$. By precisely measuring the ring radius, one can determine the particle velocity. When these rings are matched to charged particle tracks, whose momentum has been independently measured by the tracking system, the mass of the particle can be calculated using the relativistic relation between mass, momentum, and velocity:

$$p = m\beta\gamma c \rightarrow m = \frac{p}{\beta\gamma c} \quad (2.3)$$

The minimum particle velocity required to produce Cherenkov radiation in a given medium corresponds to a Cherenkov angle of $\theta_c = 0^\circ$, which occurs when the particle speed exactly equals the velocity of light in the medium. This threshold condition corresponds to $\beta = 1/n$. The maximum Cherenkov emission angle occurs when the particle speed approaches the speed of light in vacuum ($\beta \rightarrow 1$). In this limit, the Cherenkov angle reaches its maximum value, corresponding to $\theta_c^{max} = \arccos(1/n)$. Cherenkov radiation is emitted promptly as charged particle traverse the radiator. The radiation has a continuous frequency spectrum, and the number of emitted Cherenkov photons per unit path length dx and per unit wavelength $d\lambda$ is given by the Frank–Tamm formula [39]:

$$\frac{dN}{dx d\lambda} = \frac{2\pi Z^2 \alpha}{\lambda^2} \left(1 - \frac{1}{n^2 \beta^2}\right) \quad (2.4)$$

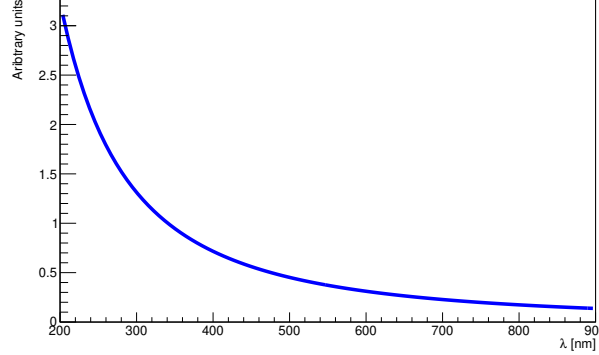


Figure 2.17: Typical Cherenkov radiation spectrum for a charged particle. The intensity follows a λ^{-2} dependence, showing stronger emission at shorter wavelengths.

Here, Z is the charge of the particle in units of the elementary charge, α is the electromagnetic fine-structure constant and λ is the photon wavelength. This equation shows that shorter wavelengths dominate the Cherenkov spectrum due to the $1/\lambda^2$ dependence, leading to a higher intensity in the ultraviolet region. From Eq. 2.4 one can derive that, in 1 cm of material with refractive index n , the number of photons emitted in the range of 1 eV, by a particle of charge Ze with a $\beta \simeq 1$, is:

$$N(\text{cm}^{-1}\text{eV}^{-1}) = 370Z^2\left(1 - \frac{1}{n^2}\right) = 370Z^2 \sin^2(\theta_c) \quad (2.5)$$

so, in general, the number of emitted photons per unit length and unit energy can be considered constant. The typical Cherenkov emission spectrum is shown in Figure 2.17. The refractive index $n = n(\nu)$ varies with photon frequency due to dispersion. An implication of dispersion is that the Cherenkov angle θ_c becomes frequency-dependent. As a result, Cherenkov photons produced by a single charged particle may have slightly different emission angles depending on their wavelength. This angular spread introduces a chromatic uncertainty in the measurement of θ_c , which must be considered when selecting radiator materials and designing RICH detectors to ensure optimal resolution and particle identification performance.

The LHCb RICH detectors, illustrated in Figure 2.18, operate within gas-tight enclosures filled with fluorocarbon gas radiators. Both RICH detectors share a similar optical design. Each employs a tilted, spherical primary mirror for focusing Cherenkov light, along with a secondary flat mirror that redirects the photons toward photodetector planes located outside the main acceptance region. This two-mirror configuration allows the detector to remain compact along the beam

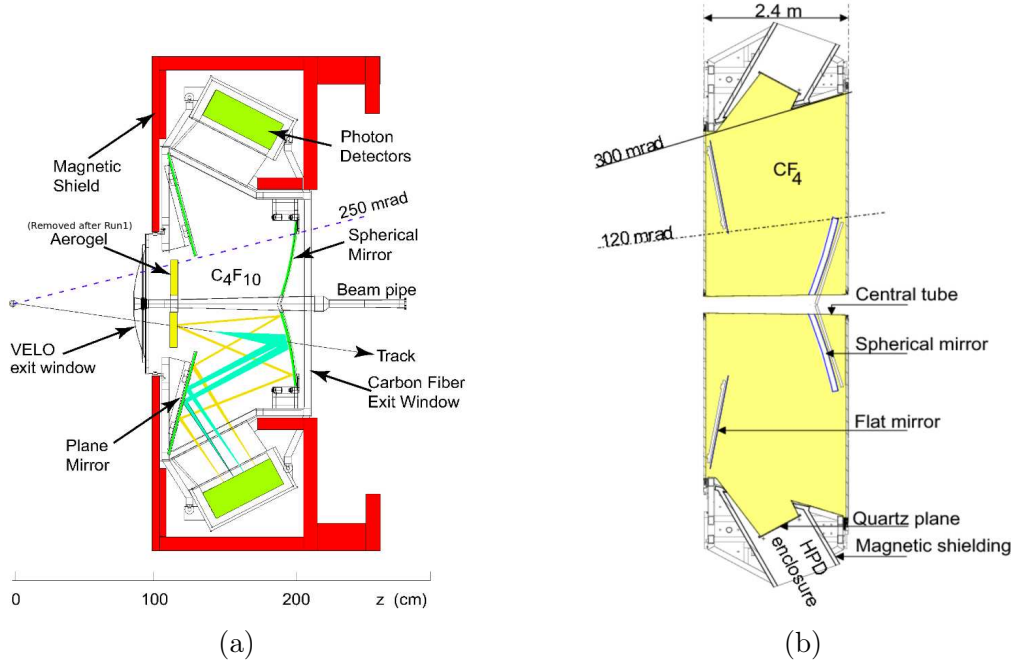


Figure 2.18: (a) Side view of RICH1. The Aerogel radiator has been removed just before the beginning of Run 2(c) Top view of RICH2.

axis. The optical systems are split into two halves symmetrically arranged around the beam pipe: RICH1 is divided vertically, while RICH2 is divided horizontally. The mirrors achieve an average reflectivity of approximately 90% across the relevant wavelength range (200–700 nm), ensuring efficient photon collection for precise Cherenkov angle measurements. RICH1 is situated between VELO and TT. A magnetic shield surrounds the detector to protect the photodetectors from residual magnetic fields that could compromise their performance. During Run 1, RICH1 utilised two Cherenkov radiators: silica aerogel and C₄F₁₀. Silica aerogel, with a refractive index of $n = 1.03$ at $\lambda = 400$ nm and standard temperature and pressure (STP), enabled positive identification in the low-momentum range of approximately 2 – 10 GeV. C₄F₁₀, with a refractive index of $n \simeq 1.0014$ at $\lambda = 400$ nm and STP, extended PID capabilities to higher momenta. In Run 2, aerogel was removed because its low clarity caused photon self-absorption, wide rings with few photons hindered pattern recognition, and it partially absorbed C₄F₁₀, reducing radiator performance. Removing it improved RICH1 performance by increasing the C₄F₁₀ volume. RICH1 covers the intermediate momentum range of approximately 3–40 GeV and angular acceptance 25–300 mrad. RICH2 is located downstream of the third tracking station and upstream of the first muon station. It uses CF₄ gas as the Cherenkov radiator, with a refractive index of

$n \simeq 1.0005$ at $\lambda = 400$ nm and STP. To suppress scintillation in the gas, approximately 5% CO₂ is added [40]. RICH2 is optimised for PID in the high-momentum range, covering approximately 15–100 GeV. Cherenkov photons in both RICH detectors are focused onto photodetector planes located outside the LHCb geometric acceptance. These planes in Run 1 and 2 are instrumented with Hybrid Photon Detectors (HPDs), vacuum photodetectors with an active diameter of 75 mm a quartz entrance window, and a multialkali photocathode. The photodetector volumes are isolated from the radiator gases by quartz windows and are maintained in a CO₂ environment to ensure detector stability by avoiding condensation.

For two particles of momentum p and masses m_1 and m_2 the condition for separation with n_σ standard deviation is:

$$n_\sigma = \frac{|m_1^2 - m_2^2| \sqrt{N}}{2p^2 \sigma_\theta \tan(\theta_c)}, \quad (2.6)$$

where N is the mean number of photon and σ_θ is the Cherenkov angle resolution. Eq. 2.6 shows that the Cherenkov angle resolution, which sets the particle discrimination power of RICH detectors, is improved by maximising the photon yield and minimising both single-photon angular uncertainty and tracking error. This becomes especially critical at high particle momenta, where the Cherenkov angles approach saturation and the separation power between particle types diminishes. Several factors contribute to the overall resolution.

- Chromatic dispersion;
- Optical aberrations and geometric effects, which cause uncertainty in the emission point;
- Pixel size of the photodetectors, which introduces granularity in the angle reconstruction;

On average, the Cherenkov angle resolution during Run 1 and 2 was approximately 1.65 mrad for RICH1 and 0.67 mrad for RICH2, reflecting their different geometries and optics. The mean photon yield is measured to be 30 ± 2 and 18.5 ± 1.2 for RICH1 and 2, respectively [41]. Figure 2.19 illustrates the dependence of the Cherenkov angle on particle momentum for various particle species across the different radiators used in the LHCb RICH system. For all tracks, under a given set of mass hypotheses (e.g., pion, kaon, proton), the expected Cherenkov photon hit distribution on the detector is calculated, based on trajectory information, optics and mass hypothesis. The consistency of the observed HPD hit pattern with each hypothesis is quantified through a global likelihood calculation. These likelihood values, when combined with information from other subdetectors, form the basis

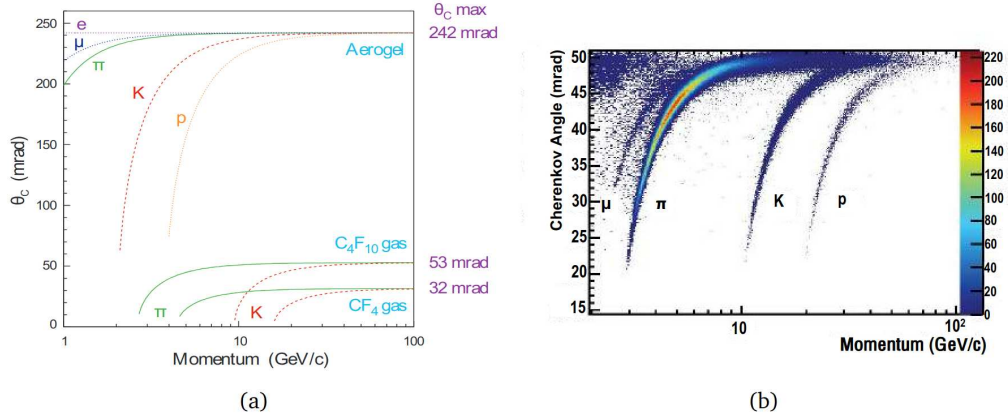


Figure 2.19: a) Cherenkov angle as a function of particle momentum for the RICH radiators (including aerogel) for different particle masses. (b) Cherenkov angle for isolated tracks, defined as tracks where the Cherenkov ring does not overlap with any other ring, in the C_4F_{10} radiator.

of PID criteria used in physics analyses. These particle estimators are calculated from the difference in log-likelihood between different mass- hypotheses for a given track, for example $\Delta LL(K, \pi)$. In addition to this likelihood-based method, an alternative and more powerful approach is employed in certain analyses. This method uses multivariate techniques to integrate PID information from multiple subsystems, while also accounting for correlations and incorporating tracking data. The output is a single, unified probability score, called ProbNN, for each particle hypothesis, significantly improving PID performance.

The performance of the RICH system is calibrated and validated using control samples of $\Lambda \rightarrow p\pi^-$ and $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$. These decays can be selected using purely kinematic selections, providing clean samples of pions, kaons, and protons for evaluating PID efficiency and misidentification rates across a range of track momenta. Figure 2.20 presents an example of performance study.

The Calorimeter system

In the calorimeter system, particles are identified based on the characteristics of the showers they produce when interacting with the detector materials. The system comprises two scintillator planes, the Scintillator Pad Detector and the Preshower, separated by a lead converter, as well as two calorimeters: the Electromagnetic Calorimeter and the Hadronic Calorimeter. The primary functions of the calorimeter system are to differentiate between hadrons, electrons/positrons, and photons through the distinct shower patterns they generate, as illustrated in Figure 2.21.

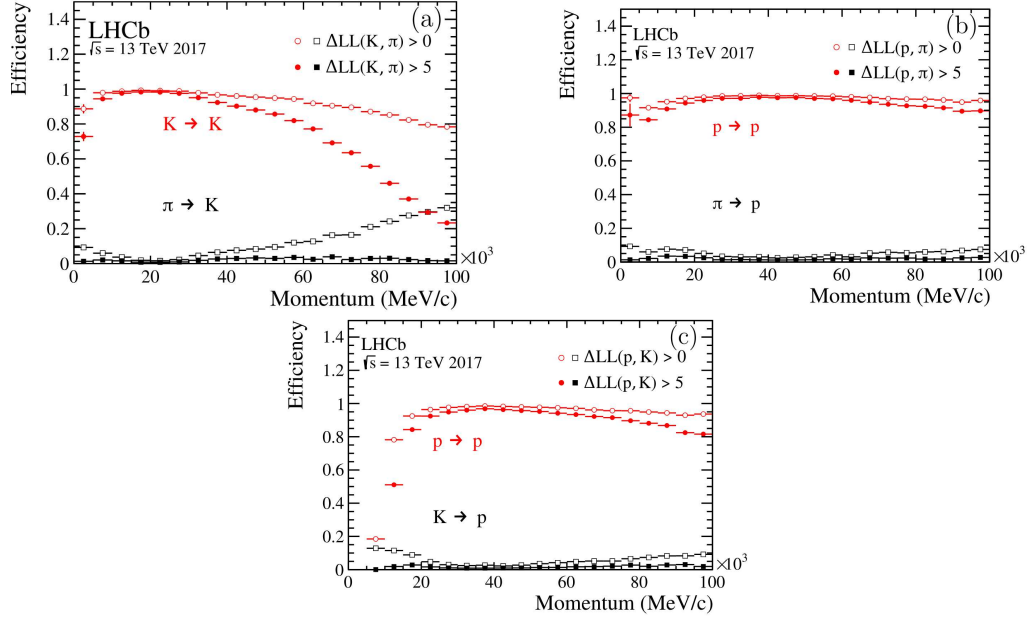


Figure 2.20: PID efficiency and misidentification rates performance in Run 2 for the discrimination of kaons from pions (a), protons from pions (b) and proton from kaons (c) as a function of the momentum. Two different ΔLL requirements have been imposed on the samples, resulting in the open and filled marker distributions, respectively

The SPD and PS detectors [42] consist of polystyrene-based scintillating planes. Positioned between them is a lead converter, with the total material corresponding to a radiation length of $6.5 X_0$ ($2 X_0$ from each scintillator plane and $2.5 X_0$ from the lead). The SPD helps distinguish showers produced by charged particles from those initiated by photons and improves the correlation with tracking information. The PS facilitate the discrimination between electromagnetic and hadronic showers exploiting the different energy deposition. The ECAL [43] is a shashlik-type sampling calorimeter composed of 66 modules, each consisting of alternating layers of 2 mm lead and 4 mm scintillator, summing to a total thickness of 25 radiation lengths. To improve the measurement of the longitudinal and transverse development of showers and to improve separation between adjacent showers, the calorimeter layers are transversely segmented. The transverse cell size increases with radial distance from the beamline to account for decreasing occupancy: $4.0 \times 4.0 \text{ cm}^2$ in the inner region, $6.1 \times 6.1 \text{ cm}^2$ in the middle, and $12.1 \times 12.1 \text{ cm}^2$ in the outer region. The ECAL achieves an energy resolution of:

$$\frac{\sigma_E}{E} = \frac{9.0\%}{\sqrt{E}} \oplus 0.8\% \oplus \frac{0.003}{E \sin \theta} \quad (2.7)$$

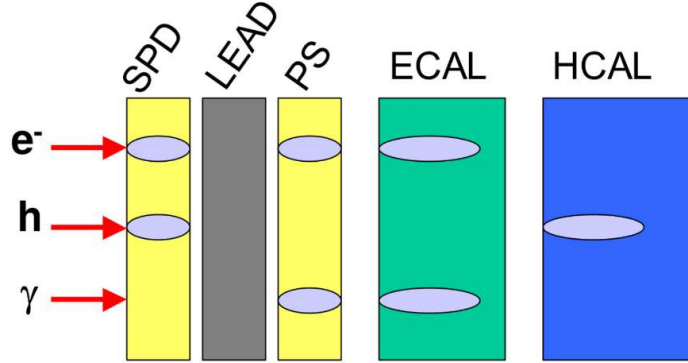


Figure 2.21: Scheme for the shower development in the LHCb calorimeter system for different particle hypotheses

where $\oplus$ denotes the sum in quadrature and the energy is measured in GeV.

The HCAL is also a sampling calorimeters, but unlike the ECAL, it uses alternating layers of iron and scintillating tiles aligned parallel to the beam axis, enhancing angular resolution. It also features transverse segmentation, with cell sizes of $13.1 \times 13.1 \text{ cm}^2$ in the inner region and $26.2 \times 26.2 \text{ cm}^2$ in the outer region. The HCAL has a total thickness of 5.6 hadronic interaction lengths, constrained by the physical space available in the LHCb cavern. As this is insufficient to fully contain hadronic showers, the resulting energy resolution is:

$$\frac{\sigma_E}{E} = \frac{67\%}{\sqrt{E}} \oplus 9\% \quad (2.8)$$

The calorimetry system is shown in Figure 2.22.

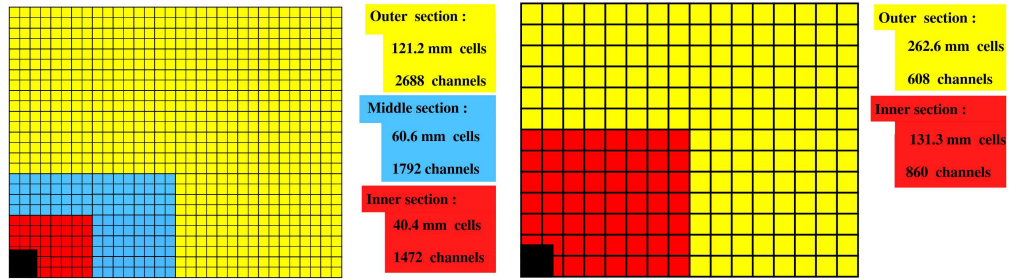


Figure 2.22: Left: segmentation of the SPD, PS and ECAL. Right: segmentation of the HCAL.

The Muon system

Five muon stations are used by the LHCb detector, labeled M1 through M5, dedicated to muon identification and reconstruction [44], as depicted in Figure 2.23. Station M1 is positioned upstream of the calorimeters, after RICH2, and enhances the early measurement of muon transverse momentum for hardware level trigger purposes. Stations M2 to M5 are located downstream of HCAL and are interleaved with iron absorbers 80 cm thick, corresponding to around 20 nuclear interaction lengths, to absorb hadronic showers. The muon system covers an angular acceptance of $[20, 306]$ mrad horizontally and $[16, 258]$ mrad vertically. To fully traverse all five stations, muons must have a minimum momentum of 6 GeV. Each muon station is segmented into four regions (R1–R4), with areas increasing in a 1:2:4:8 ratio to evenly distribute particle flux. A total of 1368 MultiWire Proportional Chambers (MWPC), filled with an Ar/CO₂/CF₄ (40/55/5 %) gas mixture, are used in regions R2–R4 of stations M2–M5, and in the outer region of M1. In the high-occupancy innermost region of M1, Gas Electron Multiplier (GEM) chambers are employed instead, offering higher radiation tolerance, granularity, and spatial resolution. The GEM chambers use a different gas mixture: Ar/CO₂/CF₄ in a 45/15/40 ratio.

Muon identification [45] is based on a likelihood function, evaluated for both the muon and non-muon hypotheses by matching the energy deposition patterns in the muon stations to the associated track. The resulting discriminant, is defined as the logarithm of the likelihood ratio between the muon and non-muon hypotheses. This scorer is further combined with information from the RICH and calorimeter systems to yield combined classifiers for improved particle identification. Using this combined likelihood approach, LHCb achieves an average muon identification efficiency well above 95%, with hadron misidentification rates below 0.6%, depending on momentum [45].

2.2.3 The LHCb Trigger system

Raw detector signals must undergo a series of processing steps [46, 47, 48] to reconstruct high-level physics objects such as tracks, vertices, and particles, and to identify events of interest based on their physics signatures. A graphical overview of the LHCb data processing workflow, executed within the Gaudi software framework [49] is shown in Figure 2.24. This diagram outlines the sequence of processing stages along with the corresponding software applications responsible for each step. The LHCb trigger system operates in two stages: the Level-0 (L0) trigger and the High-Level Trigger (HLT). The L0 trigger is implemented in hardware and reduces the detector readout rate to 1.1 MHz, which corresponds to the maximum

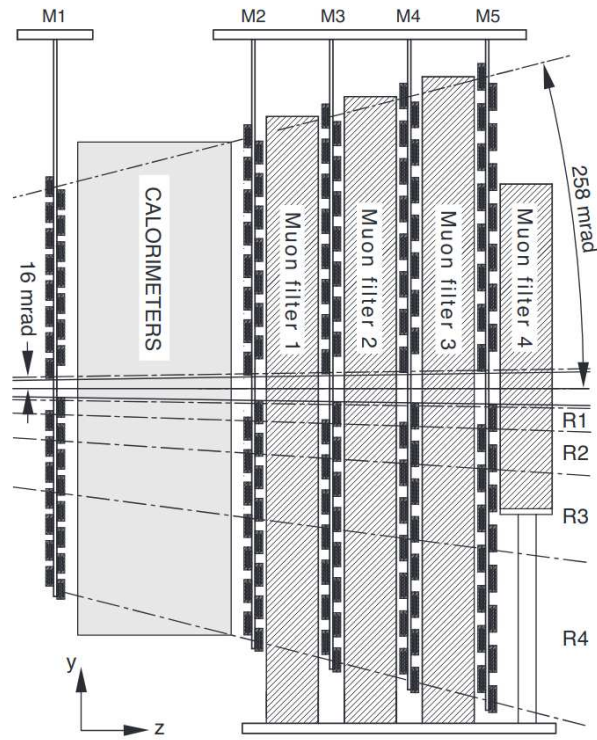


Figure 2.23: Scheme for the muon chambers location in the LHCb detector.

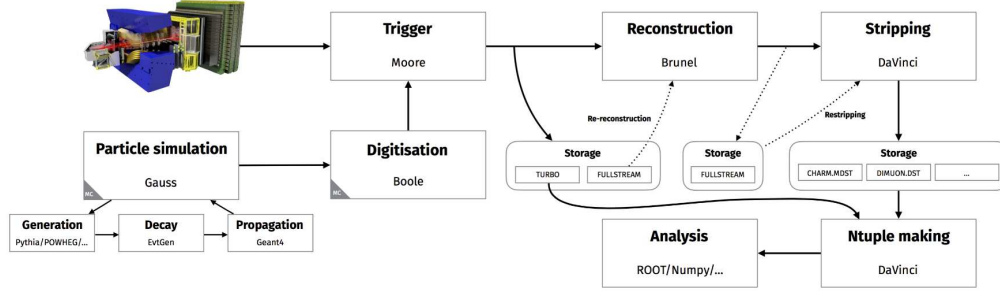


Figure 2.24: Scheme for the LHCb data processing sequence

rate supported by the front-end electronics of all subdetectors, but for the muon and calo systems having frontend electronics working at 40 MHz. The HLT is a software-based trigger system that performs full event reconstruction on a computing farm comprising approximately 25000 CPU cores distributed across around 1600 physical nodes, each equipped with at least 1 TB of local storage. The primary function of the trigger system is to reduce the raw proton-proton collision rate to a manageable output rate of 2 and 12.5 kHz, respectively for Run 1 and Run 2 [18, 35], which is suitable for permanent storage. Each individual condition or selection criterion within the trigger system is referred to as a trigger line. If an event satisfies the criteria of any trigger line, it is passed to the next stage of processing or written to storage.

The L0 Trigger

The L0 trigger is implemented using custom hardware synchronised with the 40 MHz proton-proton collision rate. It operates with a fixed processing latency of 4 μ s. The L0 system is divided into two independent subsystems: the calorimeter trigger and the muon trigger, both designed to exploit the typically high transverse energy (E_T) and p_T of decay products from b -hadron decays. Each subsystem contributes to the final trigger decision, which is made by the L0 decision unit, determining whether the full event is forwarded to the HLT. The L0-calorimeter trigger utilises signals from the SPD, PS, ECAL, and HCAL detectors to identify high- E_T electrons, photons, and hadrons. E_T is calculated independently for each subdetector by summing over clusters of 2×2 cell. The transverse energy is defined as:

$$E_T = \sum_{\text{cell}=1}^4 E_{\text{cell}} \sin \theta_{\text{cell}} \quad (2.9)$$

where E_{cell} is the energy deposited in the i -th cell and θ_{cell} is the angle between the beam axis and the line from the interaction point to the cell center. The clusters

with the highest E_T are selected, and, using SPD and PS information, they are classified as follows:

- L0Hadron: the HCAL cluster with the highest E_T . If this also corresponds to the ECAL highest- E_T cluster, the hadron E_T is taken as the sum of the HCAL and ECAL contributions.
- L0Electron: the ECAL cluster with the highest E_T that is matched to one or two hits in both the SPD and the corresponding PS cells.
- L0Photon: the ECAL cluster with the highest E_T that is matched to one or two hits in the PS but no hits in the SPD.

An event is accepted by the L0-calorimeter trigger if the E_T of any of these candidates exceeds a predefined threshold that can in general differ year by year of data-taking. To limit processing time in the HLT, high-multiplicity events are vetoed using the total number of SPD hits.

The L0-muon trigger searches for the two highest- p_T muon candidates in each detector quadrant. The muon system performs standalone track reconstruction, using hits that form straight-line tracks across all five muon stations and are consistent with originating from the interaction point. Stations M1 and M2 provide the p_T measurement. Events are retained if the highest- p_T muon exceeds a predefined threshold (L0Muon), or the product of the p_T values of the two highest- p_T muons exceeds another threshold (L0DiMuon).

The High Level Trigger

The High-Level Trigger is a software-based trigger system that operates asynchronously with respect to the 40 MHz LHC bunch-crossing rate. The HLT reduces the event rate from 1.1 MHz to 2–12.5 kHz. All events accepted by the HLT are written to permanent storage as raw data. Events reaching the HLT stage contain at least one L0 candidate and are processed by merging information from all relevant subdetectors. The HLT is structured in two stages: HLT1 and HLT2.

HLT1 performs a partial, rapid event reconstruction aimed at inclusive selections of interesting physics signatures, reducing the rate to around 50 kHz. It refines L0 candidates using tracking information from the VELO and tracking stations, retaining events that include at least one high-quality track with minimum requirements on IP, momentum, p_T and overall track quality. The L0 decision unit provides information about which L0 trigger conditions were satisfied, guiding HLT1 to execute only the relevant algorithm sequences. If multiple L0 candidates are found, their corresponding HLT1 sequences run independently. To optimise

processing, HLT1 avoids reconstructing the same track or vertex more than once. In Run 1, the most efficient HLT1 selection for hadronic decays of charm and beauty hadrons was the Hlt1TrackAllL0 line. It selected high- p_T ($p_T > 1.6$ GeV) tracks with good quality and significant displacement from the primary vertex. In Run 2, this was improved by two new selections: Hlt1TrackMVADecision and Hlt1TwoTrackMVADecision.

The HLT2 stage operates at a significantly lower input rate and therefore has the resources to run a more refined selections with respect to HLT1. It begins with complete pattern recognition to reconstruct all tracks in the event, followed by a series of inclusive and exclusive selection algorithms. Inclusive selections target generic b -hadron decays or known resonances. Exclusive selections focus on specific decay channels, aiming for maximal efficiency. Of particular importance are the inclusive topological trigger lines of HLT2. These are designed to trigger on partially reconstructed b -hadron decays with at least two charged final-state particles forming a displaced secondary vertex. Candidate tracks are selected based on fit quality, impact parameter, and in Run 2 also by using PID information. Vertices are built from combinations of two, three, or four selected tracks, and a multivariate algorithm is used to discriminate signal candidates based on kinematic and isolation variables.

The TIS and TOS categories

Trigger decisions at LHCb are categorised into three types: Triggered On Signal (TOS), Triggered Independently of Signal (TIS) and Triggered On Both (TOB). An event is classified as TOS if the signal under study alone is sufficient to satisfy the trigger requirements and retain the event. Conversely, an event is classified as TIS if it would have been triggered and recorded regardless of the presence of the specific signal being analysed. TOB events require the combined presence of both the signal and the rest of the event, with neither being sufficient on its own.

2.2.4 The LHCb data processing

To manage the data collected, LHCb employs a multi-stage data-processing pipeline designed to maximise data quality and acquisition efficiency. Each stage is controlled by specialised software processing events sequentially. The stages are:

- Triggering (Moore): Raw detector data is filtered by the High-Level Trigger (HLT), implemented via the Moore application as described in Sec. 2.2.3). During Run 1 only events passing the trigger are retained, written to tape as raw banks, containing both trigger and subdetector data. Run 2 introduced

the Turbo method [50] where only the objects involved in the trigger decision plus all the primary vertices are persisted.

- Reconstruction (Brunel): Raw data are copied and distributed across the LHC Computing Grid (LCG) and reconstructed in parallel using the Brunel application. Detector hits are converted into physics objects (e.g., tracks, clusters), with PID information assigned via likelihoods.
- Stripping and Selection (DaVinci): FullDST files, undergo further filtering in the stripping step via DaVinci. Here, physics candidates are reconstructed. Data can be written in DST format, storing the candidates created during the stripping along the full reconstructed event, or μ DST, only storing the candidates identified by the stripping algorithms.
- User Analysis (DaVinci): Users analyse μ DST/DST files with custom DaVinci algorithms to extract variables, perform vertexing, flavor tagging, decay angle computation, and momentum smearing.

An essential aspect during physics analysis is data simulation. Simulation is crucial for background subtraction, efficiency estimation, and systematic studies. It replicates the real data chain, simulating pp interaction and the detector response in the reconstruction of a decay. The generation of pp collisions, the hadronisation and the decay of the resulting particles are controlled by the Gauss application [51] responsible for calling the various Monte Carlo (MC) generators such as Pythia8 [52] (the default in LHCb), and for controlling EvtGen and Geant4. EvtGen [53] is used to describe the decays of simulated particles, while Geant4 [54, 55] is used to simulate the propagation and interaction of particles within the detector. The simulated interaction of particles with the detector and the corresponding energy depositions, are converted to signals (digitisation) that mimic the real detector electronics channels response by the Boole application [56]. In addition the Boole application allows to encode data in raw banks that have the same format of data.

Chapter 3

Theoretical and Experimental Overview of Exotic Hadrons

This chapter provides an overview of the SM with a focus on the quark model, Quantum Chromo-Dynamics (QCD) and exotic hadrons. A brief introduction to the SM is given in Sec. 3.1. Sec. 3.2 introduces the constituent quark model that was the first phenomenological framework capable of classifying, describing, and predicting hadrons composed of light quarks. In Sec. 3.3 an overview of the QCD and its connection with the quark model is presented. Sec. 3.4 introduces quarkonium states. Finally, the current understanding of exotic hadrons and the theoretical approaches developed to interpret their internal structure and formation mechanisms are reported in Sec. 3.5.

3.1 The Standard Model of Particle Physics

In the SM of elementary particle physics, the fundamental building blocks of matter are quarks and leptons, which are $\text{spin}\frac{1}{2}$ particles, called fermions, organised into three generations. Each generation includes particles with different flavours. Every particle has a corresponding antiparticle with the same mass, spin, and lifetime, but with opposite quantum numbers. In quantum field theory, all fundamental interactions between particles derive from the principle of local gauge invariance. This principle states that global symmetries of the Lagrangian, associated with the conserved quantities, must also hold locally, meaning that the symmetry transformations depend on space-time coordinates. To preserve this local invariance, additional spin-1 fields, known as gauge bosons, must be introduced. These fields mediate interactions between particles and are associated with the symmetries of

in the weak interaction. The inclusion of $U(1)_Y$, associated with hypercharge Y , enable the unification of the electromagnetic and weak forces. Since right-handed neutrinos carry no electric charge, have zero hypercharge, and do not interact via the weak or strong forces, they are not considered in the SM. Local gauge invariance under $SU(2)_L \otimes U(1)_Y$ requires the introduction of four gauge fields, which after electroweak symmetry breaking correspond to the W^+ , W^- , and Z bosons (mediators of the weak interaction), and the photon (mediator of the electromagnetic interaction). The gauge symmetry forbids explicit mass terms for the gauge bosons. Instead, masses are generated through spontaneous symmetry breaking, a mechanism where the Lagrangian remains symmetric, but the vacuum state does not. The introduction of the scalar Higgs field allows the W^+ , W^- , and Z bosons to acquire a non-zero longitudinal polarisation and therefore a mass, while the photon remains massless. The same mechanism also gives mass to fermions, except for neutrinos. Quarks and leptons are grouped into three generations, each with the same gauge structure under $SU(2)_L \otimes U(1)_Y$, differing only in mass. Within each generation, fermions are arranged in doublets. While the SM do not include direct interactions between quarks and leptons, charged current weak interactions, mediated by the W^+ and W^- bosons, allow flavour changes within each doublet. Experimental results from the decays of strange mesons revealed that flavour changing neutral currents, mediated by the Z boson, are instead suppressed in the SM and forbidden at the tree level. The Glashow-Ilioupolus-Maiani (GIM) mechanism, stating that the quark flavour eigenstates differ from their weak interaction eigenstates and predicting the charm quark, explained these experimental observations. Therefore, $SU(2)_L$ doublets of quarks are written in the form $(u, d')_L$, where d' is a linear combination of down-type quarks from different generations. This relationship is described by CKM matrix [57, 58], when at least three generations are included in the SM, which introduces a complex phase responsible for CP violation in the SM. In contrast, lepton mixing is absent in the SM due to the assumption of massless neutrinos. Consequently, lepton flavour universality holds: the couplings of leptons to $W^\pm$ are flavour-universal charged-current interactions, while those to the Z are flavour-universal neutral-current interactions with distinct coupling constants. The strong interaction is described by $SU(3)_C$ gauge symmetry, where the mediators are gluons. Only quarks participate in strong interactions, and each quark flavour appears in three colours, the quantum number associated with $SU(3)_C$. Due to the confinement property of QCD, quarks are never observed as free particles but only in bound states known as hadrons, which are colour-singlets. These bound states differ significantly from electromagnetic systems like atoms due to gluon self-interactions.

3.2 The Quark Model

The quark model was introduced in 1964 by Gell-Mann [59] and Zweig [2] to explain the spectrum of strongly interacting particles, which had been discovered since the late 1940s [60]. According to this model, hadrons are not fundamental particles but are instead composed of more elementary constituents called quarks. Initially, three types of quarks were proposed, up (u), down (d), and strange (s), which are now referred to as the light quarks. To distinguish between these quark types, a new quantum number called flavour was introduced.

3.2.1 Light-flavour quarks

Assuming an approximate symmetry among the u , d , and s quarks in the strong interaction there is an approximate invariance under unitary transformations that mix these light quark flavours. This symmetry is described by the flavour $SU(3)_f$ group and it is approximate because the degeneracy between quark flavours is broken by their different mass. In this framework, the quarks (u , d , s) are treated as components of a three-dimensional irreducible representation (denoted as $\mathbf{3}$), while the antiquarks ($\bar{u}$, $\bar{d}$, $\bar{s}$), carrying opposite quantum numbers, transform under its conjugate, denoted as $\bar{\mathbf{3}}$.

Hadrons can then be classified according to irreducible representations formed by the direct product of quark and/or antiquark multiplets. Particles belonging to the same $SU(3)_f$ representation must share the same spin and parity, and the presence of one such particle implies the existence of its $SU(3)_f$ partners within that representation.

The $SU(3)$ group has eight generators, commonly represented by the Gell-Mann matrices [61]. Two of these matrices commute with each other, meaning that within each irreducible representation, particle states can be labeled by two corresponding quantum numbers. These are I_3 , the third component of isospin and Y , the hypercharge, defined as:

$$Y = \mathcal{B} + S - \frac{C - B + T}{3} \quad (3.1)$$

where $\mathcal{B}$ is the baryon number, S is strangeness, C , B , and T represent charmness, bottomness, and topness, respectively. Mesons, composed of a quark and an antiquark, fall under the tensor product of the $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations, resulting in a direct sum of a one dimensional representation $\mathbf{1}$, and a eight dimensional representation $\mathbf{8}$:

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1}_A \oplus \mathbf{8}_S \quad (3.2)$$

where the suffixes A and S stand for antisymmetric and symmetric, respectively, and indicate the properties of the flavour wave function in the interchange of the constituents. The $SU(3)_f$ multiplets can be visualised in a two-dimensional plane using the axes I_3 and Y . When only u , d and s quarks are considered, the hypercharge coincides with the strangeness (up to a normalisation related to the baryon number). For ground-state, with zero orbital angular momentum (S-wave), mesons two different J^P configurations are allowed: the pseudoscalar mesons are in a spin-singlet configurations and have $J^P = 0^-$. The vector mesons have $J^P = 1^-$, corresponding to a spin-triplet configuration. The $SU(3)_f$ octet representation of the pseudoscalar mesons is depicted in Figure 3.2.

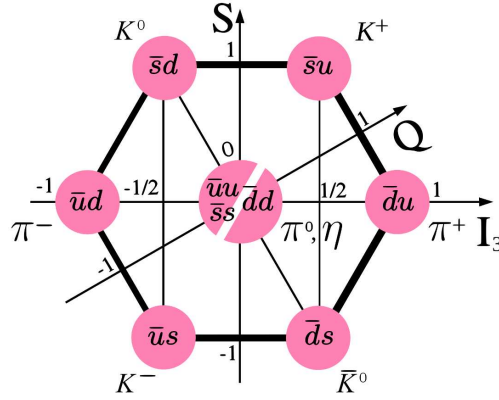


Figure 3.2: $SU(3)_f$ diagram showing the octet representation for the pseudoscalar mesons made of u , d and s quarks as a function of the isospin component I_3 and the strangeness S .

Baryons, made of three quarks, fall under the tensor product:

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_{MS} \oplus \mathbf{8}_{MS} \oplus \mathbf{1}_A \quad (3.3)$$

The flavour-space wave functions are anti-symmetric in the one-dimensional representation $\mathbf{1}$, symmetric in the ten-dimensional representation $\mathbf{10}$ and with mixed symmetry in the eight-dimensional representations $\mathbf{8}$. An example of baryon classification into the octet and decuplet representations is shown in Figure 3.3. The discovery of the Ω^- [62] a baryon composed of three strange quarks, was a major success for the quark model. However, since the Δ^{++} , Δ^- , and Ω^- consist of three identical quarks and are spin-3/2 particles, their wavefunctions could only be made antisymmetric by introducing an additional degree of freedom: colour, with the assumption that each quark flavor comes in three different colour charges. The observed pattern of a $J^P = \frac{1}{2}^+$ octet and a $J^P = \frac{3}{2}^+$ decuplet arises from the

antisymmetry of the colour wavefunction, a fundamental feature of QCD.

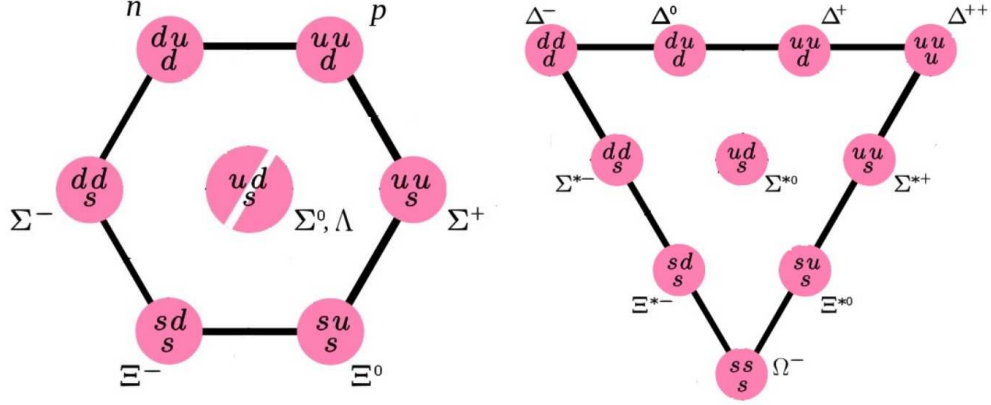


Figure 3.3: $SU(3)_f$ diagram showing the octet (left) and the decuplet (right) representations for the ground state baryons made of u , d and s quarks.

3.2.2 Heavy-flavour quarks

In addition to the light quarks, there are three heavier quark species, charm (c), beauty (b), and top (t) collectively known as heavy-flavour quarks. In 1964, Bjorken and Glashow [63] proposed the existence of a fourth quark flavour, motivated by the observation of four leptons at the time. This theoretical prediction gained strong support in 1970 through the GIM mechanism [64]. Hadrons formed by the quarks (u , d , c , s) can be grouped into irreducible representations of the $SU(4)$ symmetry group. These representations are characterised by quantum numbers such as isospin I_3 , hypercharge Y , and charm number C . For mesons, the quark–antiquark combination $4 \otimes \bar{4}$ decomposes into a singlet 1 and a fifteen-dimensional representation 15 :

$$4 \otimes \bar{4} = 1_A \oplus 15_S \quad (3.4)$$

One of the corresponding diagram is shown in Figure 3.4.

For baryons, the combination $4 \otimes 4 \otimes 4$ yields to:

$$4 \otimes 4 \otimes 4 = 20_S \oplus 20_{MS} \oplus 20_{MS} \oplus 4_A \quad (3.5)$$

corresponding to a twenty-dimensional representation 20_S with symmetric states, two twenty-dimensional representations 20_{MS} with mix-symmetric states and a four-dimensional representation 4_A with anti-symmetric states [65]. One of the

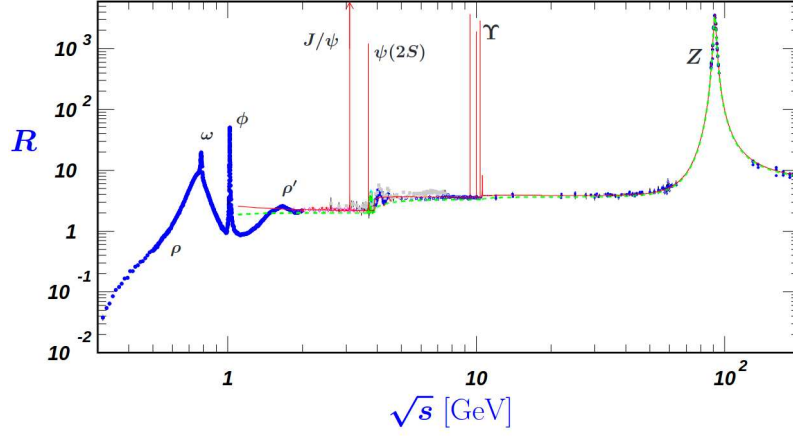


Figure 3.6: Ratio $R(\sqrt{s})$ from the Particle Data Group [66]. The green dashed line is the prediction from the quark model.

apparent inconsistency strongly indicates the presence of an additional internal quantum number that provides the necessary antisymmetric component. This degree of freedom is now known as colour charge. One of the most strongest experimental evidence for colour charge comes from measurements of the ratio of cross sections in electron–positron annihilation experiments:

$$R = \frac{\sigma_{e^+e^- \rightarrow \text{hadrons}}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}} = N_C \sum_{\text{flavour}} e_i^2, \quad (3.6)$$

where N_C is the number of colours, the sum runs over the number of flavours over threshold corresponding to the centre-of-mass energy $\sqrt{s}$ of the collision and e_i is the charge of the quark of flavour i . If only quark flavour and electric charge were to be considered, the expected value of R would be $\sum_i q_i^2$ where q_i is the electric charge of each kinematically accessible quark flavour. Figure 3.6 shows that the experimental result are in agreement with $N_C = 3$. These results, together with the explanation of the $\pi^0 \rightarrow \gamma\gamma$ decay amplitude provided by the introduction of the colour degree of freedom, led to the interpretation of the colour as the charge of the strong interaction.

QCD has two fundamental properties that define the behavior of strong interactions. The first one is asymptotic freedom which describes that, at high energy scales, the strength of the strong force decreases, allowing quarks and gluons to behave almost as free particles. This enables the use of perturbative techniques for high-energy processes. The second property is colour confinement, that describes that all free particles are colour singlets, and coloured particles (quarks and gluons) cannot be observed as isolated objects. Instead, they are confined

within colour-neutral hadrons. Both features are deeply connected to the running coupling of QCD, which characterises the behavior of the strong interaction as a function of the energy scale. The intensity of the strong interaction is characterised by the coupling constant g_s in the QCD Lagrangian, with an equivalent expression of $\alpha_s = \frac{g_s^2}{4\pi}$. The effective value of α_s is determined through renormalisation, a technique used to regulate and absorb divergencies that appear in loop-level quantum field theory calculations. To analyse the scale dependence of α_s , the assumption that it is small enough is made, in order to allow a perturbative expansion. The evolution of α_s with the renormalisation scale μ is governed by the QCD beta function $\beta(\alpha_s) = \mu \frac{d}{d\mu} \alpha_s$. This function can be expressed as a power series in α_s , giving the renormalisation group equation (RGE):

$$\beta(\alpha_s) = -\epsilon\alpha_s - 2\alpha_s\left[\frac{\alpha_s}{4\pi}\beta_0 + \left(\frac{\alpha_s}{4\pi}\right)^2\beta_1 + \left(\frac{\alpha_s}{4\pi}\right)^3\beta_2 + \dots\right], \quad (3.7)$$

if the value of μ is taken as the momentum transfer Q^2 of a given process, the effective coupling of strong interactions is given by the value of $\alpha_s(\mu)$.

Setting $\epsilon = 0$ for simplicity, the one-loop approximation of the RGE becomes:

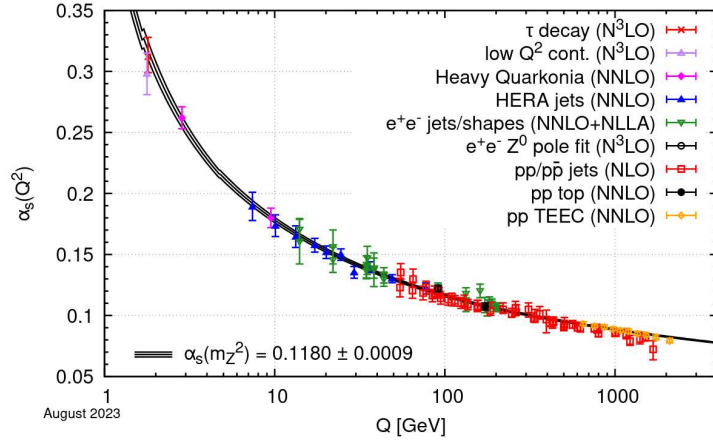
$$\mu \frac{d}{d\mu} \alpha_s = -\frac{\alpha_s^2}{2\pi} \beta_0 \quad (3.8)$$

Solving this differential equation yields the one-loop running coupling constant:

$$\alpha_s(\mu) = -\frac{2\pi}{\beta_0} \frac{1}{\ln \frac{\mu}{\Lambda_{\text{QCD}}}} \quad (3.9)$$

here $\beta_0 = 11 - \frac{2}{3}n_f$, where n_f is the number of active quark flavours contributing at the scale μ . Λ_{QCD} is an integration constant known as the QCD scale, marking the transition between perturbative and non-perturbative QCD regime. Its value depends on the renormalisation scheme and typically lies in the range of 300 to 1100 MeV for $n_f = 3$ [67]. When $Q^2 \gg \Lambda_{\text{QCD}}^2$, α_s becomes small, leading to asymptotic freedom. In the other case, $Q \rightarrow 0$, α_s grows rapidly, entering a non-perturbative regime. This increase is responsible for colour confinement, as it prevents the isolation of individual quarks and gluons. Figure 3.7 shows the variation of α_s as a function of the energy scale.

Since the mass differences between hadrons and their constituent quarks are generally comparable to the QCD scale Λ_{QCD} , an accurate description of hadron spectroscopy cannot be obtained directly from the QCD Lagrangian using perturbative methods. Instead, non-perturbative approaches such as lattice QCD and effective models have been developed to describe and predict hadron properties.

Figure 3.7: Variation of α_s depending on the energy scale Q

Lattice QCD is a non-perturbative technique in which space-time is discretised into a lattice, allowing hadron spectrum to be computed through numerical simulations [68]. Phenomenological models have been formulated to effectively describe the interactions between quarks and gluons within hadrons. One such example is the potential model, which incorporates both short- and long-range features of strong interactions, as well as spin-spin interactions at the quark level. This model has successfully predicted the spectrum of hadrons containing charm–anticharm ($c\bar{c}$) quark pairs [69]. Experimental studies of hadron spectroscopy provide essential tests for theoretical understanding of strong interactions at the hadronic energy scale, as obtained from lattice QCD or effective models.

3.4 Quarkonium states

A special class of mesons, called quarkonia, consists of a quark and its corresponding antiquark of the same flavour. Specifically, a quarkonium composed of a charm–anticharm pair ($c\bar{c}$) is referred to as charmonium, while one made of a beauty–antibeauty pair ($b\bar{b}$) is called bottomonium. Quarkonium states are commonly labeled using spectroscopic notation $n^{2S+1}L_J$, where n is the radial excitation quantum number, S the total spin, L the orbital angular momentum between the quark and antiquark and $J = L \oplus S$ the total angular momentum. The allowed quantum numbers $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, \dots$ where the parity is defined as $P = (-1)^{L+1}$ and the charge-conjugation parity is $C = (-1)^{L+S}$. Each quarkonium system has multiple states with different masses and quantum numbers. The ground state has $L = S = 0$ and no radial excitations. It is denoted as $\eta_c(1S)$ for charmonium and $\eta_b(1S)$ for bottomonium. The next higher states,

with $S = 1$ and $L = 0$, are the vector mesons J/ψ for charmonium and $\Upsilon(1S)$ for bottomonium. Higher-mass states differ due to variations in spin couplings, orbital angular momentum, and radial excitations. The study and classification of these states is known as quarkonium spectroscopy. Quarkonium states with masses below the threshold for decays into open-flavour hadrons are typically narrow states, decaying via electromagnetic or strong interactions into lower-mass quarkonia, light hadrons, or charged leptons. This behavior can be well understood through the Okubo-Zweig-Iizuka (OZI) rule, which states that decays where the initial and final quark lines are connected are favoured, whereas those with disconnected quark lines are suppressed. In QCD terms, hadrons are color singlets, so such decays require at least two gluons, with the number depending on the J^{PC} quantum numbers of the decaying quarkonium. States above the open-flavour threshold, however, tend to have much larger widths dominated by strong decays into open-flavour hadrons. Because the charm and bottom quarks are much heavier than Λ_{QCD} ($m_b > m_c \gg \Lambda_{\text{QCD}}$), charmonium and bottomonium provide an important system for studying the strong interaction, bridging perturbative and non-perturbative QCD. The large quark masses also allow for a non-relativistic treatment, such as using potential models where the quark-antiquark interaction is modeled by a radial potential. A widely used example is the Cornell potential [70]:

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \sigma r + \text{constant} \quad (3.10)$$

where r is the effective radius of the quarkonium state, α_s is the QCD running coupling, σ is the QCD string tension and is a constant of $\sim 0.18 \text{ GeV}^2$ [71]. Starting from this simple potential, numerous models have been developed and refined to match experimental data. Incorporating relativistic corrections further improves the accuracy of predictions, resulting in excellent agreement with observations as shown in. This makes the quarkonium system one of the most precisely understood examples within the non-perturbative regime of QCD. The mesons with a minimal $c\bar{c}$ and $b\bar{b}$ quark content are shown in Figure 3.8.

3.5 Exotic hadrons

The quark model, phenomenologically successful, cannot be rigorously derived from QCD. The hadron spectrum predicted by the quark model does not necessarily coincide with that of QCD, which may allow for a much richer spectrum of hadronic states. In QCD, gluons not only mediate the strong force between quarks but also interact with each other, as they carry colour charge. These self-interactions can give rise to bound states of gluons, known as glueballs. Furthermore, gluons can participate as valence constituents by coupling with a quark-

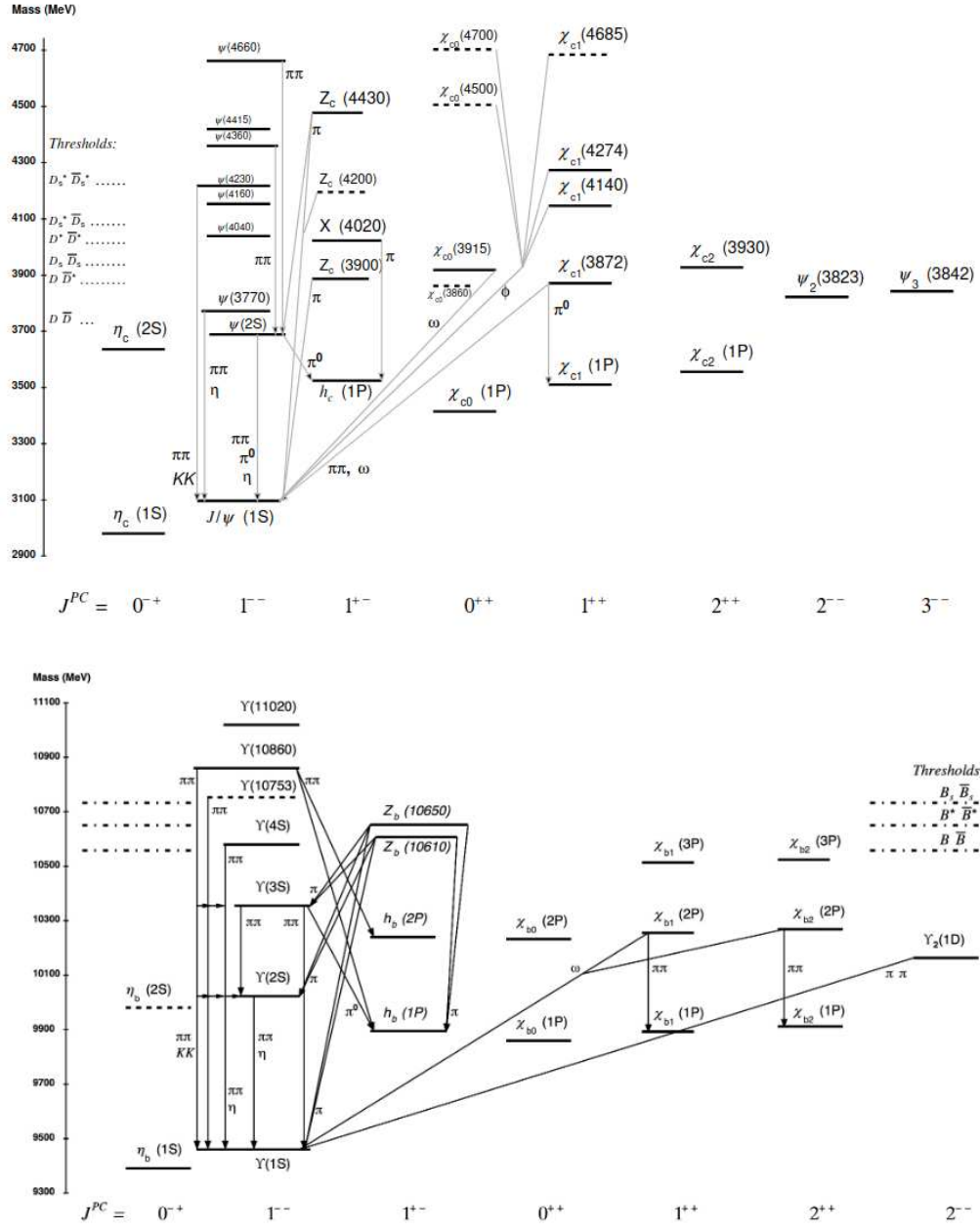


Figure 3.8: The level scheme of meson states containing a minimal quark content of (left) $c\bar{c}$ (right) $b\bar{b}$.

antiquark pair to form hybrid mesons. Both the proponent of the quark model introduced not only the existence of mesons and baryons, but also suggested the possibility of more exotic configurations such as $q\bar{q}q\bar{q}$ and $qqq\bar{q}\bar{q}$. This early recognition of the possibility of multiquark states predates the formal development of QCD. Multiquark states can be further categorised into tetraquarks ($q\bar{q}q\bar{q}$), pentaquarks ($qqq\bar{q}\bar{q}$) and others. Charged quarkonium-like states cannot be described by conventional quark-antiquark mesons. Also, certain quantum numbers, such as $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}$, and 2^{+-} , cannot be achieved either. States possessing these so-called exotic quantum numbers are clear indicators of non-conventional structures. These exotic quantum numbers can, however, be realised by hybrid mesons, tetraquarks, or glueballs. In contrast, three-quark baryons can accommodate all possible J^P combinations such as $\frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}$, and so on. The past 15 years have seen a huge number of evidence suggesting the existence of hadrons that fall outside the simple $q\bar{q}$ framework. Notable among these are the discoveries of two pentaquark candidates reported by the LHCb Collaboration [72], the discovery of one of the narrowest exotic state observed [73] and a series of charmonium-like and bottomonium-like states, that cannot be accommodated within the conventional spectra [74].

A new naming scheme for exotic hadrons is under development by the Particle Data Group with input from LHCb. In this thesis, the most recent names of the exotic hadrons will be used, in line with the PDG prescriptions.

3.5.1 Observations of exotic candidates

The $\chi_{c1}(3872)$ was the first firmly established exotic hadron and remains the most extensively studied to date. It was discovered by the Belle collaboration in 2003 in the $J/\psi\pi^+\pi^-$ system originating from B^+ decays [3], as shown in Figure 3.9. Subsequent analyses by CDF [75], Belle [76], and LHCb [77, 78] determined its quantum numbers to be $J^{PC} = 1^{++}$, consistent with the $\chi_{c1}(2P)$ charmonium assignment. However, its measured mass is about 100 MeV lower than quark model expectations [79], and its unusually narrow width is puzzling for a state lying above the $D\bar{D}$ threshold. Further anomalies arise in its decay pattern. The observation of comparable branching fractions to $\rho(770)J/\psi$, $\omega(782)J/\psi$, and $\pi^0\chi_{c1}(1P)$ indicates isospin violation at the ~ 0.3 level, much larger than the ~ 0.05 typically seen in conventional charmonia [80]. This enhanced violation is plausibly linked to the 8.7 MeV splitting between the $D^0\bar{D}^{*0}$ and D^+D^{*-} thresholds. The $\chi_{c1}(3872)$ lies extraordinarily close to the $D^0\bar{D}^{*0}$ threshold (-0.04 ± 0.12 MeV) and dominantly decays into $D^0\bar{D}^{*0}$, strongly suggesting a $D\bar{D}^*$ molecular nature [81]. At the same time, the preference for $\gamma\psi(2S)$ over $\gamma J/\psi$ decays observed by BaBar [82] and LHCb [83, 84], though contested by Belle [85] and BESIII [86], points to a sizable

charmonium mixture [87]. Such a component, however, does not preclude a predominantly molecular structure [88]. An alternative interpretation is a compact tetraquark. The absence of an experimentally observed charged partner challenges this picture, but it could be reconciled if production of the isovector states is suppressed relative to the isoscalar $\chi_{c1}(3872)$ [89]. Production properties of the $\chi_{c1}(3872)$ in pp and $p\bar{p}$ collisions have also been invoked to argue for the presence of a compact component [90, 91], though these arguments remain debated [92, 93, 94].

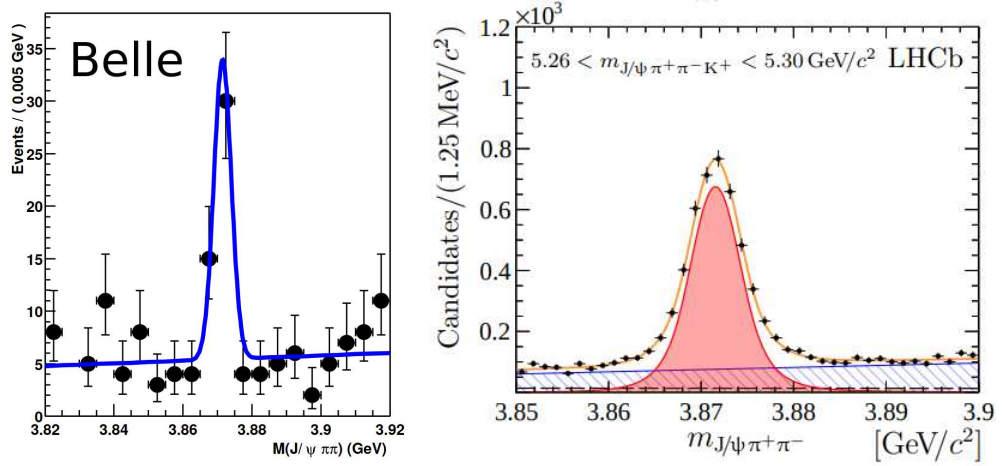


Figure 3.9: (Left) $J/\psi\pi^+\pi^-$ invariant-mass distribution from Belle with the results of the unbinned fit superimposed. (Right) $\chi_{c1}(3872) \rightarrow J/\psi\pi^+\pi^-$ observed at LHCb [95].

Several exotic hadron candidates share the quantum numbers of χ_{cJ} states (J^{++}), most notably the $\chi_{c1}(3872)$ discussed earlier. Belle first observed the $\chi_{c0}(3915)$ and $\chi_{c2}(3930)$ in $B^+ \rightarrow \omega J/\psi K^+$ [5] and $\gamma\gamma \rightarrow D\bar{D}$ [96], with confirmations by BaBar [97, 98]. Both collaborations also reported signals of the $\chi_{c0}(3915)$ in $\gamma\gamma \rightarrow \omega J/\psi$ [99, 100], shown in Figure 3.10, while Belle claimed evidence for either $\chi_{c0}(3915)$ or $\chi_{c2}(3930)$ in $\gamma\gamma \rightarrow \gamma\psi(2S)$ [101]. BESIII found hints of the $\chi_{c0}(3915)$ in $e^+e^- \rightarrow \gamma\omega J/\psi$ [102]. LHCb observed the $\chi_{c2}(3930)$ in prompt production decaying to both D^+D^- and $D^0\bar{D}^0$ [103], shown in Figure 3.10. LHCb also found contributions from both $\chi_{c0}(3915)$ and $\chi_{c2}(3930)$ in $B^+ \rightarrow D^+D^-K^+$ decays [104]. Belle also reported the $\chi_{c0}(3860)$ in $e^+e^- \rightarrow J/\psi D\bar{D}$ [105], with a much larger width than $\chi_{c0}(3915)$ and $\chi_{c2}(3930)$, 100–400 MeV and 20–35 MeV respectively. LHCb, however, saw no evidence in $B^+ \rightarrow D^+D^-K^+$. Interpreting these states as the $\chi_{cJ}(2P)$ multiplet is problematic. The small mass gap between $\chi_{c0}(3915)$ and $\chi_{c2}(3930)$, plus the OZI-suppressed $\chi_{c0}(3915) \rightarrow \omega J/\psi$, disfavors their assignment as $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$. The $\chi_{c0}(3860)$ may be a better $\chi_{c0}(2P)$

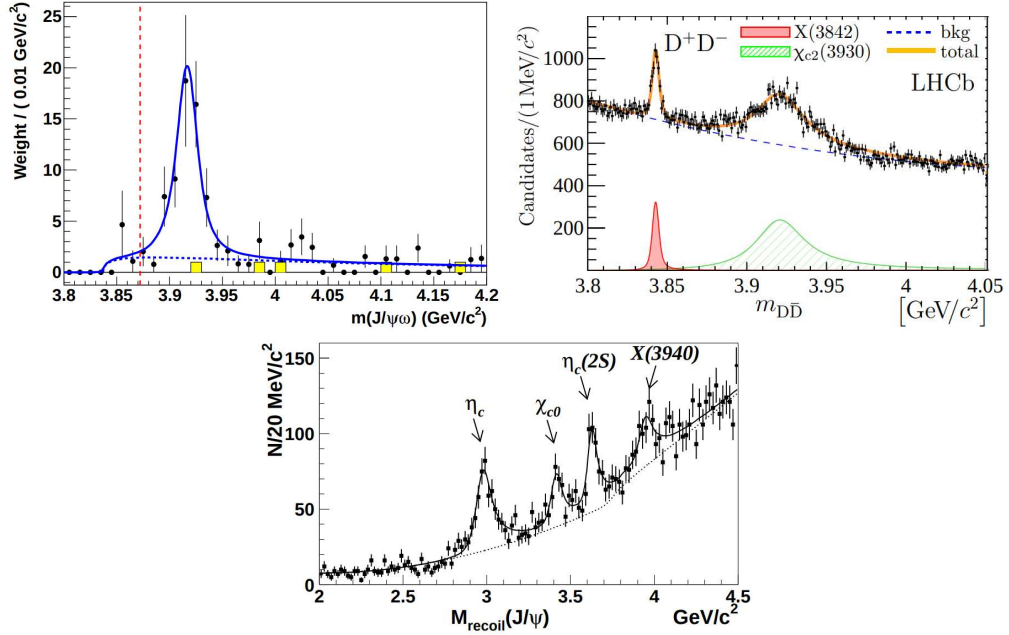


Figure 3.10: (Top left) $\chi_{c0}(3915) \rightarrow J/\psi\omega$ at BaBar. (Top right) The $\chi_{c2}(3930) \rightarrow D^+D^-$ at LHCb (bottom) and $X(3940)$ in $e^+e^- \rightarrow J/X$ at Belle.

candidate but remains unconfirmed. No solid $\chi_{c1}(2P)$ has been established, the $X(3940)$ observed by Belle [106], shown in Figure 3.10 could be a candidate, but its J^P is unknown and no other experiment has confirmed it. Since the $\chi_{c1}(2P)$ mass should fall between $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$, further clarification is crucial. A recent LHCb report of a possible $h_c(2P)$ near 4 GeV [107], if confirmed, would make conventional assignments of $\chi_{c0}(3915)$ and $\chi_{c2}(3930)$ even more difficult.

A broad peak observed in $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi\pi J/\psi$ by BaBar [108], was later resolved by BESIII [109] into two structures, $\psi(4230)$ and $\psi(4360)$. The latter had already appeared in $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi\pi\psi(2S)$ at BaBar [110] and Belle [111]. Their spin-parity is fixed by the production process. Many $e^+e^- \rightarrow (c\bar{c})(q\bar{q})$ processes show large structures between 4.2–4.4 GeV, often involving intermediate $T_{c\bar{c}}$ states, suggesting decays like $\psi(4230)$ or $\psi(4360) \rightarrow T_{c\bar{c}}\pi$, though resonance parameters vary by final state. In collision systems other than e^+e^- , the D0 collaboration sees a $T_{c\bar{c}1}(3900)^{\pm,0}$ signal in inclusive b -decays for $J/\psi\pi^+\pi^-$ masses 4.2–4.3 GeV [112, 113], unconfirmed by LHCb or Belle. If $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$ are assigned to $\psi(3S)$, $\psi(2D)$, and $\psi(4S)$, the $\psi(4230)$ and $\psi(4360)$ cannot be conventional charmonia [114], consistent with their prominence in OZI-suppressed hidden-charm decays. Better understanding of vector states above open-charm thresholds is needed, and coupled-channel analyses [115, 116, 117,

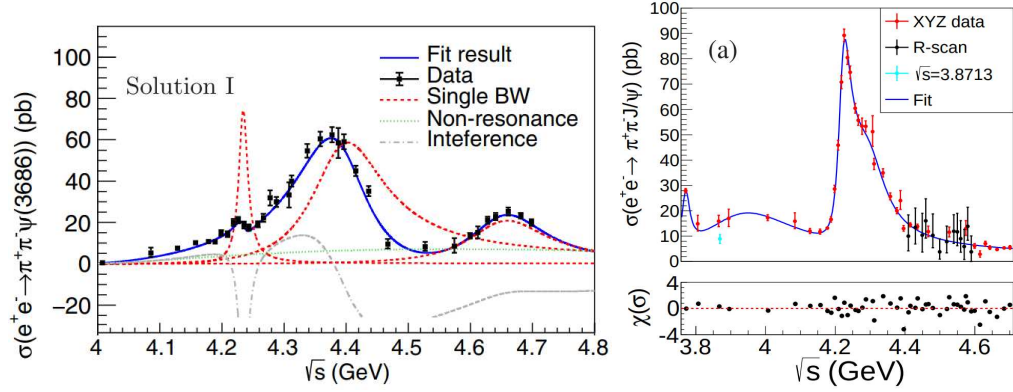


Figure 3.11: (Left) $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ cross-section measured at BESIII. (Right) $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ cross-section at BESIII [130].

118, 119, 120, 121] could help. The $\psi(4230)$ may unify several exotic states, given its decays to $\pi^\pm T_{c\bar{c}1}(3900)^\mp$ and $\gamma\chi_{c1}(3872)$, potentially interpretable as a $D_1(2420)\bar{D}$ molecule [122, 123]. Higher-energy e^+e^- structures are usually attributed to $\psi(4660)$, seen in $\pi^+\pi^-\psi(2S)$ by Belle [111], BaBar [124], and BESIII [125], shown in Figure 3.11. A near-threshold $\Lambda_c^+\Lambda_c^-$ peak seen by Belle [126] is unconfirmed by BESIII [127], while recent BESIII data on $K\bar{K}J/\psi$ [128] and $D_s^{*+}D_s^{*-}$ [129] indicate additional structures near 4.5–4.7 GeV.

In 2009, CDF observed a near-threshold $J/\psi\phi$ structure at 4140 MeV in $B^+ \rightarrow J/\psi\phi K^+$, the $\chi_{c1}(4140)$ [131], later confirmed by CDF[7], CMS [132], and D0 [133], along with a possible second state near 4270 MeV. D0 also reported a similar structure in prompt $p\bar{p}$ production [134], with hints from BaBar [135], though mass and width measurements varied. Early searches by Belle [136, 137], BESIII [138], and LHCb [139] did not confirm the $\chi_{c1}(4140)$. High-statistics amplitude analyses by LHCb [140, 141] then observed both $\chi_{c1}(4140)$ and $\chi_{c1}(4274)$, with different masses and widths and determined quantum numbers, as shown in Figure 3.12. Additional states were found: $\chi_{c0}(4500)$ and $\chi_{c0}(4700)$ ($J^{PC} = 0^{++}$), $X(4630)$ ($J^{PC} = 1^{-+}$ or 2^{-+}), $\chi_{c1}(4685)$ ($J^{PC} = 1^{++}$), evidence for $X(4150)$ [142] ($J^P = 2^-$), and a potential $X(4740)$ [143]. LHCb also observed $\chi_{c1}(4274)$ and $\chi_{c0}(4500)$ in the final state $J/\psi\phi$ in diffractive processes in proton-proton collisions [144]. Some of these may correspond to conventional charmonia: $\chi_{c1}(4140)$ or $\chi_{c1}(4274)$ as $\chi_{c1}(3P)$, and $\chi_{c0}(4500)$ or $\chi_{c0}(4700)$ as $\chi_{c0}(4P/5P)$, though decays to $J/\psi\omega$ remain unexplored. Molecular interpretations have been suggested: $D_s^+D_s^-$ ($J^{PC} = 0^{++}$ or 2^{++}) is ruled out for $\chi_{c1}(4140)$ by its $J^P = 1^+$, while $D_s^+D_s^{*-}$ molecules [145] or kinematic cusps [146, 147] remain possible.

The $T_{c\bar{c}1}(3900)^\pm$ was first observed by Belle and BESIII in $e^+e^- \rightarrow T_{c\bar{c}1}(3900)^\pm\pi^\mp$ [148,

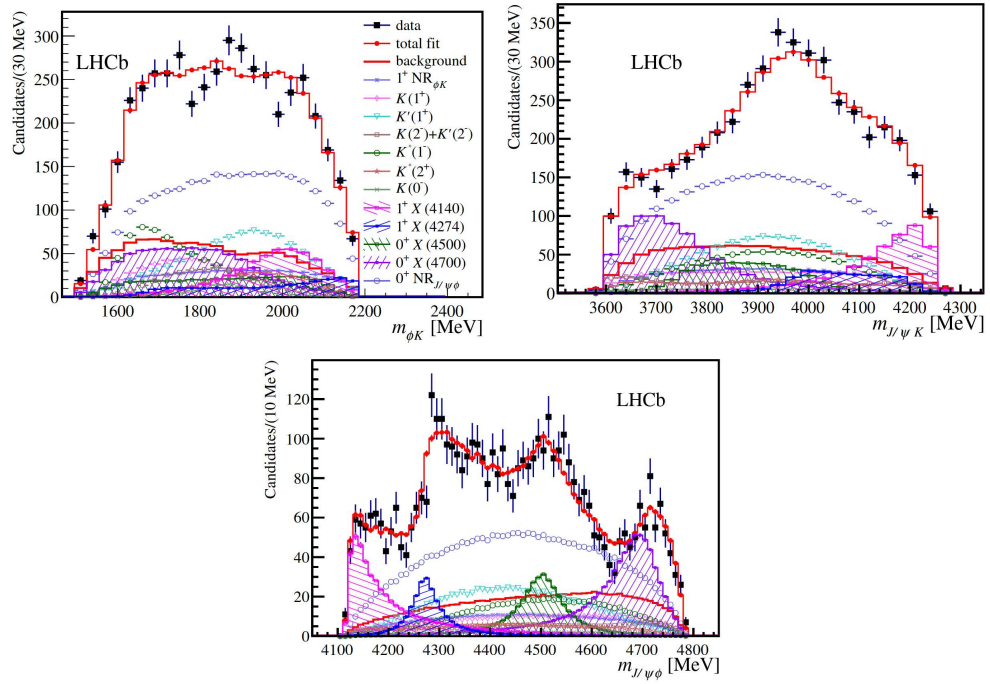


Figure 3.12: Distributions of (top left) ϕK^+ , (top right) $J/\psi K^+$ and (bottom) $J/\psi \phi$ invariant masses for the $B^+ \rightarrow J/\psi \phi K^+$ candidates (black data points) compared with the amplitude fit results (red points). Individual fit components are also shown.

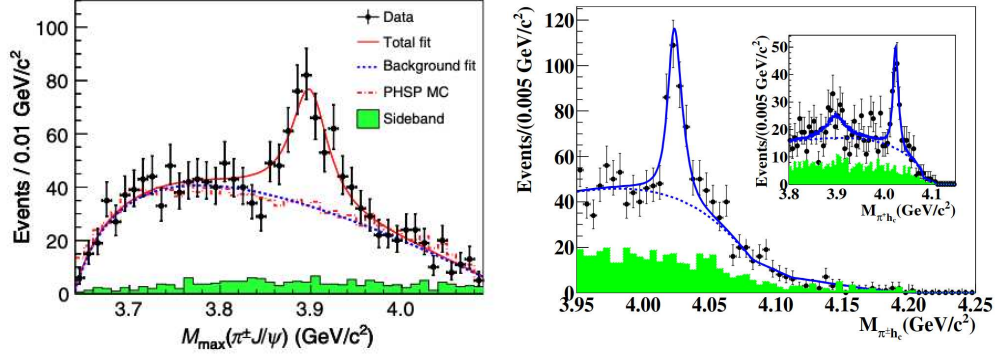


Figure 3.13: (Left) $T_{cc1}(3900)$ and (right) $T_{cc}(4200)$ signal at BESIII.

149], with Belle using initial state radiation. CLEO-c soon confirmed and reported evidence of the neutral partner[150], later established with high statistics by BESIII [151, 152]. BESIII amplitude analysis determined $J^{P(C)} = 1^{+(-)}$ [153]. Its mass lies near the $(D^*\bar{D})^{\pm,0}$ threshold, its dominant decay[154, 155]. Since it is electrically charged, it is manifestly exotic, requiring $c\bar{c}$ plus light quarks. A related state, $T_{cc}(4020)^{\pm,0}$, is found near the $(D^*\bar{D}^*)^{\pm,0}$ threshold, decaying to $h_c\pi^{\pm,0}$ [156, 157] and $(D^*\bar{D}^*)^{\pm,0}$ [158, 159], and is likely connected to the $T_{cc1}(3900)^{\pm,0}$, as shown in Figure 3.13. Both are often interpreted as hadronic molecules, though diquark–antidiquark models and triangle-singularity explanations remain viable [160, 122, 161, 162, 163, 164, 165, 166, 167].

The first charged charmonium-like state, the exotic $T_{cc1}(4430)^-$, was discovered by Belle in $\bar{B} \rightarrow \psi(2S)\pi^-\bar{K}$ as a peak in $\psi(2S)\pi^-$ [168]. A later Belle amplitude analysis favored $J^P = 1^+$ at 3.4σ [169]. LHCb confirmed the state with much larger data, measuring a width of ~ 180 MeV and $J^P = 1^+$ shown in Figure 3.14, supported by clear Argand phase motion [170]. In the same study, LHCb reported a second resonance, $T_{cc}(4240)^+$, at 4240 MeV with width ~ 220 MeV and preferred $J^P = 0^-$ [170]. Belle also observed the broad $T_{cc1}(4200)^+$ in $\bar{B}^0 \rightarrow J/\psi\pi^+K^-$ [171], which interferes with $T_{cc1}(4430)^+$, producing a dip in the $J/\psi\pi^+$ spectrum. Further candidates include $T_{cc}(4050)^+$ and $T_{cc}(4250)^+$ in $\chi_{c1}\pi^+$ from Belle[172], and a possible $T_{cc}(4100)^-$ in $\eta_c\pi^-$ from LHCb [173], with significance just above 3σ and favored $J^P = 0^+$ or 1^- . While the $T_{cc1}(4430)^+$ shows resonant phase motion [170], such effects may also arise kinematically. Resolving their true nature will require independent tests, e.g. photo-production.

First open-strange hidden-charm candidates were reported by BESIII in $e^+e^- \rightarrow D^*\bar{D}_sK$ and $D\bar{D}_s^*K$ [174] and by LHCb in $B \rightarrow J/\psi\phi K$ [141]. BESIII observed an enhancement near the $D_s^-D^{*0}$ and $D_s^{*-}D^0$ thresholds in the K^+ recoil

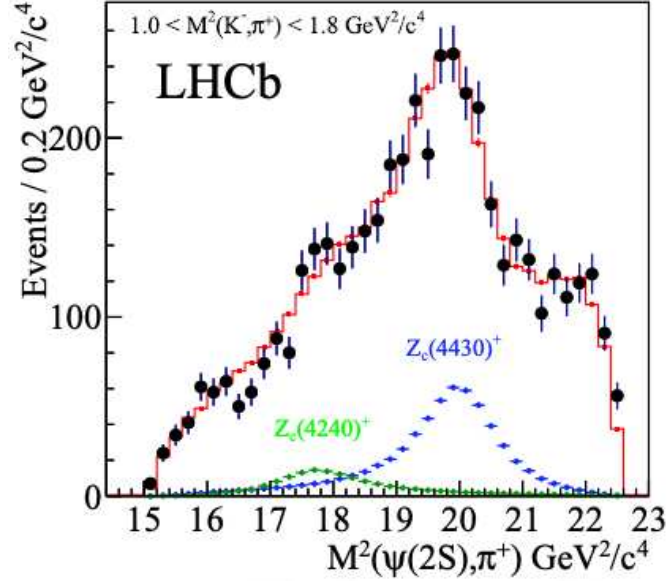


Figure 3.14: $T_{cc1}(4430)^+$ and $T_{cc0}(4240)^+$ signal at LHCb.

mass, named $T_{ccs}(3985)^-$ [174]. Fitted with a relativistic Breit–Wigner, it has a width of ~ 13 MeV. Spin-parity is assumed 1^+ due to S -wave dominance, though other assignments are not excluded. A neutral partner was later seen in $e^+e^- \rightarrow (D_s^+ D^{*-} K_S^0$ and $e^+e^- \rightarrow D_s^{*+} D^- K_S^0$) [175], strengthening but not proving a resonant interpretation. In $B^+ \rightarrow J/\psi \phi K^+$, LHCb identified two states, $T_{ccs1}(4000)^+$ and $T_{ccs1}(4220)^+$, with $> 5\sigma$ significance [141], shown in Figure 3.15. Amplitude analysis established $J^P = 1^+$ for the $T_{ccs1}(4000)^+$, while 1^+ is preferred over 1^- for the $T_{ccs1}(4220)^+$. An isospin partner of $T_{ccs1}(4000)^+$ was later observed in $B^0 \rightarrow J/\psi \phi K_S^0$ with 4σ significance [176], with consistent mass and width. Despite similar masses, the $T_{ccs}(3985)^+$ and $T_{ccs1}(4000)^+$ differ greatly in width, ~ 10 and ~ 130 MeV), possibly indicating distinct states or coupled-channel effects [177]. In the diquark model, they correspond to different but mass-degenerate $J^{PC} = 1^{++}$ and 1^{+-} states [178].

The $\Upsilon(10860)$ and $\Upsilon(11020)$ were first observed as structures in the $e^+e^- \rightarrow$ hadrons cross section by CUSB [179] and CLEO [180]. They were later seen in exclusive decays to $\pi\pi\Upsilon(nS)$ ($n = 1, 2, 3$) and $\pi\pi h_b(nP)$ ($n = 1, 2$) [181, 182], see Figure 3.16. In $e^+e^- \rightarrow \pi^+\pi^-\Upsilon(nS)$, Belle reported an additional structure, $\Upsilon(10753)$, near the $\Upsilon(10860)$ [181], confirmed by Belle II with higher precision [183]. Belle II also observed $\Upsilon(10753)$ in $\omega\chi_{b1,2}(1P)$ production [184], suggesting that earlier signals attributed to $\Upsilon(10860) \rightarrow \omega\chi_{b1,2}(1P)$ [185] may have been due to its tail, shown in Figure 3.16. A coupled-channel analysis indicates

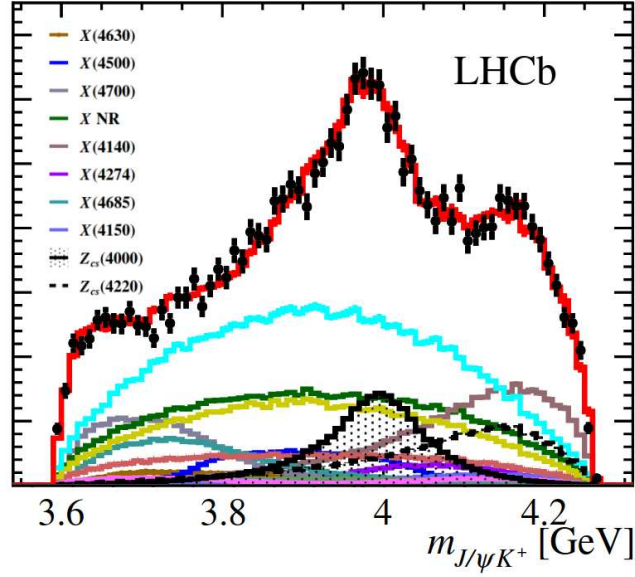


Figure 3.15: $T_{c\bar{c}s}(4000)^+$ and $T_{c\bar{c}s}(4220)^+$ signal at LHCb.

that large fractions of $\Upsilon(10860)$ and $\Upsilon(11020)$ decays remain unobserved, with $B^*\bar{B}^{(*)}\pi$ likely dominant[186]. Above threshold, conventional bottomonia are expected to decay mainly to $B\bar{B}$ -type final states. The $\Upsilon(10753)$, $\Upsilon(10860)$ and $\Upsilon(11020)$ dominate the $e^+e^- \rightarrow \pi\pi\Upsilon(nS)$ cross sections. While $\Upsilon(10860)$ and $\Upsilon(11020)$ are usually identified as $\Upsilon(5S)$ and $\Upsilon(6S)$, alternatives remain. The $\Upsilon(10753)$ could be a $\Upsilon(nD)$ [187, 186] or a tetraquark[188]. Hybrid bottomonium states are also predicted in this mass range, with all three structures suggested as candidates [189, 190, 191, 192].

The $T_{b\bar{b}}(10610)$ and $T_{b\bar{b}}(10650)$ were first observed by Belle in $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ and $e^+e^- \rightarrow h_b(nP)\pi^+\pi^-$ [4], as shown in Figure 3.17, with a neutral partner of the $T_{b\bar{b}}(10610)$ identified soon after [193]. Partial-wave analysis at $\sqrt{s} = 10.866$ GeV favors $J^P = 1^+$ [194]. Like the $T_{c\bar{c}}(3900)$ and $T_{c\bar{c}}(4020)$ at the $D^*\bar{D}$ and $D^*\bar{D}^*$ thresholds, the $T_{b\bar{b}}(10610)$ and $T_{b\bar{b}}(10650)$ lie at the $B^*\bar{B}$ and $B^*\bar{B}^*$ thresholds and decay mainly into these channels [195]. They also correlate with the $\Upsilon(10860)$ and $\Upsilon(11020)$ structures in $e^+e^- \rightarrow \Upsilon(nS)\pi\pi$ and $e^+e^- \rightarrow h_b(nP)\pi\pi$ [182], though these are usually assigned as $\Upsilon(5S)$ and $\Upsilon(6S)$. A molecular picture is commonly favored [196, 197, 198, 199, 167], though it predicts spin partners not yet observed [200, 201, 202]. Compact tetraquark interpretations remain possible [203, 204]. Compositeness studies suggest strong molecular components [205], and lattice QCD finds attractive $B^*\bar{B}$ interactions consistent with this picture [206, 207,

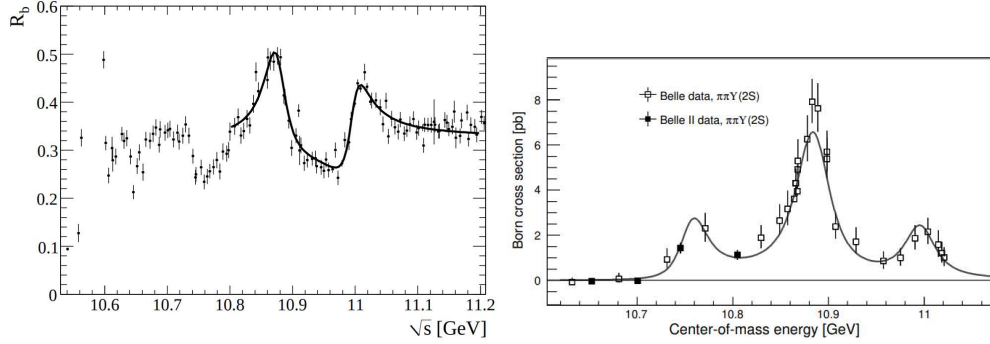


Figure 3.16: (Left) $\Upsilon(10860)$ and $\Upsilon(11020)$ peaks in the $e^+e^- \rightarrow \text{hadrons}$ cross-section as measured by BaBar and (right) in the $e^+e^- \rightarrow \Upsilon(2S)\pi^+\pi^-$ cross-section with also the $\Upsilon(10753)$ measured by Belle and Belle II.

208].

Resonant structures in $J/\psi J/\psi$ were first reported by LHCb in 2020 [209], with a clear peak at 6.9 GeV and hints near 6.5 and 7.2 GeV. CMS [210] and ATLAS [211] later confirmed the 6.9 GeV state; CMS additionally observed a second resonance around 6.6 GeV ($> 6\sigma$) and evidence for another at 7.29 GeV ($\sim 4\sigma$). ATLAS also reported hints of these states. The $J/\psi J/\psi$ spectra of the three experiments are depicted in Figure 3.19. Masses, widths, and yields of the $T_{cc\bar{c}\bar{c}}$ candidates might be highly sensitive to interference effects in one-dimensional fits, and theory predicts a richer spectrum of possible resonances [212, 213, 214, 215]. The experimental values, especially for the 6.6 and 7.3 GeV structures, remain with pending confirmation in other channels. The states have been interpreted as compact tetraquarks, molecular states, or more exotic configurations [216, 214, 215, 217, 218], with early predictions dating back to 1975 [219] and 1982 [220]. A broader overview can be found in Ref. [221]. CMS recently measured the spin and parity of these structures [222].

The $D_{s0}^*(2317)^+$ and $D_{s1}^*(2460)^+$ were first observed by BaBar [223, 224] and CLEO [225], and later confirmed by Belle [226] and BESIII [227]. Both have positive parity and could be conventional P -wave D_s excitations, but their masses are 50–200 MeV below expectations and their widths remain unusually small, < 3.7 MeV, [228]. This has motivated exotic interpretations as tetraquarks [229, 9] or $DK^{(*)}$ molecules [230], though conventional assignments influenced by nearby $DK^{(*)}$ thresholds [228] and suppressed, isospin-violating decays remain viable.

The T_{cs} and $T_{c\bar{s}}$ states have been identified at LHCb in amplitude analyses of $B^+ \rightarrow D^+ D^- K^+$ and $B \rightarrow \bar{D} D_s \pi$ decays [231, 104, 232, 233, 107], shown in Figure 3.20. The decays of $T_{cs0,1}(2900)^0 \rightarrow D^- K^+$, $T_{c\bar{s}0}(2900)^{++} \rightarrow D_s^+ \pi^+$, and

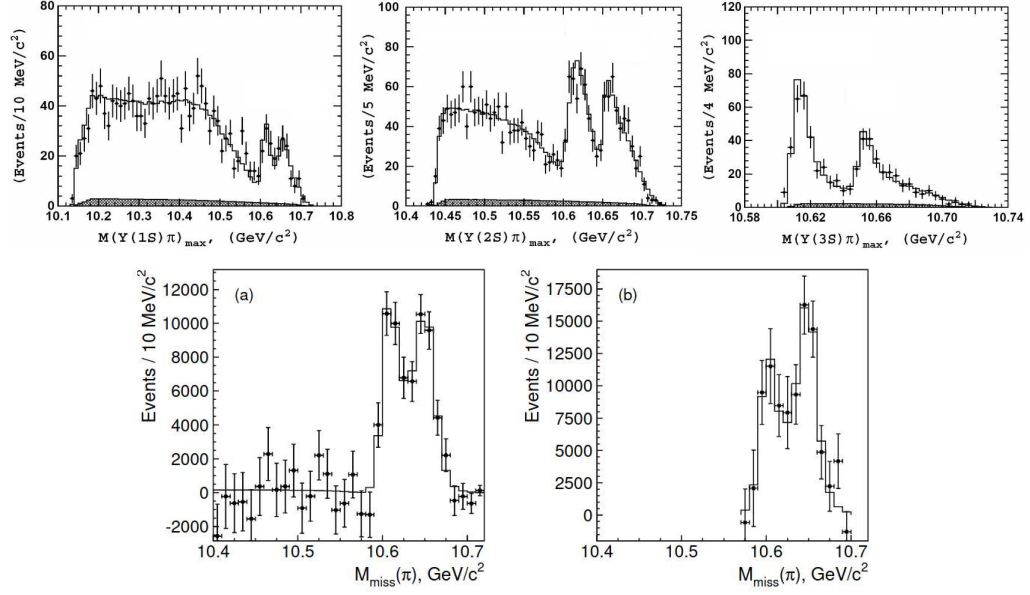


Figure 3.17: (Top) The T_{bb}^+ peaks seen in the $\Upsilon(1S/2S/3S)\pi^+$ and (bottom) $h_b(1P/2P)\pi^+$ modes by Belle.

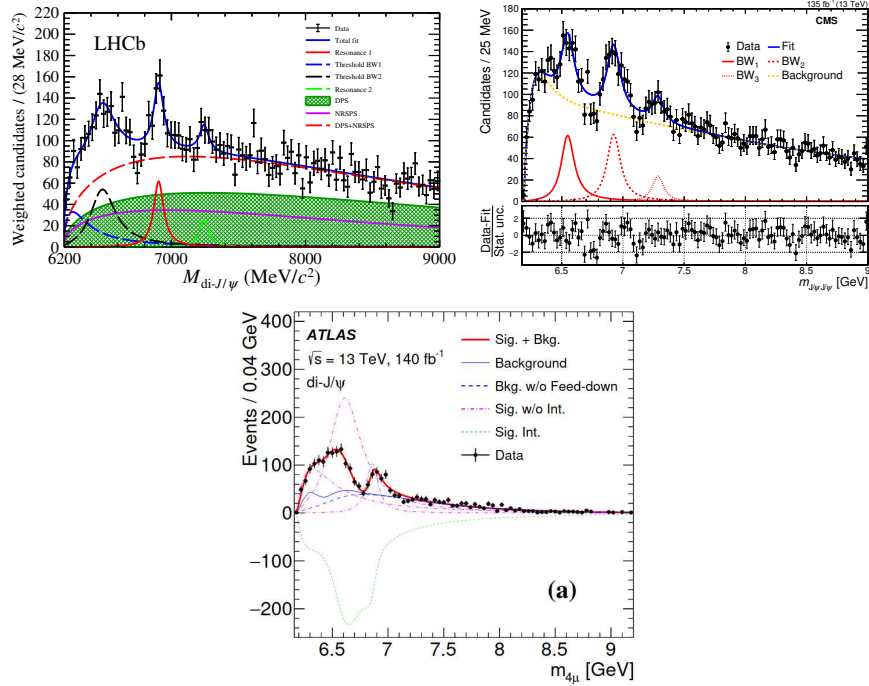


Figure 3.18: $J/\psi J/\psi$ mass spectra in (top left) LHCb, (top right) CMS and (bottom) ATLAS.

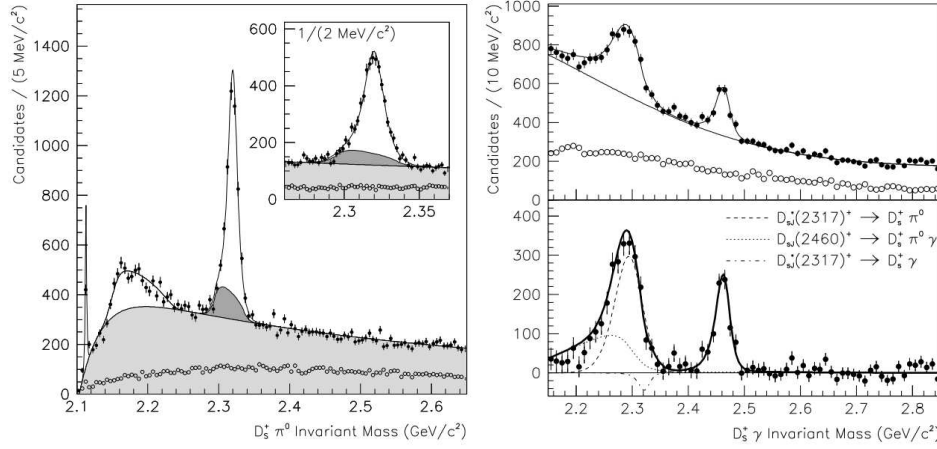


Figure 3.19: (Left) $D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0$ and (right) $D_{s1}^*(2460)^+ \rightarrow D_s^+ \gamma$ at BaBar.

$T_{c\bar{s}0}(2900)^0 \rightarrow D_s^+ \pi^-$ establish minimal quark contents of $[cs\bar{u}\bar{d}]$, $[c\bar{s}u\bar{d}]$, and $[c\bar{s}d\bar{u}]$, respectively, making all manifestly exotic states with open charm. The $T_{c\bar{s}0}(2900)^{0/++}$ may be $SU(3)_f$ partners of $T_{c\bar{s}0}(2900)^0$, though differences in mass and especially width suggest a more complex relation [234]. Proposed interpretations include compact tetraquarks [235, 236], hadronic molecules such as $D^* K^*$ or $D_1 K$ [237], and kinematic effects like threshold cusps or triangle singularities [238, 239].

LHCb observed a narrow structure, the $T_{cc}(3875)^+$, in the $D^0 D^0 \pi^+$ spectrum from pp collisions [240, 241], as shown in Figure 3.21. The measured binding energy is $\delta m = -359 \pm 40_{-6}^{+9}$ keV, corresponding to a width of $48 \pm 2_{-14}^{+0}$ keV, orders of magnitude narrower than typical exotic hadrons. The analysis assumes $I(J^P) = 0(1^+)$, consistent with expectations for a $[cc\bar{u}\bar{d}]$ ground state, decaying via DD^{*+} and $D^+ D^{*0}$ with an off-shell D^* . The data strongly support the interpretation of the T_{cc}^+ as the $[cc\bar{u}\bar{d}]$ ground state with $I(J^P) = 0(1^+)$, in agreement with theoretical predictions [242, 243, 244, 245, 246, 247]. The excellent fit with a pure DD^* decay model, combined with the tiny binding energy, favors a molecular interpretation [243], though a compact component cannot be excluded. Finally, models predict that the $[bc\bar{u}\bar{d}]$ partner may lie below strong and electromagnetic thresholds, rendering it long-lived, with realistic prospects for discovery in the near future [248].

The first pentaquark candidates were reported by LHCb in 2015 in $\Lambda_b \rightarrow J/\psi, pK^-$ decays [72], requiring a minimal quark content of $c\bar{c}uud$. An amplitude analysis revealed two resonances, $P_{c\bar{c}}(4380)^+$ and $P_{c\bar{c}}(4450)^+$, with widths of $205 \pm 18 \pm$

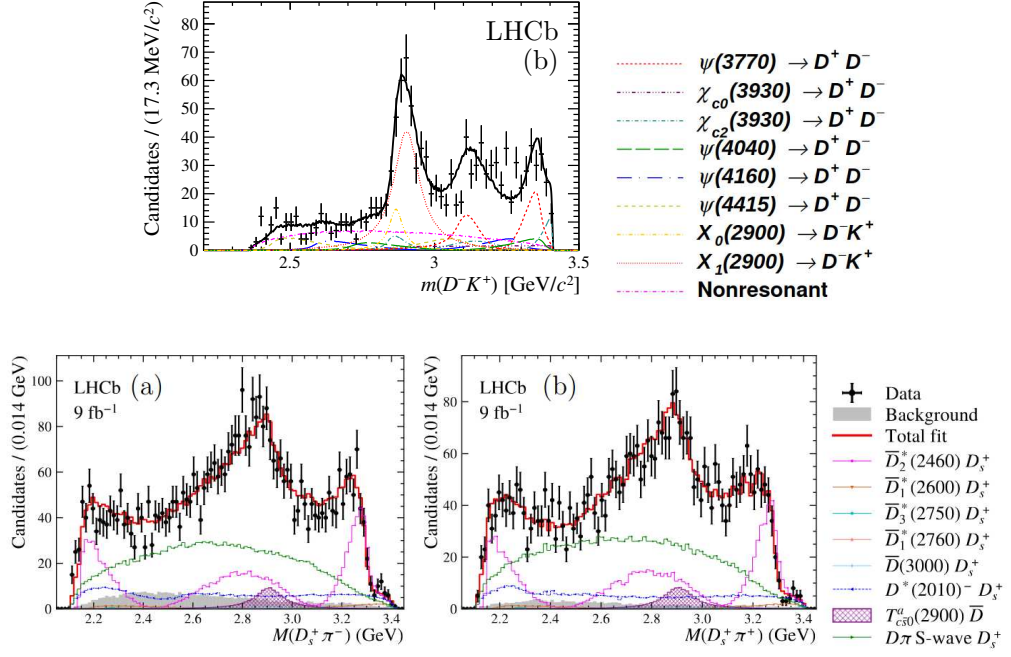


Figure 3.20: (Top) $T_{cs0,1}(2900)^0$ in $B^+ \rightarrow D^+ D^- K^+$, (bottom left) $T_{c\bar{s}0}(2900)^0$ in $B^0 \rightarrow \bar{D}^0 D_s^+ \pi^-$ and (bottom right) $T_{c\bar{s}0}(2900)^+$ in $B^+ \rightarrow D^- D_s^+ \pi^+$ seen by LHCb.

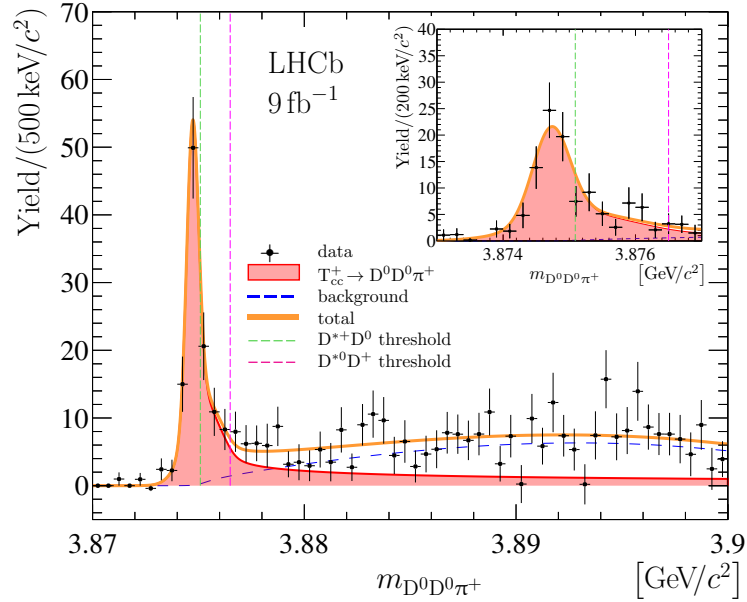


Figure 3.21: $T_{cc}(3875)^+ \rightarrow D^0 D^0 \pi^+$ observed by LHCb.

86 MeV and $39 \pm 5 \pm 19$ MeV and preferred combination $J^P = (3/2^-, 5/2^+)$, though alternative assignments were not excluded. A 2019 update with the full 9 fb^{-1} dataset resolved the 4450 peak into two states at 4440 and 4457 MeV and uncovered a new state at 4312 MeV [249], as shown in Figure 3.22. Their masses lie just below $\Sigma_c^+ \bar{D}^{(*)0}$ thresholds, suggesting a molecular nature [250, 251, 8, 252], though compact pentaquarks [253, 254, 255, 256] or triangle singularities [257] have also been discussed. LHCb disfavors the cusp interpretation for $P_{c\bar{c}}(4312)^+$ and $P_{c\bar{c}}(4440)^+$ [249]. A further candidate, $P_{c\bar{c}}(4337)^+$, was observed in $B_s^0 \rightarrow J/\psi p \bar{p}$ decays with a mass of 4337 MeV and width of 30 MeV [258], as displayed in Figure 3.22. This state does not match those seen in Λ_b decays. Its mass, lying 20–40 MeV above nearby thresholds, disfavors a molecular picture; instead, it may be a compact pentaquark [259, 260] with $J^P = 1/2^+$ or a cusp [261] with $J^P = 1/2^-$. Future LHCb amplitude analyses can discriminate between these options. Complementary searches are performed via photoproduction, $\gamma p \rightarrow J/\psi p$, at GlueX [262, 263, 264, 265, 266].

$SU(3)_f$ flavour symmetry implies that hidden-charm pentaquarks should also appear with strangeness [251, 267, 268, 269, 270, 271]. The first such candidate, $P_{c\bar{c}s}(4338)$, was observed by LHCb in $B^- \rightarrow J/\psi \Lambda \bar{p}$ decays [272], as shown in Figure 3.23. An amplitude analysis revealed a narrow resonance in $J/\psi \Lambda$ at 4338.2 MeV with width ~ 7 MeV and preferred $J^P = 1/2^-$, located just below the $\Xi_c^+ D^-$ threshold. Its proximity to threshold and narrow width support a molecular interpretation [273], though a triangle singularity has also been proposed [274]. A second candidate, $P_{c\bar{c}s}(4458)$, was reported in $\Xi_b \rightarrow J/\psi \Lambda K^-$ with $\sim 3\sigma$ significance [275], displayed in Figure 3.23. The structure lies 19 MeV below the $\Xi_c^0 \bar{D}^{*0}$ threshold and is consistent with a possible two-peak pattern with $J^P = 1/2^-$ and $3/2^-$, as predicted by molecular models [268, 269, 270, 271, 273], analogous to the $P_{c\bar{c}}(4440/4457)$ doublet. In this picture, $P_{c\bar{c}s}(4338)$ could be the $SU(3)_f$ partner of $P_{c\bar{c}}(4312)$. Current statistics, however, are insufficient to confirm the two-peak structure.

In Table 3.24 is reported the list of exotics hadron candidates up to date.

3.5.2 Models for the exotic candidates

Many models have been proposed to explain the exotic nature of such states [8, 9, 10, 11, 12, 13] but a coherent and complete picture for these states is not yet available. Different theoretical approaches have successfully described some of the observed states but fail or become inapplicable for others. This inconsistency comes from an incomplete understanding of the underlying nature of these states. It is possible that an underlying mechanism, not yet observed, governs their be-

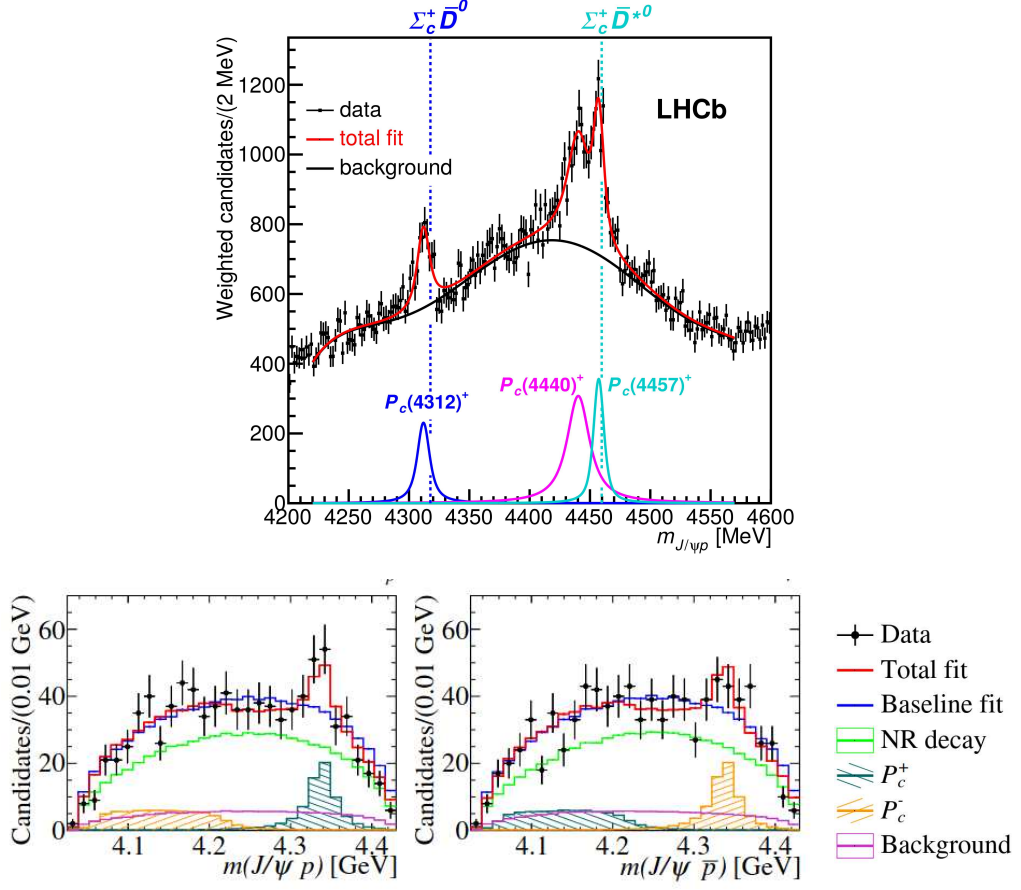


Figure 3.22: (Top) $P_{c\bar{c}}$ states in $\Lambda_b \rightarrow J/\psi p K^-$ at LHCb. (Bottom) $P_{c\bar{c}}(4337)^\pm$ state in $B_s^0 \rightarrow J/\psi p \bar{p}$

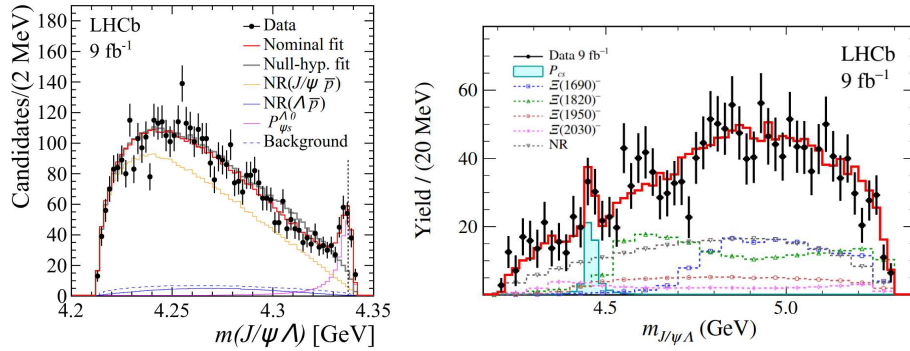


Figure 3.23: (Left) $P_{c\bar{c}s}(4338)^0$ and (right) $P_{c\bar{c}s}(4458)^0$ states at LHCb.

Category		States / Candidates	
Meson-like (incl. tetraquarks)	Hidden Charm	$I = 0$	χ -like: $\chi_{c1}(3872)$, $\chi_{c0}(3860)$, $\chi_{c0}(3915)$, $\chi_{c2}(3930)$, $X(3940)$ ψ -like: $\psi(4230)$, $\psi(4360)$, $\psi(4660)$ with $s\bar{s}$: $\chi_{c1}(4140)$, $\chi_{c1}(4274)$, $\chi_{c1}(4685)$, $\chi_{c1}(4500)$, $\chi_{c1}(4700)$ $X(4150)$, $X(4630)$, $X(4740)$
		$I = 1$	seen in e^+e^- : $T_{c\bar{c}1}(3900)^{+0}$, $T_{c\bar{c}}(4020)^{+0}$, $T_{c\bar{c}}(4055)^+$ seen in B decays: $T_{c\bar{c}}(4050)^+$, $T_{c\bar{c}}(4100)^+$, $T_{c\bar{c}1}(4200)^+$, $T_{c\bar{c}}(4240)^+$, $T_{c\bar{c}}(4250)^+$, $T_{c\bar{c}1}(4430)^+$
		$I = 1/2$	$T_{c\bar{c}s}(3985)^-$, $T_{c\bar{c}s1}(4000)^{-0}$, $T_{c\bar{c}s1}(4220)^-$
		$I = 0$	$\Upsilon(10753)$, $\Upsilon(10860)$, $\Upsilon(11020)$
	Hidden Bottom	$I = 1$	$T_{bb1}(10610)^+$, $T_{bb1}(10650)^+$
	Hidden Double Charm		$T_{c\bar{c}c\bar{c}}(6550)$, $T_{c\bar{c}c\bar{c}}(6900)$, $T_{c\bar{c}c\bar{c}}(7290)$
	Open Single Charm		D_s^* -like: $D_{s0}^*(2317)^+$, $D_{s1}(2460)^+$
			$T_{cs/c\bar{s}}$: $T_{cs0}(2900)^0$, $T_{cs0}(2900)^{0/+}$, $T_{cs1}(2900)^0$
	Open Double Charm		$T_{cc}(3875)^+$
Baryon-like (incl. pentaquarks)	Hidden Charm	$I = 1/2(3/2)$	$P_{c\bar{c}}(4312)^+$, $P_{c\bar{c}}(4440)^+$, $P_{c\bar{c}}(4457)^+$ $P_{c\bar{c}}(4380)^+$, $P_{c\bar{c}}(4337)^+$
		$I = 0(1)$	$P_{c\bar{c}s}(4458)^0$, $P_{c\bar{c}s}(4338)^0$

Figure 3.24: Known exotic hadron candidates up to date. Underscored candidates are well-established states.

havior, or alternatively, that multiple, unrelated mechanisms are responsible. A key feature of these exotic candidates lies in their distinctive decay patterns. For example, while several known charmonium states lie above the open-charm threshold and predominantly decay into open-charm pairs approximately twice as many charmonium-like states are found in the same mass region but decay preferentially into hidden-charm final states. This discrepancy indicates that conventional charmonium models are effective only below the open-charm threshold and fail at higher masses, where new degrees of freedom likely become relevant. Identifying and understanding these additional degrees of freedom remains one of the most pressing challenges for theoretical models of hadronic structure. Although bottomonium spectroscopy is more experimentally demanding and requires larger data samples, several unconventional states have been observed in the $b\bar{b}$ system as well. These states exhibit properties similar to those seen in the $c\bar{c}$ sector, consistent with expectations from heavy-quark symmetry. Several theoretical frameworks aim to explain the internal quark structure of these exotic states:

- Hadronic molecules [8]: These states are proposed to be loosely bound meson–antimeson molecules, with a spatial separation greater than ~ 1 fm. Their binding arises from nuclear-force-like interactions, mediated by pion and light meson exchange.
- Diquark–antidiquark configurations[9, 10]: QCD allows for the formation of diquarks, quark pairs in a spin 0 configuration, with a binding strength roughly half that of a quark–antiquark pair. These diquarks can pair with corresponding antidiquarks to form compact tetraquark states through gluon exchange.
- QCD hybrids [276]: These are exotic hadrons composed by a quark–antiquark pair and a valence gluon. The gluon contributes to the quantum numbers, enabling combinations such as $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}$, which are forbidden in conventional quarkonia.
- Hadroquarkonium [11]: In these models the heavy quark pair forms a compact, colour-singlet core resembling a conventional quarkonium state, embedded in a cloud of light-quark matter. The interaction between the core and the light mesonic field is similar to a QCD analogue of the van der Waals force. This framework provides an explanation for the prevalence of hidden-flavour decays in many states lying above the open-flavour thresholds.
- Threshold effects [12, 13]: In three-body decays, a virtual S-wave meson pair can produce a sharp enhancement or cusp—near the threshold in the final-state invariant-mass distribution. These effects have in general a small

coupling but can mimic resonances.

If a resonance-like structure is confirmed to be a genuine hadronic state, its wave function is likely a quantum admixture of several configurations, including a conventional quarkonium component. The key distinction between competing models lies in the assumed dominant configuration, while the others contribute smaller admixtures. Determining the leading component can yield valuable insights into the strong dynamics governing multi-quark systems. A notable example is the $\chi_{c1}(3872)$, whose internal structure is widely believed to be a superposition of a compact $c\bar{c}$ core in the 2^3P_1 configuration and a loosely bound $D\bar{D}^*$ S-wave molecular component. The molecular part accounts for the observed isospin violation, while the charmonium core explains the prompt production and production via B decays. Also, the compact interpretations expect the existence of multiplets, similarly to what occurs for conventional hadrons.

Distinguishing between genuine exotic hadrons and kinematic artifacts demands further experimental input. In this thesis, a search for charged charmonium-like mesons is conducted using the dataset collected by the LHCb detector, as described in the following chapters.

Chapter 4

Amplitude analysis of $B^0 \rightarrow \eta_c(1S)K^+\pi^-$ decay

This chapter reports the analysis of $B^0 \rightarrow \eta_c(1S)K^+\pi^-$ decay. In Sec. 4.1 are summarised the motivations for this analysis, while the analysis strategy is briefly described in Sec. 4.2. The data samples used, corresponding to the full Run 1 and Run 2 dataset, are described in Section 4.3. In the same Section are also described the MC samples used for the selection procedure. The selection procedure is described in Section 4.4. The fit procedure to extract the $B^0 \rightarrow \eta_c(1S)K^+\pi^-$ candidates is described in Section 4.5. Sec.4.6 reports the measurements of the branching fraction of the $B^0 \rightarrow \eta_c(1S)K^+\pi^-$ decays. The strategy to take into account efficiency variations across the phase space is described in Section 4.7. The **Laura++** package [277] has been utilised to perform the amplitude fit. The Dalitz plot fit using a default model only comprising the K^* contributions is described in Section 4.8. The addition of exotic amplitudes and the determination of their significance is reported in Section 4.9. Finally, the study of systematic effects have been reported in Section 4.10 .

4.1 Physics motivation

BESIII reported the observation of a distinct peak [149], named $T_{c\bar{c}1}(3900)^-$, as shown in Figure 4.1, in the $J/\psi\pi$ final state. This structure was subsequently confirmed by the Belle [148] and CLEO [150] collaborations. In a related analysis of $e^+e^- \rightarrow D\bar{D}^*\pi$, BESIII observed a prominent near-threshold enhancement [154]. Although its measured mass was approximately 2σ lower than that of the $T_{c\bar{c}1}(3900)^-$, this discrepancy may be attributed to potential biases from the

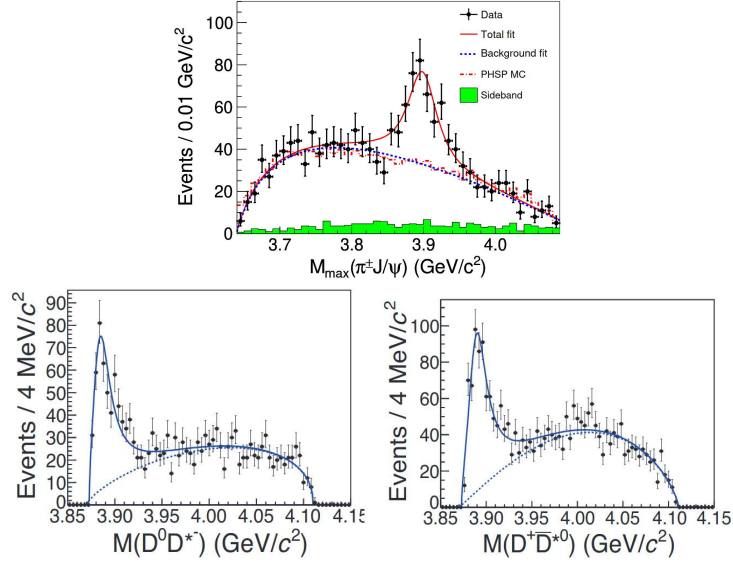


Figure 4.1: The $J/\psi\pi^\pm$ invariant-mass distribution in $e^+e^- \rightarrow J/\psi\pi^+\pi^-$ events collected by BESIII at $\sqrt{s} = 4.26\text{GeV}$ (top). The $m_{D^0 D^{*-}}$ (bottom left) and $m_{D^+ \bar{D}^{*0}}$ (bottom right) distributions for selected events

Analysis strategy

simple one-dimensional fitting method. This state is likely the same $T_{c\bar{c}1}(3900)^-$, observed in an alternative decay mode. The strong signal observed in this new decay mode, shown in Figure 4.1, enabled BESIII to determine the spin-parity of the state as $J^P = 1^+$.

Theoretical studies of this state, that manifestly require the presence of two quarks and two antiquarks in its composition, lead to the prediction of some states that can decay to $\eta_c(1S)\pi^{-1}$. The authors of [278] interprets the $T_{c\bar{c}1}(3900)^-$ as a hadrocharmonium state, where the J/ψ is embedded in a light-quark excited state. This interpretation leads to the prediction of a isovector four-quark resonance where the embedded J/ψ is replaced with the η_c with dominant decay mode to $\eta_c\pi^-$. A feature of this model is the relation of the $T_{c\bar{c}1}(3900)^-$ with the $T_{c\bar{c}1}(4430)^-$, which decays into $\psi(2S)\pi^-$. One can suppose that the $T_{c\bar{c}1}(4430)^-$ is a radial excitation of the $c\bar{c}$ core over the $T_{c\bar{c}1}(3900)^-$, like the $\psi(2S)$ is a radial excitation over the J/ψ . The mass difference between $T_{c\bar{c}1}(4430)^-$ and $T_{c\bar{c}1}(3900)^-$ is ~ 55 MeV lower than the mass difference between $\psi(2S)$ and J/ψ : this deviation can be attributed to the different interaction with the light-quarks and gluons cloud due to a larger spatial size of the $\psi(2S)$ meson. In the limit of heavy quark spin symmetry, the mass splitting between the $T_{c\bar{c}1}(3900)^-$ and the state decaying

¹The simplified notation η_c to refer to the $\eta_c(1S)$ meson is used throughout.

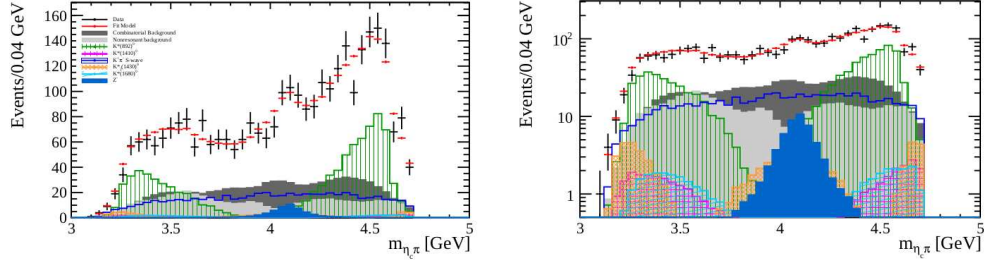


Figure 4.2: The $\eta_c\pi^-$ mass distribution collected by LHCb in the 2011-2012-2016 data taking.

to $\eta_c\pi^-$ should be the same as between the J/ψ and η_c masses: the expected mass of this resonance would be therefore ~ 3785 MeV while its quantum numbers would be $I^G(J^P) = 1^-(0^+)$. This state should decay strongly into a $D\bar{D}$ pair, but in the hadrocharmonium model the decay into open charm mesons is suppressed because the probability of the reconnection of the bindings between heavy and light quarks is suppressed. The authors of [279] interpreted states like the $T_{c\bar{c}1}(3900)^-$ as analogues of quarkonium hybrids. The excitation of the gluon field (the valence gluon) is replaced by an isospin-1 excitation of the gluon and light-quark fields. This interpretation is based on lattice QCD and predicts different multiplets of charmonium tetraquarks, some of these states allowing the decay into the $\eta_c\pi^-$ system. The mass of these charmonium tetraquarks are estimated from lattice QCD calculations for the energy splitting between charmonium mesons [280]. The masses of these states are 4025 ± 49 MeV for the quantum numbers $I^G(J^P) = 1^-(0^+)$, 3770 ± 42 MeV for $I^G(J^P) = 1^-(1^-)$ and 4045 ± 44 MeV for $I^G(J^P) = 1^-(2^+)$. The diquark model [9] also predict a possible exotic candidate decaying to the $\eta_c\pi^-$ system. The diquark model predicts a $J^P = 0^+$ candidate below the open-charm threshold that could decay into the $\eta_c\pi^-$ final state.

An LHCb analysis searching for this state has been performed, using data collected in 2011, 2012 and 2016. The evidence of a state decaying to $\eta_c\pi^-$ has been found in the $B^0 \rightarrow \eta_c K^+\pi^-$ decay [173], see Figure 4.2. The mass and width of this resonance are $m_{T_{c\bar{c}}^-} = 4096 \pm 20_{-22}^{+18}$ MeV and $\Gamma_{T_{c\bar{c}}^-} = 152 \pm 58_{-35}^{+60}$ MeV. After this evidence has been found various theoretical studies have been done to understand its nature. The authors of [281] calculated the mass, width and coupling of this resonance treating it as a scalar four-quark system with a diquark–antidiquark structure. The calculated mass and width are respectively $m_{T_{c\bar{c}}(4100)} = (4080 \pm 150)$ MeV and $\Gamma_{T_{c\bar{c}}(4100)} = (147 \pm 19)$ MeV.

Other studies found the $T_{c\bar{c}}(4100)$ compatible with the tetraquark hypothesis favouring the quantum number $J^{PC} = 0^{++}$ considering the measured width [282].

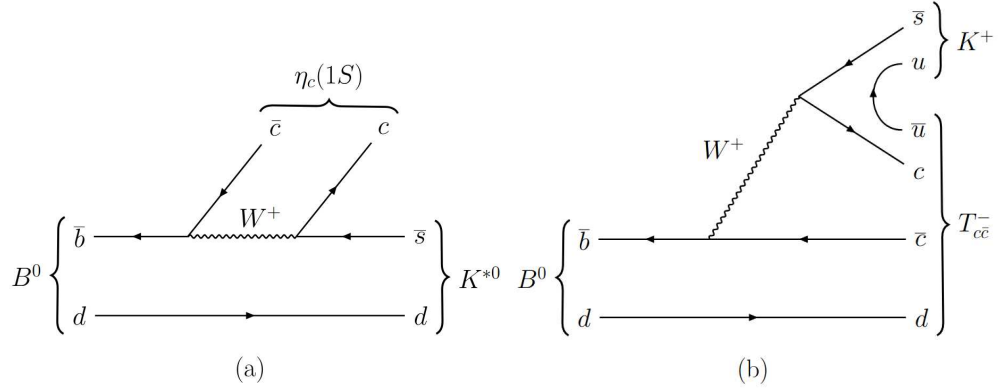


Figure 4.3: Feynman diagrams for the (a) $B^0 \rightarrow \eta_c K^{*0}$ and (b) $B^0 \rightarrow T_{c\bar{c}}(4100)^- K^+$ decay modes.

The study [283] suggest that this is a four quark state of the hadrocharmonium type where the charmonium core η_c is embedded in an S-wave inside the excited light-quark state X with the quantum numbers of a pion $I^G(J^P) = 1^-(0^-)$, suggesting also that the difference of the mass between $T_{c\bar{c}}(4200)$ and $T_{c\bar{c}}(4100)$ should be close to that between the J/ψ and η_c and that the widths of their dominant decays, into $J/\psi\pi$ and $\eta_c\pi$ respectively, should be the same. There are also studies [284] proposing that $T_{c\bar{c}}(4100)$ is the charge conjugate state of $T_{c\bar{c}}(4050)^+$ found by the Belle collaboration [285]. It could also be a kinematic effect arising from an S-wave $D^*\bar{D}^*$ rescattering with $I^G(J^{PC}) = 1^-(0^{++})$ or a resonance produced by the P-wave $D^*\bar{D}^*$ interaction [286].

The $B^0 \rightarrow \eta_c K^+ \pi^-$ decay can proceed through the Feynman diagram shown in Figure 4.3 and it is expected to be dominated by the $K^{*0}(892) \rightarrow K^+ \pi^-$ resonance, with some contributions from other $K^{*0} \rightarrow K^+ \pi^-$ resonances. The decay could also proceed through exotic intermediate states, as indicated by the diagram shown in Figure 4.3, which could result in resonant structures in the $\eta_c \pi^-$ system. Therefore, conducting a new study of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay with the full Run 1 and Run 2 dataset, approximately twice the size of the one used in [173], will lead to more precise assessments about the presence and eventually the properties of exotic contributions in this decay.

4.2 Analysis strategy

In order to investigate possible exotic contributions in the $\eta_c \pi^-$ spectrum, an amplitude analysis of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay channel is carried out. The $B^0 \rightarrow \eta_c K^+ \pi^-$ channel is reconstructed with the η_c decaying to a proton-antiproton pair,

in order to remove the need to distinguish between kaons and pions originating from the B^0 decay, reducing associated systematic uncertainties. The $B^0 \rightarrow \eta_c K^+ \pi^-$ involves pseudoscalar mesons in the initial and in the final state. The phase space of the decay of a spin-zero particle to three spin-zero particles can be completely described by the Dalitz plot of any pair of the two-body square invariant-masses. Therefore the decay is fully described by two independent variables. An angular analysis is not needed since it would not add any discriminant power to the results. A Dalitz plot analysis of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay channel is carried out.

The kinematics of three-body decays involving spin-0 particles are completely determined by two variables. These are typically chosen to be two of the three possible squared invariant-masses of the two-body subsystems, which then form the coordinate axes of the Dalitz plot (DP). The third squared invariant-mass is not independent but constrained by energy and momentum conservation:

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = M^2 + m_1^2 + m_2^2 + m_3^2 \quad (4.1)$$

Here, m_{ij}^2 are the squared invariant-masses of particle pairs, m_i are the masses of the final state particles, and M is the mass of the decaying parent particle.

In terms of these variables, the differential partial decay width is given by:

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} |\overline{\mathcal{A}}|^2 dm_{12}^2 dm_{23}^2 \quad (4.2)$$

Here, $|\overline{\mathcal{A}}|^2$ is the squared matrix element (or squared decay amplitude), averaged over the initial and summed over final states, containing the dynamics of the decay. If $|\overline{\mathcal{A}}|^2$ is constant, the DP is uniformly populated, the event density per unit area $dm_{12}^2 dm_{23}^2$ is uniform. However, if $|A|^2$ varies with momenta and angles, the event density reflects the underlying decay dynamics. Thus, the DP provides a comprehensive picture of the decay, including intermediate resonant or non-resonant states, their spins, and possible interference patterns. The Dalitz plot is a powerful tool to visualise and study decay sequences. Intermediate resonant states, which are often produced via weak decays followed by strong decays, manifest as bands on the DP, as can be seen from Figure 4.4. Decays may also proceed through non-resonant or quasi-resonant intermediate states, often characterised by final state interactions in a given two-particle subsystem. All contributions with the same spin J in the same pair of children can be grouped into a partial wave of spin J . The DP is essentially a scatter plot in the $m_{12}^2 - m_{23}^2$ plane, where each point represents a single decay event. The distribution is confined within a kinematic boundary defined by the available phase space, as illustrated in Figure 4.4.

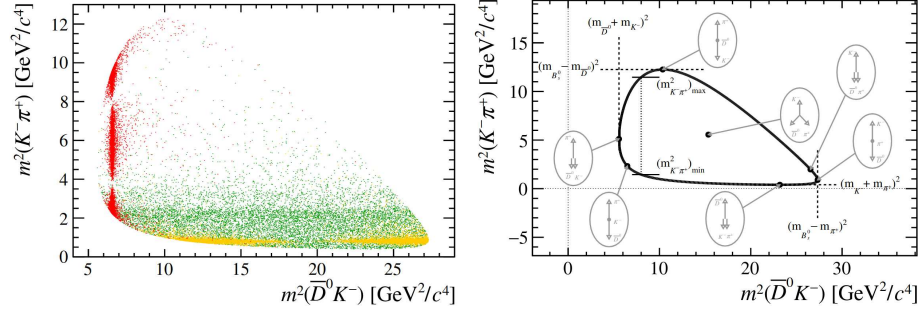


Figure 4.4: Left: examples of the resonances which may appear a the Dalitz plot, in this case: (red) $D_{s2}^{*}(2573)^{-}$, (orange) $K^{*}(892)^0$, (green) $K\pi$ S-wave. Right: kinematic boundaries of the three-body phase space for the decay $B_s^0 \rightarrow \bar{D}^0 K^- \pi^+$. The insets indicate the configuration of the final-state particle momenta in the parent rest frame at various different DP positions.

4.3 Data samples

The data sample used has been collected by LHCb at $\sqrt{s} = 7$ TeV in 2011, at $\sqrt{s} = 8$ TeV in 2012 and $\sqrt{s} = 13$ TeV in 2015-2018, corresponding to an integrated luminosity of about 9 fb^{-1} .

Monte Carlo samples have been used to optimise the event selection, evaluate the efficiency and to validate the parameterisation of the components in fit to the data. The decay modes have been generated with Pythia 8. Run 1 MC samples used in the analysis have been summarised in Tab. 4.1 for B^0 and B_s^0 decay modes. Run 2 MC samples have been summarised in Tab. 4.2. All the samples have been generated uniformly in the phase space, with the exception of the 11134004 event type, where the events have been generated uniformly in the square Dalitz Plot (SDP) space.

Decay mode	Event type	Num events
$B^0 \rightarrow p\bar{p}K^+\pi^-$	11104070	12009296
$B^0 \rightarrow p\bar{p}\pi^+\pi^-$	11104076	12005998
$B_s^0 \rightarrow p\bar{p}K^+K^-$	13104061	12003999
$B_s^0 \rightarrow p\bar{p}K^+\pi^-$	13104060	9177632
$B^0 \rightarrow \eta_c K^+\pi^-$	11134003	12017998
$B^0 \rightarrow \eta_c K^+\pi^-$ (SDP)	11134004	24017996
$B^0 \rightarrow J/\psi K^+\pi^-$	11134010	12015997

Table 4.1: Run 1 MC samples with the B^0 and B_s^0 meson as initial state. η_c and J/ψ decay into $p\bar{p}$ final state. SDP stands for events generated flat in the Square Dalitz Plot.

Decay mode	Event type	Num events
$B^0 \rightarrow p\bar{p}K^+\pi^-$	11104070	28019948
$B^0 \rightarrow p\bar{p}\pi^+\pi^-$	28089947	12005998
$B_s^0 \rightarrow p\bar{p}K^+K^-$	13104061	28023967
$B_s^0 \rightarrow p\bar{p}K^+\pi^-$	13104060	16142545
$B^0 \rightarrow \eta_c K^+\pi^-$	11134003	28035961
$B^0 \rightarrow \eta_c K^+\pi^-$ (SDP)	11134004	56221916
$B^0 \rightarrow J/\psi K^+\pi^-$	11134010	28039965

Table 4.2: Run 2 MC samples with the B^0 and B_s^0 meson as initial state. η_c and J/ψ decay into $p\bar{p}$ final state. SDP stands for events generated flat in the Square Dalitz Plot.

From the hardware level trigger responses the logical OR between the L0HadronDecision TOS (Trigger On Signal) and the L0Physics TIS (Trigger Independent of Sig-

nal) lines, where L0Physics is the OR between L0Electron, L0Photon, L0Muon, L0DiMuon and L0Hadron lines, is chosen. Events that passed the L0 lines are also requested to pass the logical OR between the B_Hlt1TrackMVADecision TOS line and B_Hlt1TwoTrackMVADecision TOS. At the HLT2 trigger level the events are required to satisfy at least one of the topological B decay trigger lines.

A dedicated stripping line has been developed in order to study $B_{(s)}^0 \rightarrow p\bar{p}h_1^+h_2^-$ decays, where $h_{1,2} = K, \pi$. The B2pphh_KpiLine whose requirements are listed in Tab. 4.3 is used. There are common cuts on the momentum p and the transverse momentum p_T of the children. The track quality is ensured by cutting on the track χ^2/ndf and on the ghost probability associated to the track. In Run 2, the ghost probability classifier became faster and it was then implemented directly in the reconstruction. A combined cut on the track χ^2/ndf and on the ghost probability at the reconstruction level has been implemented, improving the tracking performance and the background rejection. Loose PID requirements are applied to each final state particle. In order to exclude particles originating from the PV there are requirements on the IP χ^2 for the bachelors with respect to the PV. Cuts are also applied to the $p\bar{p}$ pair and to the $p\bar{p}K^+$ combination. The particles in the combinations are required to come from the same vertex through a requirement on the χ^2 distance of closest approach (DOCA). There are requirements on the invariant-masses of the combinations. Specific kinematic cuts are applied to the $p\bar{p}$ pair. A further requirement is applied to the product of the PID quantities of the proton and antiproton candidates. The reconstructed B^0 mesons are selected with topological requirements as the maximum DOCA of the bachelors, the cosine of the angle (DIRA) between the momentum of the B^0 and the direction of flight from the PV to the secondary vertex (SV), and the minimum IP to the PV. A good fit quality of the vertex is required too. There are also kinematic requirements on the reconstructed B^0 and a global cut on the maximum number of long tracks reconstructed in the event.

Variable	Run 1	Run 2
Kaons		
p	> 1500 MeV	
p_T	> 300 MeV	
Track χ^2 / ndof	< 3.0	$< 4.0^{(*)}$
Track Ghost Prob	< 0.35	$< 0.4^{(*)}$
ProbNNk	> 0.05	
IP χ^2	> 4.0	> 5.0
Pions		
p	> 1500 MeV	
p_T	> 300 MeV	
Track χ^2 / ndof	< 3.0	$< 4.0^{(*)}$
Track Ghost Prob	< 0.35	$< 0.4^{(*)}$
ProbNN π	> 0.05	> 0.1
IP χ^2	> 6.0	> 8.0
Protons		
p	> 1500 MeV	
p_T	> 300 MeV	
Track χ^2 / ndof	< 3.0	$< 4.0^{(*)}$
Track Ghost Prob	< 0.35	$< 0.4^{(*)}$
ProbNNp	> 0.05	
IP χ^2	> 2.0	> 3.0
$p\bar{p}$ combination		
$M_{p\bar{p}}$	< 5000 MeV	
Σp_T	> 750 MeV	
Σp	> 7000 MeV	
Minimum p_T	> 400 MeV	
Minimum p	> 4000 MeV	
ProbNNp $\cdot$ ProbNN $\bar{p}$	> 0.05	> 0.01
χ^2 DOCA	< 20	
$p\bar{p}K^+$ combination		
$M_{p\bar{p}K^+}$	< 5600 MeV	
χ^2 DOCA	< 20	
B^0		
Max DOCA	< 0.3 mm	< 0.25 mm
χ^2 DOCA	< 20	
DIRA	> 0.9999	
Vertex χ^2	< 30	< 25
Min IP	> 0.2 mm	
p_T	> 1000 MeV	
Σp_T of the bachelors	> 3000 MeV	
Mass	$[5.05, 5.55]$ GeV	
Number of long tracks < 200		

Table 4.3: Stripping line cuts. The Track Ghost Probability and Track χ^2 /ndf in Run 2 (marked with $(*)$) are set a priori of the stripping line at the reconstruction level.

4.4 Offline selection

In order to obtain the optimal signal to background ratio an offline selection is applied. Preliminary cuts are applied to the particle momenta and on the pseudorapidity as described in Sec. 4.4.1. Multivariate analysis (MVA) based selection has been applied on Run 1 and Run 2 separately as described in Sec. 4.4.2. Due to the limited size of the available sample, PID resampling cannot be applied on Run 1 PID variables: a dedicated optimisation is described in Sec. 4.4.3. The optimisation of the selection for Run 2 is described in Sec. 4.4.4. Other particles that can be identified as signal have been vetoed as described in Sec. 4.4.5. In Sec. 4.4.6 the elimination of the multiple candidates and of the clones is described.

4.4.1 Preselection

The preselection applied on the final state particles used in this analysis is described in the following. A fiducial cut is applied on the pseudorapidity requiring for the final state particles to lie in the RICH acceptance with $2.0 < \eta < 4.9$. Cuts on the particle momenta have also been applied to take into account the radiator momentum thresholds and to avoid a suboptimal discrimination from the RICH system at high momenta, requiring therefore $10 \text{ GeV} < p_{p/\bar{p}} < 150 \text{ GeV}$ and $3 \text{ GeV} < p_{K^+/\pi^-} < 150 \text{ GeV}$.

4.4.2 MVA selection

A multivariate based selection to optimise the purity of the sample using the TMVA package [287] is used. MC samples of $B^0 \rightarrow p\bar{p}K^+\pi^-$ events are utilised, generated according to the phase space of the decay, as signal training and test samples. The region $[5450, 5550] \text{ MeV}$ of the invariant-mass $m_{p\bar{p}K^+\pi^-}$ is used as background training and test sample, to avoid regions containing partially reconstructed or B_s^0 events. Preselection, trigger and stripping requirements have been applied on the events used for the training and testing. Several distributions have been studied in order to find the set of variables that better discriminate between signal and background. The variables used are:

- minimum IP χ^2 of the B^0 to the PV;
- maximum DOCA of the tracks to the B^0 vertex;
- the χ^2 distance of the B^0 vertex from the PV;
- pointing variable, defined as:

$$\frac{|\vec{p}_B| \sin \theta_{DIRA}}{|\vec{p}_B| \sin \theta_{DIRA} + \sum_i |\vec{p}_i| \sin \theta_i} \quad (4.3)$$

where $\vec{p}_B$ is the momentum of the B^0 candidate, θ_{DIRA} is the angle between the direction of $\vec{p}_B$ and the displacement vector from PV to SV, $\vec{p}_i$ is the momentum of particle i , θ_i is the angle between the direction $\vec{p}_i$ and the displacement vector from PV to SV and the sum runs over the candidate B^0 children;

- χ^2 of the B^0 vertex;
- the minimum p_T between the two protons;
- the maximum p_T between the two protons;
- the minimum p between the two protons;
- the maximum p between the two protons;

Only for Run 2:

- ProbNNp variable for the proton candidate;
- ProbNNp variable for the antiproton candidate;
- ProbNNk variable for the kaon candidate;
- ProbNNpi variable for the pion candidate;

Since there are discrepancies between data and MC on the PID response, data driven tools are available at LHCb within the PIDCalib package [288] in order to obtain variables distributed as in data. For this analysis the PID resampling is used, where the PID response on the MC is completely replaced by one randomly generated from calibration probability density functions (PDF). The PID resampling is applied on the Run 2 data sample but cannot be applied on the Run 1 data sample because the sample available for the generation of the PDFs of the proton and antiproton does not cover the full kinematic region needed (very low statistics for $p_T > 4$ GeV), as shown in Figure. 4.5. The usage of the ProbNN to train the MVA is not possible for the Run 1 sample due to the fact that the resampling technique cannot be applied in this sample. The calibration sample used are:

- $D^0 \rightarrow K^- \pi^+$ from $D^{*+} \rightarrow D^0 \pi^+$ decays for kaons and pions ProbNN.
- $\Lambda \rightarrow p \pi^-$ decays for protons and antiprotons ProbNN.

show no significant differences between 2011 and 2012 and between 2015-2016-2017-2018. The subsamples have been merged in two samples corresponding to Run 1 and Run 2, training a different MVA for each sample to take into account

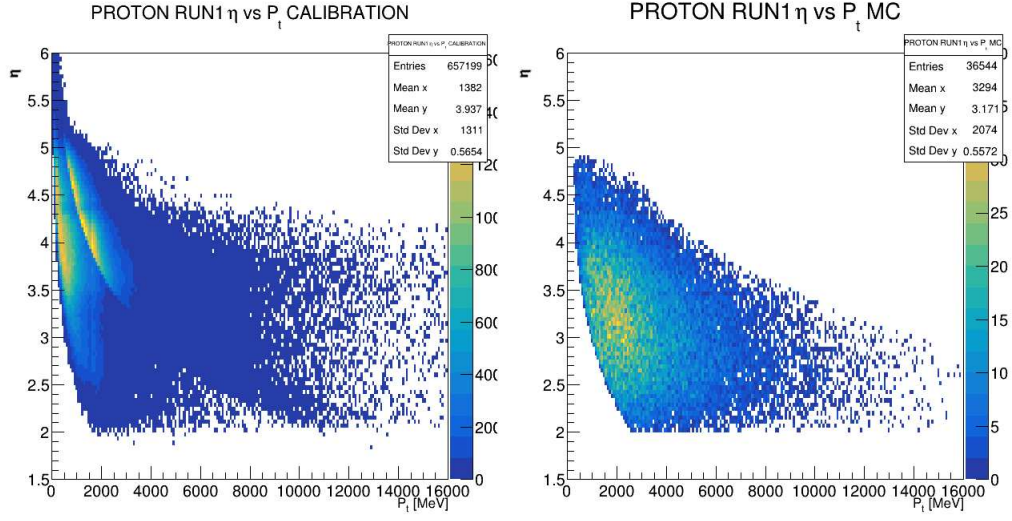


Figure 4.5: The calibration sample (left) and MC (right) distribution of η VS p_T

differences in the kinematics due to the different centre of mass energy and substantial changes at the reconstruction level between Run 1 and 2. The comparison are shown in Figure 4.6.

The comparison between signal and background discriminating variables for Run 1 and Run 2 is shown in Figure 4.7, 4.8 respectively. The correlation matrices for signal and background variables for Run 1 and Run 2 are shown in Figure 4.9, 4.10 respectively.

BDT and MLP classifier have been tested. The corresponding Receiver Operating Characteristic (ROC) curves are shown in Figure 4.11 for Run 1 and in Figure 4.12 for Run 2. The result of the Kolmogorov-Smirnov test are reported in Figure 4.13 for Run 1 and in Figure 4.14 for Run 2. There is no evidence of over-training.

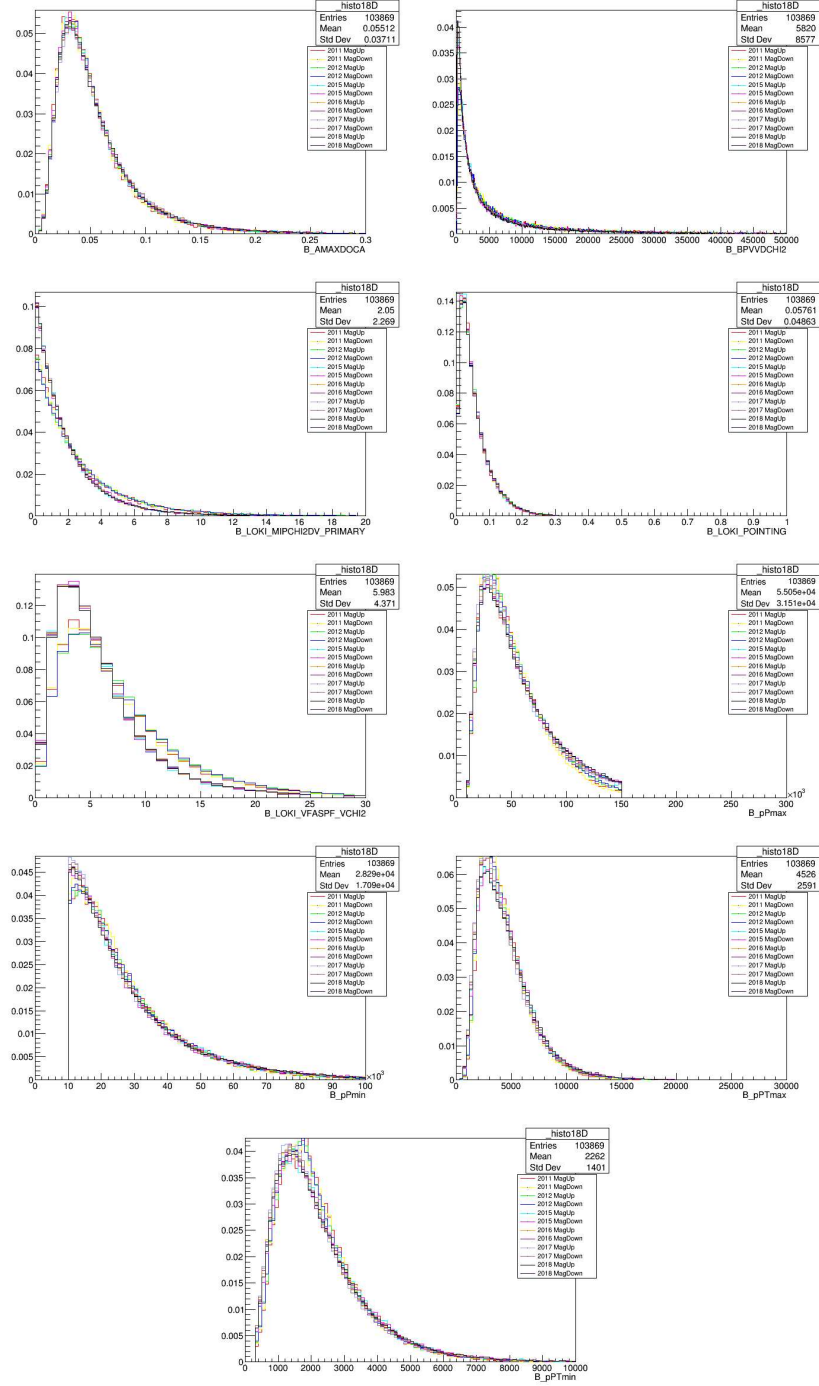


Figure 4.6: Comparison of MVA variables between 2011, 2012, 2015, 2016, 2017, 2018 MC subsamples.

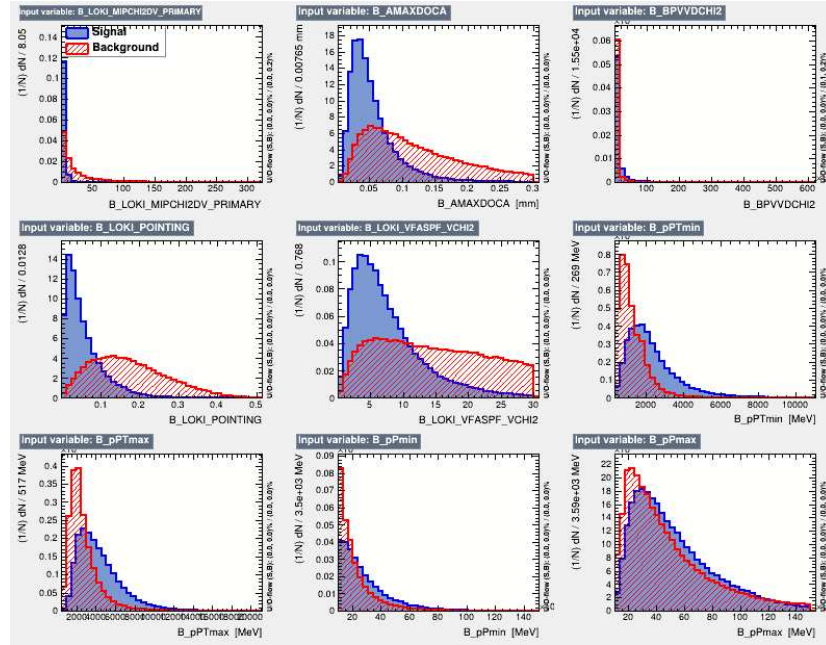


Figure 4.7: Comparison of Run 1 signal (blue filled) and background (red dashed) distributions used as MVA discriminating variables.

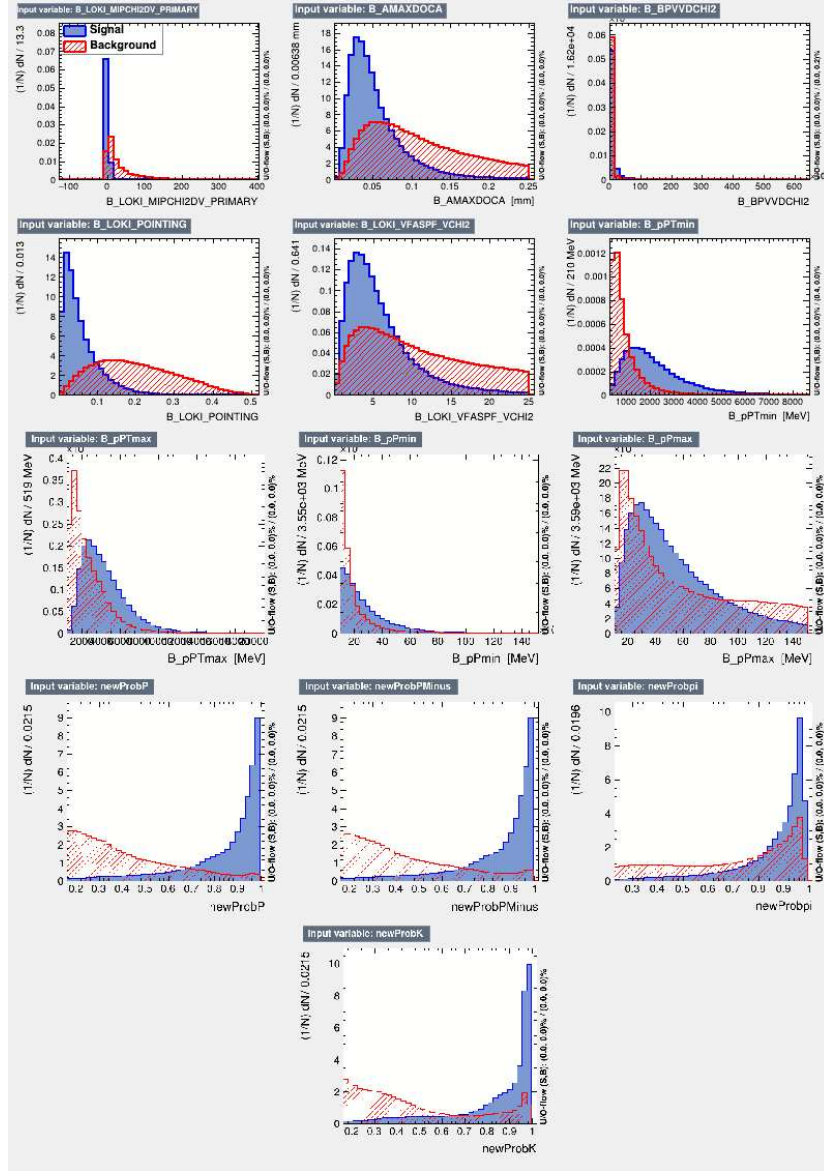


Figure 4.8: Comparison of Run 2 signal (blue filled) and background (red dashed) distributions used as MVA discriminating variables.

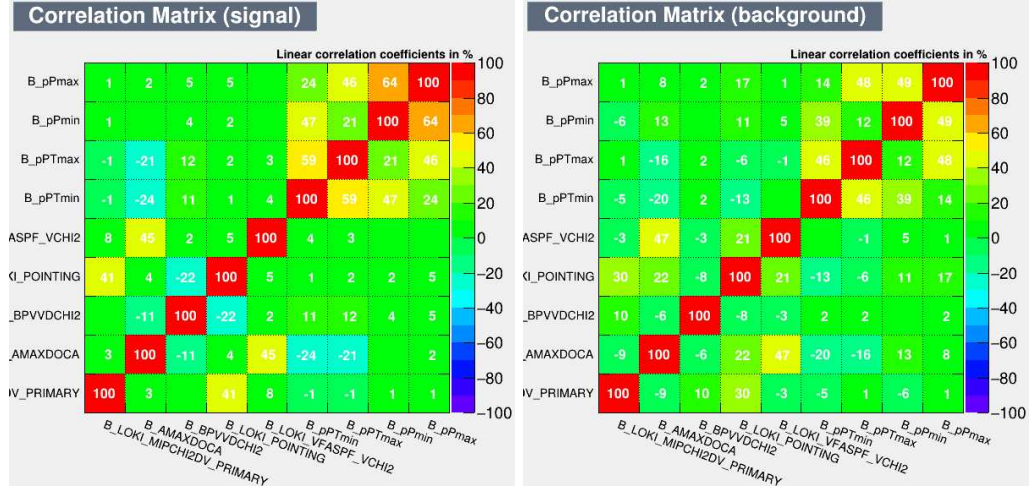


Figure 4.9: Correlation matrices for Run 1 signal (left) and background (right) MVA input variables.

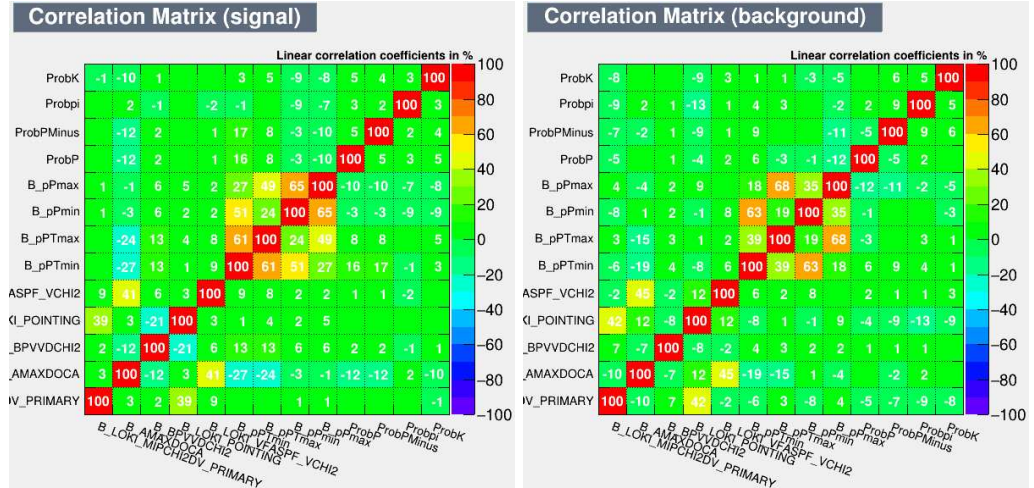


Figure 4.10: Correlation matrices for Run 2 signal (left) and background (right) MVA input variables.

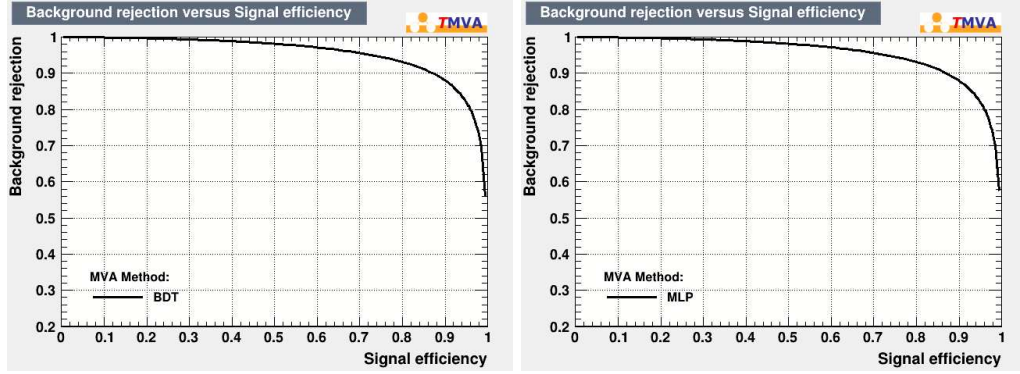


Figure 4.11: ROC curves for the algorithms used in the Run 1 MVA analysis, BDT (left) and MLP (right)

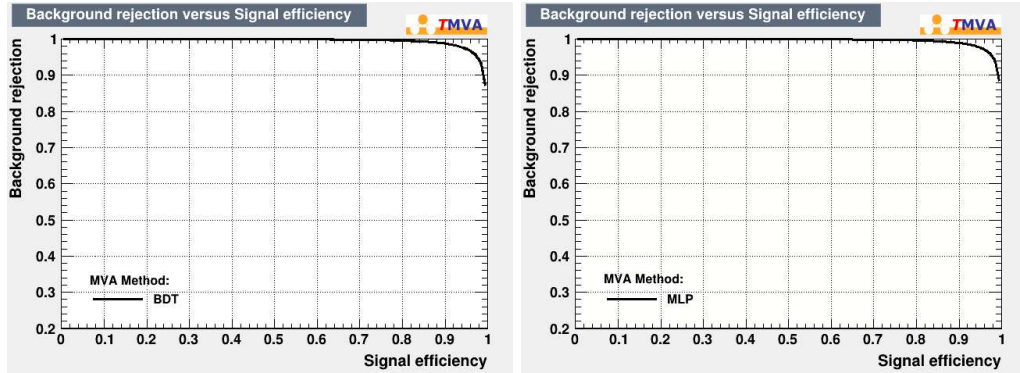


Figure 4.12: ROC curves for the algorithms used in the Run 2 MVA analysis, BDT (left) and MLP (right)

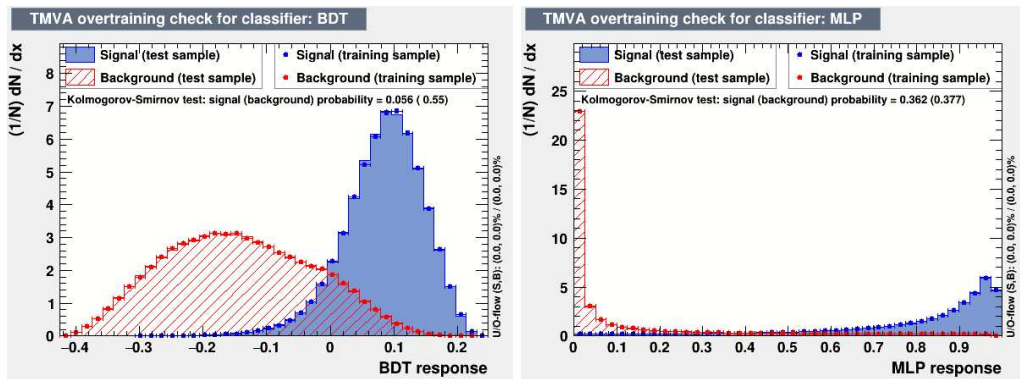


Figure 4.13: Results of the Kolmogorov-Smirnov test for the BDT to check for possible over-training in Run 1 MVA analysis, BDT (left) and MLP (right).

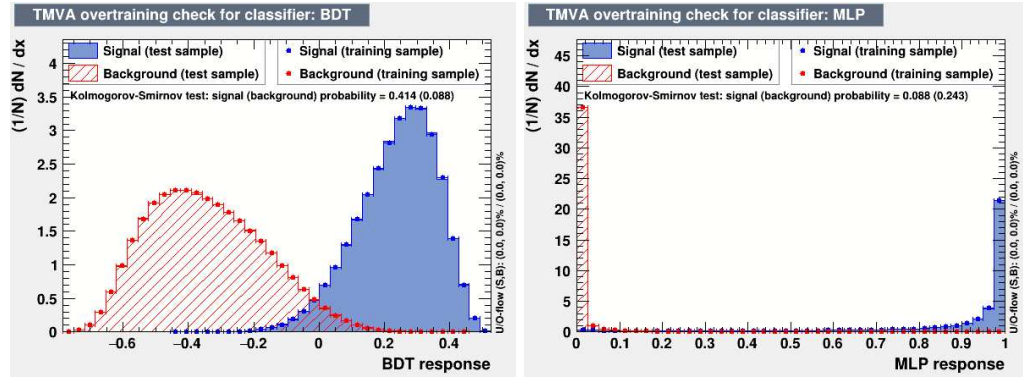


Figure 4.14: Results of the Kolmogorov-Smirnov test for the BDT to check for possible over-training in Run 2 MVA analysis, BDT (left) and MLP (right).

4.4.3 PID and MVA optimisation for Run 1

The background sample has ~ 300000 events, while the signal sample has ~ 200000 entries. Following the usual procedure, the samples have been randomly split in two equal parts as training and testing samples. The training sample comprising both the signal and background events contains approximately 250000 events. PID cuts are applied alongside the MVA selection to improve the rejection of combinatorial background. These PID cuts are kept loose to minimise potential biases arising from discrepancies between the MC simulation and data, as described before. The MVA and PID cuts combination providing the best performance in terms of a figure of merit (FOM) is chosen. The FOM is defined as:

$$f = \frac{N_{sig} \cdot \epsilon_{MC}}{\sqrt{N_{sig} \cdot \epsilon_{MC} + B}} \quad (4.4)$$

where N_{sig} and B are respectively the number of signal before the MVA cut and background events from the data and ϵ_{MC} is the efficiency of the cut obtained from the MC of $B^0 \rightarrow p\bar{p}K^+\pi^-$ defined as:

$$\epsilon_{MC} = \frac{N_{MC}^{cut}}{N_{MC}^{tot}} \quad (4.5)$$

where N_{MC}^{cut} is the number of MC events after the cut and N_{MC}^{tot} is the number of MC events before the cut.

Different cuts in the p , K and π ProbNN variables have been tested. The proton and antiproton cuts are the same. After selecting the PID cut, a scan on the MVA output between 0 and 0.18 for the BDT and 0.6 and 0.98 for the MLP has been done by fitting the $m_{p\bar{p}K\pi}$ distribution from the data in the range [5.2, 5.36] GeV. The B^0 signal has been parameterised with a gaussian for the signal and an exponential for the background. The signal yield N_{sig} and background yields were extracted within the $\pm 3\sigma$ region around the mean of the Gaussian. The procedure has been repeated for every combination of the ProbNN cuts shown in Tab. 4.4. The combination of MVA and PID cuts that provided the best performance is:

$$\text{BDT} > 0.04 \ \& \ \text{pProbNNp} > 0.225 \ \& \ \text{kProbNNk} > 0.225 \ \& \\ \text{piProbNNpi} > 0.125$$

The BDT is chosen for consistency with what used in the Run 2 sample, as reported in Sec. 4.4.4, also considering that the difference with respect to the optimal point when using the MLP algorithm is negligible, as shown in Figure 4.15.

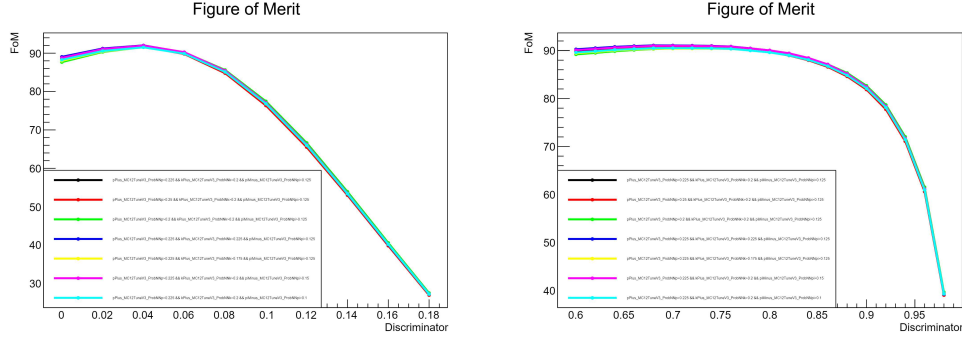


Figure 4.15: Run 1 FOM for BDT (left) and MLP (right) obtained for different PID cuts shown in the legend. Only the combinations of PID cuts with best performance are shown.

Variable	Cut
pProbNNp	$> 0.225, > 0.25, > 0.2$
kProbNNk	$> 0.225, > 0.2, > 0.175$
piProbNNpi	$> 0.15, > 0.125, > 0.1$

Table 4.4: Cuts studied for the ProbNN variables during PID selection.

4.4.4 Optimisation of the selection for Run 2

The background sample has ~ 6000000 events, while the signal sample has ~ 750000 entries. The sample are randomly split in two equal parts as training and testing samples. The training sample comprising both the signal and background events contains ~ 3300000 events. The strategy is similar to the one used for Run 1. The ProbNN variable are also used to train the MVA. The FOM is the same used for Run 1, described in Eq. 4.4. The B^0 signal is parameterised with a gaussian for the signal and an exponential for the background. The background and signal yields are extracted in the $\pm 3\sigma$ region around the mean of the gaussian. The MVA choosen is the BDT and the cut that provides the best performance is:

$$\text{BDT} > 0.19$$

The BDT is choosen over the MLP because, as can be seen in Figure 4.16, the FOM have similar values but the MLP maximum is near an abrupt change of the values (above 0.95).

4.4.5 Other background vetoes

After the application of the MVA cut, the invariant-mass distribution of the combination of the child particles of the B^0 have been studied. $\bar{D}^0 \rightarrow K^+\pi^-$,

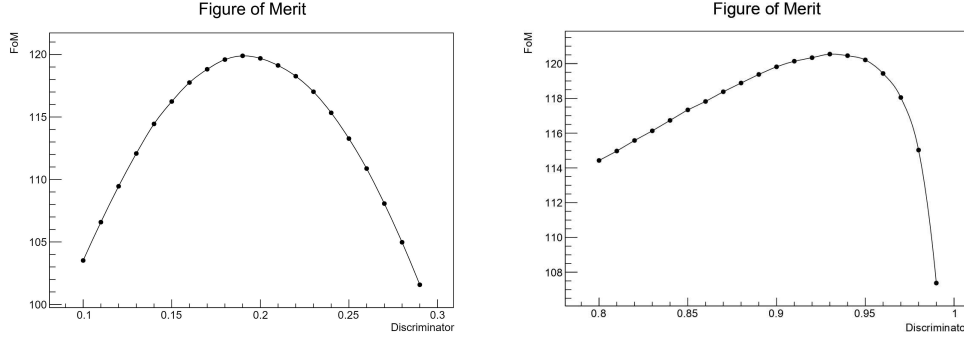


Figure 4.16: Run 2 FOM of the BDT (left) and MLP (right).

$\Lambda_c^+ \rightarrow pK^-\pi^+$ and $\phi \rightarrow K^+K^-$ candidates are still present in the sample, as can be seen in Figure 4.17.

The cuts applied in order to veto these particles are the following:

$$\begin{aligned}
 m_{K^+\pi^-} < 1835 \text{ MeV} & \quad || \quad m_{K^+\pi^-} > 1880 \text{ MeV} \\
 m_{\bar{p}K^+\pi^-} < 2250 \text{ MeV} & \quad || \quad m_{\bar{p}K^+\pi^-} > 2305 \text{ MeV} \\
 m_{K\pi} \text{ }_{K\text{mass hypothesis}} < 1010 \text{ MeV} & \quad || \quad m_{K\pi} \text{ }_{K\text{mass hypothesis}} > 1028 \text{ MeV}
 \end{aligned}$$

The range of the cuts are chosen by applying fits on the distribution shown in Figure 4.17 and selecting the interval $(\mu - 3\sigma, \mu + 3\sigma)$, the distribution are parameterised with gaussians.

4.4.6 Multiple candidates and clones

Multiple candidates, sequential events in the ntuples having the same values for the topological variables used in the MVA selection, have been found in the samples. Multiple candidates differ for the swapping in the assignment of momenta between tracks. Multiple candidates are mainly due to an ambiguous K/π or K/p mass hypothesis assigned to a track. In case of multiple candidates for the same event, randomly, one event is retained. For Run 2 the number of multiple candidates vetoed is 5366, which corresponds to 2.6% of the events up to that point of the selection. For Run 1 the vetoed events are 1731, which correspond to 3.9% of the events up to that point of the selection.

In LHCb the tracking pattern recognition sequence comprises a series of algorithms. Some of these pattern recognition algorithms use information output by others in order to "build" more complete tracks. As a result, it may happen that

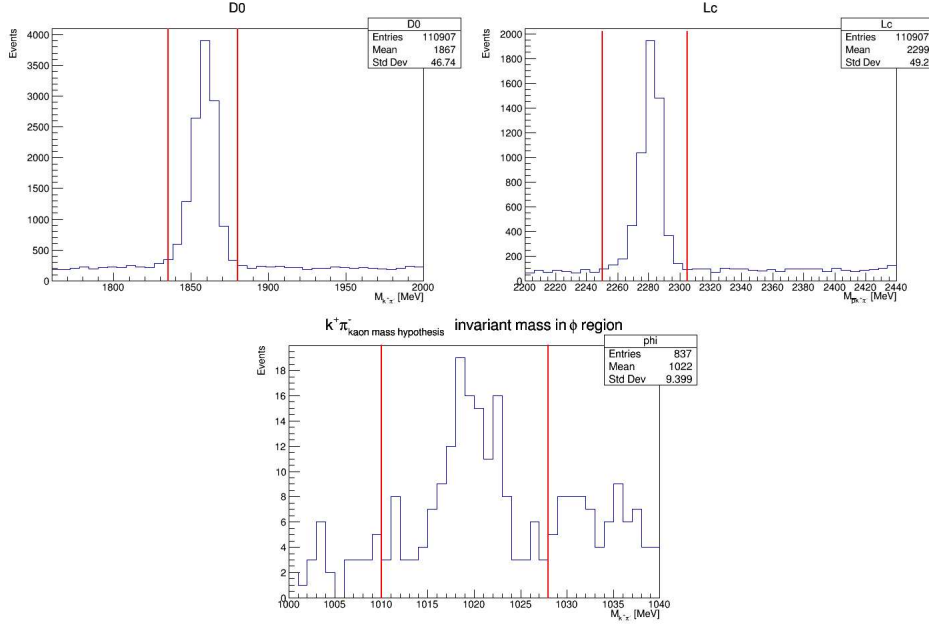


Figure 4.17: Distribution of the background particles, $D^0 \rightarrow K^-\pi^+$ (top left), $\Lambda_c^+ \rightarrow pK^-\pi^+$ (top right) and $\phi \rightarrow K^+K^-$ (bottom). The red lines represents the veto borders.

the set of tracks made at the end of the pattern recognition sequence contain in fact clone tracks. Two tracks are considered clones of each other if they share at least 70% of the hits in the VELO and at least 70% of the hits in the T-stations seeding region. These events can be eliminated by reconstructing the angles between tracks and by applying cuts on them. The angles between tracks have been reconstructed as the angle between the 3-momentum of the two particles. In Figure 4.18 and 4.19 are shown the distribution of the angles between the tracks for Run 1 and Run 2 respectively. The peak at 0rad in each distribution is due to the presence of the clones in the sample. A cut is applied at 0.5 mrad to eliminate the clones.

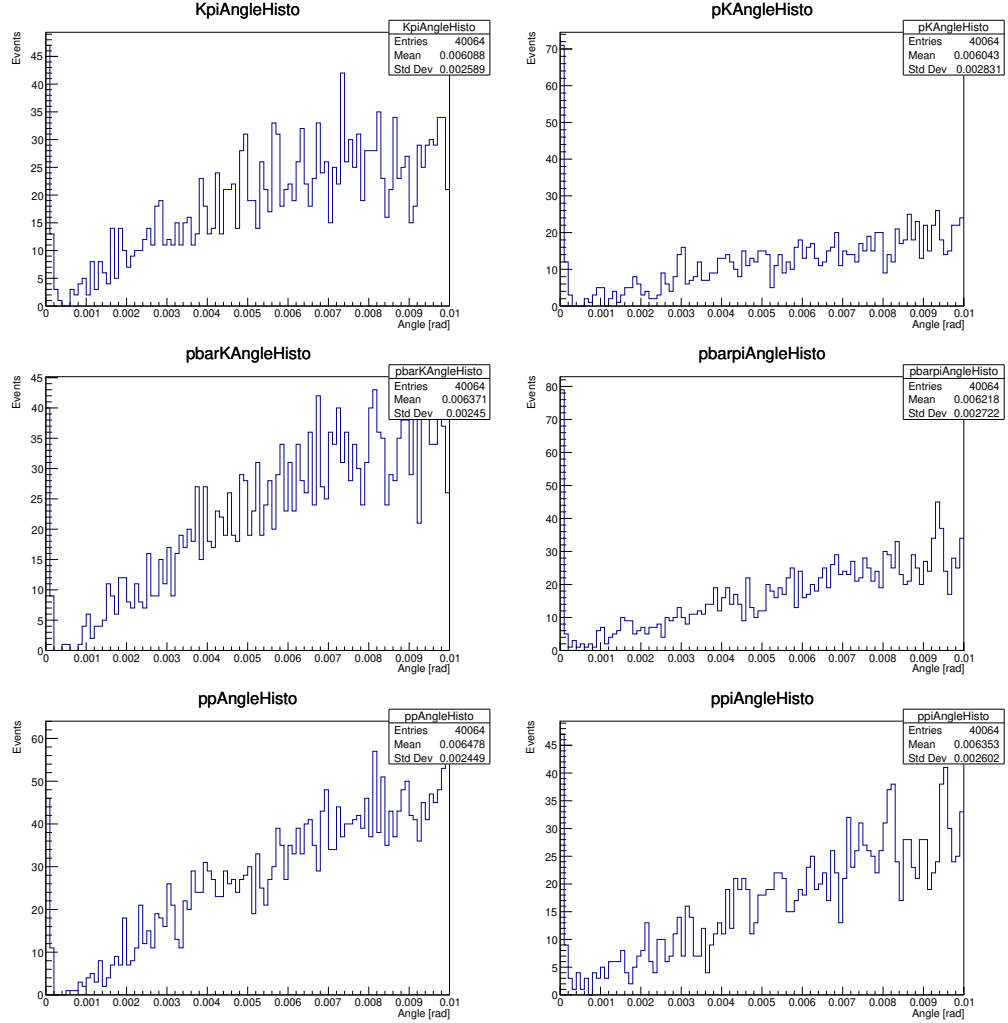


Figure 4.18: Distribution of the angles between tracks for Run 1. $K^+\pi^-$ (top left), pK^+ (top right), $\bar{p}K^+$ (mid left), $\bar{p}\pi^-$ (mid right), $p\bar{p}$ (bottom left) $p\pi^-$ (bottom right)

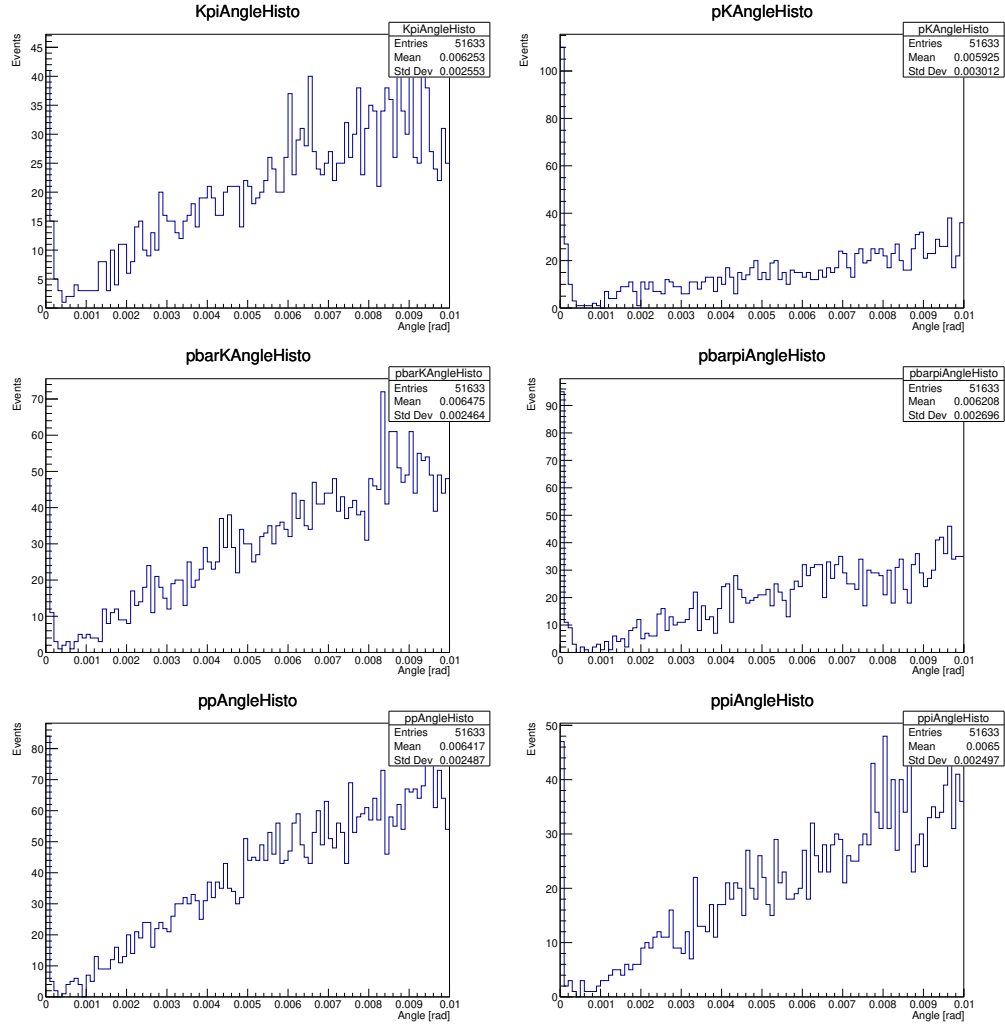


Figure 4.19: Distribution of the angles between tracks for Run 2. $K^+\pi^-$ (top left), pK^+ (top right), $\bar{p}K^+$ (mid left), $\bar{p}\pi^-$ (mid right), $p\bar{p}$ (bottom left) $p\pi^-$ (bottom right).

4.5 Mass fit models and extraction of the $B^0 \rightarrow \eta_c K^+ \pi^-$ candidates

The $m_{p\bar{p}K^+\pi^-}$ spectra for the Run 1 and Run 2 data samples after the selection procedure previously described are shown in Figure 4.20. The B^0 peak is clearly visible. The events on the right sideband are due to the $B_s^0 \rightarrow p\bar{p}K^+\pi^-$ [289], $B^0 \rightarrow p\bar{p}\pi^+\pi^-$ and $B_s^0 \rightarrow p\bar{p}K^+K^-$ events, where the positive pion is misidentified (MisID) with a kaon and the negative kaon is misreconstructed as a pion, respectively.

The PDFs used to build the fit model have been validated on the corresponding MC sample, as described in Sec. 4.5.1. Run 1 and Run 2 are fitted separately with the extended unbinned maximum likelihood formalism provided by the RooFit package [290]. First the B^0 distribution is fitted, as described in section Sec. 4.5.2 to extract the B^0 yield and to validate the model. The nonresonant $p\bar{p}$ contribution under the η_c region is not negligible. For this reason a 2D fit is performed to the $m_{p\bar{p}K^+\pi^-}$ and $m_{p\bar{p}}$ invariant-mass distributions and to use the corresponding joint PDF as discriminant variable to compute sWeights, through the sPlot technique [291], to unfold the $B^0 \rightarrow \eta_c K^+ \pi^-$ events, as described in Sec. 4.5.3. The extracted signal and background yields are used in the fit to the Dalitz plot variables.

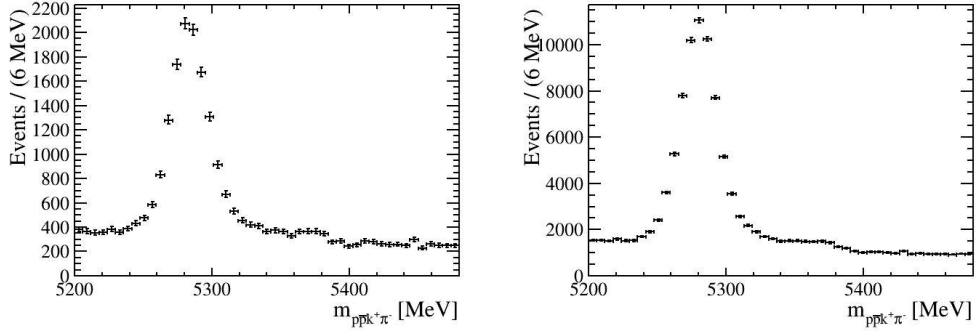


Figure 4.20: The $m_{p\bar{p}K^+\pi^-}$ Run 1 (left) and Run 2 (right) spectra after selection requirements.

4.5.1 Validation of the model components with MC sample

The PDFs used to build the fit model rely on the corresponding simulated sample. All the samples are reconstructed as $B^0 \rightarrow p\bar{p}K^+\pi^-$ and the selection is applied to each MC sample. The fit on the MC sample are used not only to validate the PDF for the single component but also to extract the shape parameters that will

be fixed in the fit on the data sample. The $m_{p\bar{p}}$ distribution has also been studied in order to validate the model used to fit the components corresponding to the η_c and J/ψ mesons.

For all the samples a truth-matching check have been applied requiring that:

- All the bachelor particle are the product of the same particle. If there is a intermediate resonance (η_c and J/ψ) the p and $\bar{p}$ are required to be the product of that resonance and their grandparent has to be the parent of the K^+ and π^- .
- All the particle true ID are required to be equal to their PDG code value.

The functions used to describe the components, together with their fit results are presented in Appendix D.

4.5.2 Fit to the $p\bar{p}K^+\pi^-$ invariant-mass data distribution

The fit model consists of:

- $B^0 \rightarrow p\bar{p}K^+\pi^-$
- $B^0 \rightarrow p\bar{p}\pi^+\pi^-$
- $B_s^0 \rightarrow p\bar{p}K^+K^-$
- $B_s^0 \rightarrow p\bar{p}K^+\pi^-$
- combinatorial background

The $B^0 \rightarrow p\bar{p}K^+\pi^-$ and $B_s^0 \rightarrow p\bar{p}K^+\pi^-$ components are parameterised with a Hypatia [292] PDF while the $B^0 \rightarrow p\bar{p}\pi^+\pi^-$ and $B_s^0 \rightarrow p\bar{p}K^+\pi^-$ are parameterised with a Crystal Ball function [293]. The combinatorial background is parameterised with an exponential function. Firstly, the fit is applied in the range $5200 < m_{p\bar{p}K^+\pi^-} < 5360$ MeV, as shown in Figure 4.21. In order to get the full $B_s^0 \rightarrow p\bar{p}K^+\pi^-$ distribution to improve the fit performance the range has been extended to $5200 < m_{p\bar{p}K^+\pi^-} < 5480$ MeV. The fit with the extended range are shown in Figure 4.22 In this fit the yields of the misID components are constrained to be

$$N_{misID} = N_{B^0 \rightarrow p\bar{p}K^+\pi^-} \cdot eff_{misID}, \quad (4.6)$$

where $N_{B^0 \rightarrow p\bar{p}K^+\pi^-}$ is the yield of $B^0 \rightarrow p\bar{p}K^+\pi^-$ and eff_{misID} is defined as

$$eff_{misID} = \frac{f \cdot \epsilon_{misID} \cdot \mathcal{B}_{misID}}{\epsilon_{B^0 \rightarrow p\bar{p}K^+\pi^-} \cdot \mathcal{B}_{B^0 \rightarrow p\bar{p}K^+\pi^-}}, \quad (4.7)$$

where f takes into account the different fragmentation factor between B^0 and B_s^0 [294], ϵ_i is the acceptance of the selection calculated from the MC and $\mathcal{B}_i$ is the branching ratio of the decay channel. In Table 4.5 are listed the values used to calculate ϵ_{misID} for all the backgrounds. In Table 4.6 are listed the extracted parameters of the fit on the B^0 spectrum.

To suppress the backgrounds, the fit is applied only on the events that satisfy the condition $2700 < m_{pp} < 3300$ MeV. This cut is also useful, as a fit will be applied to the m_{pp} spectrum in the range of η_c and J/ψ , in order to select the events coming from $\eta_c \rightarrow p\bar{p}$. With this new configuration the acceptances of the background are very small so the yields are left floating. The fits obtained with this last configuration are shown in Figure 4.23, the parameters extracted are listed in Table 4.7. The $m_{p\bar{p}K^+\pi^-}$ distribution is used as discriminant variable to compute the sWeights of the $B^0 \rightarrow p\bar{p}K^+\pi^-$ signal component. These sWeights are then used to fit the m_{pp} sPlot.

4.5. Mass fit models and extraction of the $B^0 \rightarrow \eta_c K^+ \pi^-$ candidates

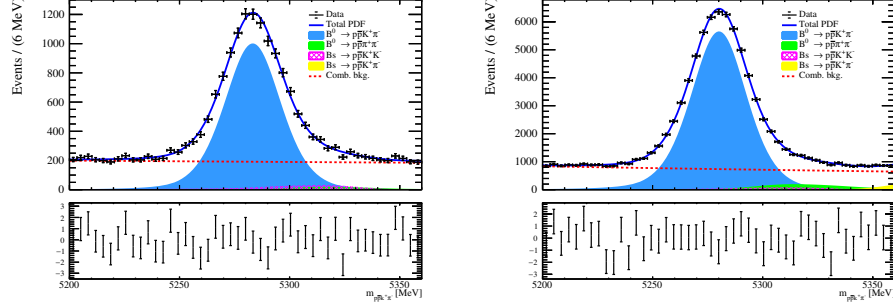


Figure 4.21: Fit to the Run 1 (left) and Run 2 (right) $B^0 \rightarrow p\bar{p}K^+\pi^-$ invariant-mass distribution in the range (5200,5360)MeV. The pull distribution are also shown.

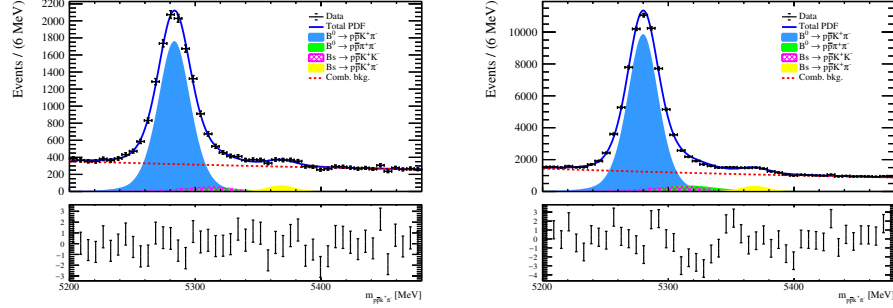


Figure 4.22: Fit to the Run 1 (left) and Run 2 (right) $B^0 \rightarrow p\bar{p}K^+\pi^-$ invariant-mass distribution in the range (5200,5480)MeV. The pull distribution are also shown.

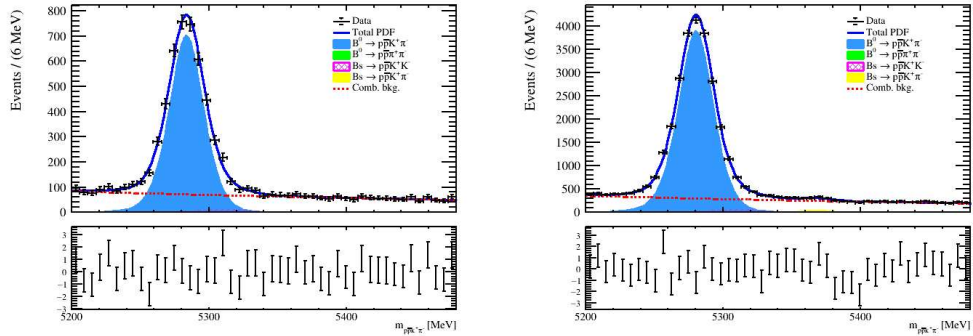


Figure 4.23: Fit to the Run 1 (left) and Run 2 (right) $B^0 \rightarrow p\bar{p}K^+\pi^-$ invariant-mass distribution in the range (5200,5480)MeV and with $2700 < m_{pp} < 3300$ MeV. The pull distribution are also shown.

4.5. Mass fit models and extraction of the $B^0 \rightarrow \eta_c K^+ \pi^-$ candidates

Parameter	Value [MeV]
$\mathcal{B}_{B^0 \rightarrow p\bar{p}K^+\pi^-}$	$(6.3 \pm 0.5) \cdot 10^{-6}$
$\mathcal{B}_{B^0 \rightarrow p\bar{p}\pi^+\pi^-}$	$(2.9 \pm 0.2) \cdot 10^{-6}$
$\mathcal{B}_{B_s^0 \rightarrow p\bar{p}K^+\pi^-}$	$(1.4 \pm 0.3) \cdot 10^{-6}$
$\mathcal{B}_{B_s^0 \rightarrow p\bar{p}K^+K^-}$	$(4.5 \pm 0.5) \cdot 10^{-6}$
Run 1 in range (5200,5360)MeV	
$\epsilon_{B^0 \rightarrow p\bar{p}K^+\pi^-}$	$(1.18 \pm 0.01) \cdot 10^{-2}$
$\epsilon_{B^0 \rightarrow p\bar{p}\pi^+\pi^-}$	$(7.53 \pm 0.08) \cdot 10^{-4}$
$\epsilon_{B_s^0 \rightarrow p\bar{p}K^+\pi^-}$	$(3.94 \pm 0.02) \cdot 10^{-3}$
$\epsilon_{B_s^0 \rightarrow p\bar{p}K^+K^-}$	$(4.23 \pm 0.02) \cdot 10^{-3}$
Run 2 in range (5200,5360)MeV	
$\epsilon_{B^0 \rightarrow p\bar{p}K^+\pi^-}$	$(2.00 \pm 0.01) \cdot 10^{-2}$
$\epsilon_{B^0 \rightarrow p\bar{p}\pi^+\pi^-}$	$(2.35 \pm 0.01) \cdot 10^{-3}$
$\epsilon_{B_s^0 \rightarrow p\bar{p}K^+\pi^-}$	$(9.73 \pm 0.02) \cdot 10^{-3}$
$\epsilon_{B_s^0 \rightarrow p\bar{p}K^+K^-}$	$(7.56 \pm 0.01) \cdot 10^{-3}$
Run 1 in range (5200,5480)MeV	
$\epsilon_{B^0 \rightarrow p\bar{p}K^+\pi^-}$	$(1.18 \pm 0.01) \cdot 10^{-2}$
$\epsilon_{B^0 \rightarrow p\bar{p}\pi^+\pi^-}$	$(1.01 \pm 0.01) \cdot 10^{-3}$
$\epsilon_{B_s^0 \rightarrow p\bar{p}K^+\pi^-}$	$(1.44 \pm 0.01) \cdot 10^{-1}$
$\epsilon_{B_s^0 \rightarrow p\bar{p}K^+K^-}$	$(4.32 \pm 0.02) \cdot 10^{-3}$
Run 2 in range (5200,5480)MeV	
$\epsilon_{B^0 \rightarrow p\bar{p}K^+\pi^-}$	$(2.00 \pm 0.01) \cdot 10^{-2}$
$\epsilon_{B^0 \rightarrow p\bar{p}\pi^+\pi^-}$	$(3.22 \pm 0.01) \cdot 10^{-3}$
$\epsilon_{B_s^0 \rightarrow p\bar{p}K^+\pi^-}$	$(3.68 \pm 0.01) \cdot 10^{-1}$
$\epsilon_{B_s^0 \rightarrow p\bar{p}K^+K^-}$	$(7.71 \pm 0.01) \cdot 10^{-3}$

Table 4.5: Values used to constrain the backgrounds yields. The branching ratios are the ones reported by the PDG[66].

4.5. Mass fit models and extraction of the $B^0 \rightarrow \eta_c K^+ \pi^-$ candidates

Parameter	Value
Run 1 in range (5200,5360)MeV	
μ	5283.60 ± 0.09
σ	14.73 ± 0.09
$N_{B2ppkpi}$	9989 ± 136
N_{B2pppi}	429 ± 8
$N_{Bs2ppkk}$	351 ± 6
$N_{Bs2ppkpi}$	15 ± 47
N_{bkg}	9014 ± 186
τ_{bkg}	$(-4.73 \pm 3.72) \cdot 10^{-4}$
Run 2 in range (5200,5360)MeV	
μ	5283.60 ± 0.09
σ	14.73 ± 0.09
$N_{B2ppkpi}$	55147 ± 349
N_{B2pppi}	2951 ± 12
$N_{Bs2ppkk}$	3789 ± 32
$N_{Bs2ppkpi}$	1020 ± 119
N_{bkg}	32685 ± 413
τ_{bkg}	$(-2.37 \pm 0.20) \cdot 10^{-3}$
Run 1 in range (5200,5480)MeV	
μ	5283.60 ± 0.19
σ	14.97 ± 0.23
$N_{B2ppkpi}$	10121 ± 137
N_{B2pppi}	394 ± 41
$N_{Bs2ppkk}$	627 ± 86
$N_{Bs2ppkpi}$	413 ± 61
N_{bkg}	13920 ± 177
τ_{bkg}	$(-1.36 \pm 0.12) \cdot 10^{-3}$
Run 2 in range (5200,5480)MeV	
μ	5280.50 ± 0.08
σ	14.50 ± 0.09
$N_{B2ppkpi}$	54524 ± 295
N_{B2pppi}	3993 ± 413
$N_{Bs2ppkk}$	3804 ± 520
$N_{Bs2ppkpi}$	1946 ± 126
N_{bkg}	53197 ± 358
τ_{bkg}	$(-1.76 \pm 0.06) \cdot 10^{-3}$

Table 4.6: Parameters extracted from the fit on the B^0 spectrum

Parameter	Value [MeV]
Run 1	
μ	5283.90 ± 0.32
σ	14.75 ± 0.38
$N_{B2ppkpi}$	3993 ± 102
$N_{B2pppipi}$	0 ± 103
$N_{Bs2ppkk}$	159 ± 164
$N_{Bs2ppkpi}$	11 ± 29
N_{bkg}	3133 ± 90
τ_{bkg}	$(-1.49 \pm 0.24) \cdot 10^{-3}$
Run 2	
μ	5281.00 ± 0.13
σ	15.09 ± 0.16
$N_{B2ppkpi}$	22476 ± 242
$N_{B2pppipi}$	86 ± 125
$N_{Bs2ppkk}$	264 ± 64
$N_{Bs2ppkpi}$	738 ± 370
N_{bkg}	12935 ± 192
τ_{bkg}	$(-1.91 \pm 0.14) \cdot 10^{-3}$

Table 4.7: Parameters extracted from the fit on the B^0 spectrum in the range (5200,5480)MeV and with $2700 < m_{pp} < 3300$ MeV

4.5. Mass fit models and extraction of the $B^0 \rightarrow \eta_c K^+ \pi^-$ candidates

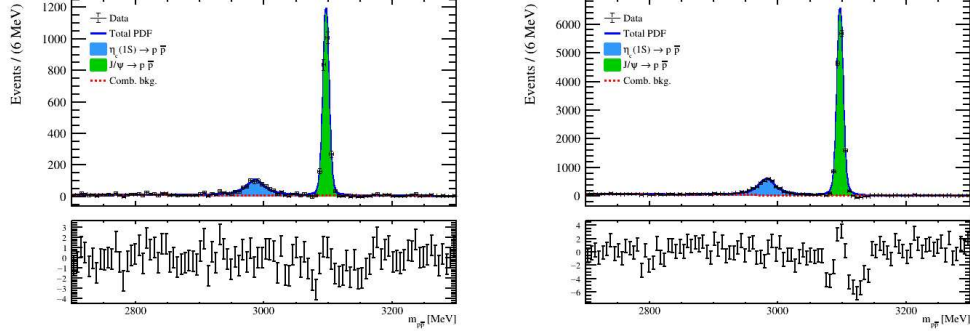


Figure 4.24: Fit to the weighted Run 1 (left) and Run 2 (right) m_{pp} invariant-mass distribution. The pull distribution are also shown.

Fit to the m_{pp} sPlot

The control variable is the m_{pp} DTF (see Appendix B) distribution in the range [2700, 3300] MeV. The η_c has been parameterised using a relativistic Breit-Wigner convoluted with a double Crystall Ball PDF, while the J/ψ has been parameterised through an Hypatia PDF. The shape parameters of the η_c and J/ψ have been fixed from the MC fits. Also, an exponential PDF is added for the nonresonant $p\bar{p}$ contribution. The fits result are shown in Figure 4.24. The parameter extracted are listed in Table 4.8

Study of the interference between η_c and the non-resonant $p\bar{p}$ S-wave

The presence of a possible interference term between the η_c and the non-resonant $p\bar{p}$ S-wave has been investigated. An amplitude fit is performed where the η_c amplitude is parameterised as a Breit-Wigner function

$$\mathcal{A}_{\eta_c} = \frac{1}{m - m_{\eta_c} - i\Gamma_{\eta_c}/2} \quad (4.8)$$

and the non-resonant amplitude, $\mathcal{A}_{\text{nonRes}}$, corresponding to the $p\bar{p}$ S-wave, is a real-valued exponential. The probability density function takes the form

$$\mathcal{P} = f_{J/\psi} \mathcal{P}_{J/\psi} + f_{\text{nonRes}} \mathcal{P}_{\text{nonRes}} + f_{\eta_c} \mathcal{P}_{\eta_c} + (1 - f_{J/\psi} - f_{\text{nonRes}} - f_{\eta_c}) \mathcal{P}_{\text{int}}, \quad (4.9)$$

where $\mathcal{P}_{J/\psi}$ is the Hypatia PDF used to parameterise the J/ψ component, $\mathcal{P}_{\text{nonRes}} = |\mathcal{A}_{\text{nonRes}}|^2$, $\mathcal{P}_{\eta_c}$ is defined as

4.5. Mass fit models and extraction of the $B^0 \rightarrow \eta_c K^+ \pi^-$ candidates

Parameter	Value [MeV]
Run 1	
$\mu_{J/\Psi}$	3096.6 ± 0.1
σ	5.20 ± 0.10
$N_{J/\Psi}$	2297 ± 49
N_{η_c}	828 ± 38
Γ_{η_c}	33 ± 2
μ_{η_c}	2986.4 ± 0.9
N_{bkg}	881 ± 40
τ_{bkg}	-0.0016 ± 0.0001
Run 2	
$\mu_{J/\Psi}$	3096.52 ± 0.05
σ	5.01 ± 0.04
$N_{J/\Psi}$	12958 ± 115
N_{η_c}	4514 ± 89
Γ_{η_c}	30.7 ± 0.9
μ_{η_c}	2982.2 ± 0.4
N_{bkg}	4978 ± 94
τ_{bkg}	-0.0023 ± 0.0001

Table 4.8: Parameters extracted from the fit on the m_{pp} spectrum in the range (2700,3300)MeV weighted with sWeights.

$$\mathcal{P}_{\eta_c} = |c_F|^2 |\mathcal{A}_{\eta_c}|^2 \otimes G(0, \sigma), \quad (4.10)$$

where $c_F = m e^{i\delta}$ is the isobar coefficient incorporating the relative phase between the η_c and the non-resonant contributions while $G(0, \sigma)$ is the gaussian PDF parameterising the resolution. Finally, $\mathcal{P}_{\text{int}}$ is the function corresponding to the interference term defined as

$$\mathcal{P}_{\text{int}} = 2[\mathcal{A}_{\text{nonRes}}(c_F \mathcal{A}_{\eta_c})^*]. \quad (4.11)$$

Taking into account the normalisation of the coefficients and functions/PDFs entering the RooRealSumPDF method in RooFit, this parameterisation allows to define the fit fractions as

$$\text{FF}_i = \frac{f_i \int dm(p\bar{p}) \mathcal{P}_i}{\int dm(p\bar{p}) \mathcal{P}} \quad (4.12)$$

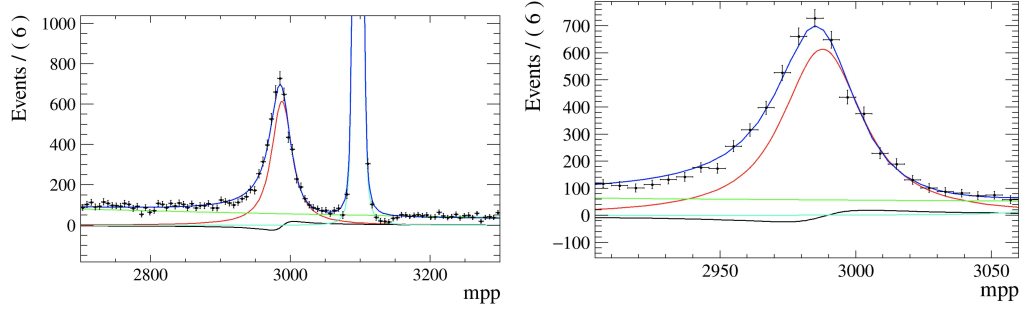


Figure 4.25: Fit to the $m_{p\bar{p}}$ for the combined Run 1 and Run 2 including the interference term. The right plot is a zoom on the $m_{p\bar{p}}$ region used in the Dalitz plot analysis.

Figure 4.25 shows the fit results. The fit parameters, listed in Tab. 4.9, are consistent with the ones determined in the previous subsection. The fit fractions, determined as reported above, are reported in the same table. The interference term, whose fit fraction is determined to be 0.8%, does not help to improve the description of data. The negative log-likelihood value is -132852, to be compared with the value -132917 when not including the interference term, despite the largest number of free parameters in the former case. The interference fit fraction is therefore not significant and not considered in the remainder part of the analysis.

Parameter	Value
$\mu_{J/\psi}$	3096.80 ± 0.05 MeV
μ_{η_c}	2988 ± 41 MeV
δ	0 ± 50
m	0.9 ± 6.7
τ_{nonRes}	-0.0012 ± 0.0019 1/MeV
σ	5.15 ± 0.20 MeV
FF_{η_c}	19.4 %
FF_{int}	0.8 %
$\text{FF}_{\text{nonRes}}$	21.5 %
$\text{FF}_{J/\psi}$	58.3 %

Table 4.9: Parameters extracted from the fit on the $m_{p\bar{p}}$ spectrum for the combined Run 1 and Run 2 with interference.

Background studies using $B^0 \rightarrow p\bar{p}K^+\pi^-$ sWeight

A check for possible intermediate states is performed, in any invariant-mass combination of the final state particles (with the exception of the $K^+\pi^-$ system), falling in the η_c - J/ψ region, defined in the $2700 < m_{p\bar{p}} < 3300$ MeV range. After having

applied the vetoes reported in Sec. 4.4.5, there are no evidence for intermediate states as it can be seen in Figure 4.26 except in the $m_{\bar{p}K^+}$ invariant-mass where the $\Lambda(1520)$ peak is visible. This contribution is removed once the 2D fit, described in subsection 4.5.3, is applied as can be seen in the signal reweighted distribution shown in Figure 4.27.

4.5.3 2D fit to $m_{p\bar{p}K^+\pi^-}$ and $m_{p\bar{p}}$

The $B^0 \rightarrow \eta_c K^+ \pi^-$ events have been unfolded performing a two dimensional fit to the $m_{p\bar{p}K^+\pi^-}$ and the DTF $m_{p\bar{p}}$ invariant-mass distributions. Using this pair of masses as discriminant variables, it is possible to use the sPlot technique to compute the sWeights for the $B^0 \rightarrow \eta_c K^+ \pi^-$ component. The sWeights can then be used to build the corresponding Dalitz plot and to build the combinatorial and non-resonant background histograms to be used later on the amplitude fit. The fit is applied in the $[5220, 5340]$ MeV $m_{p\bar{p}K^+\pi^-}$ range and in the $[2908, 3058]$ MeV $m_{p\bar{p}}$ range. The components used are:

- $B^0 \rightarrow \eta_c K^+ \pi^-$ signal ($B^0 \cdot \eta_c$);
- $B^0 \rightarrow p\bar{p}K^+\pi^-$ non resonant ($B^0 \cdot \exp1$);
- Combinatorial background component ($\exp2 \cdot \exp3$);

The component describing η_c combined with spurious K^+ and π^- has been investigated and added to the fit ($\exp \cdot \eta_c$). Since the yield is compatible with zero, it is then removed from the nominal fit model, the results from this fit are shown in G. The B^0 and η_c signal have been parameterised with the same PDF described in the previous section, with their shape parameter fixed. The combinatorial background and the non-resonant component have been parameterised using exponential functions ($\exp2$ in the $m_{p\bar{p}K^+\pi^-}$ projection and $\exp1, \exp3$ in the $m_{p\bar{p}}$ projection). In figures 4.28 are shown the projections on the two invariant-masses of the fit for Run 1 and Run 2.

The extracted parameters are listed in Table 4.10.

To investigate the goodness of the 2D mass fits, a 2D pull plot has been computed. The 2D pull histogram are computed from the data histogram and the histogram filled with the values obtained generating $N_{toy}^{tot} = 1000 \cdot N_{fit}$ events (where N_{fit} is the total yield from the fit) from the PDF resulting from the fit. An adaptive binning algorithm has been used to build the 2D histogram from the data in order to have at least 20 events per bin. The pull plot has been computed as:

$$\frac{N_{toy} - N_{data}}{\sqrt{N_{data}}} \quad (4.13)$$

4.5. Mass fit models and extraction of the $B^0 \rightarrow \eta_c K^+ \pi^-$ candidates

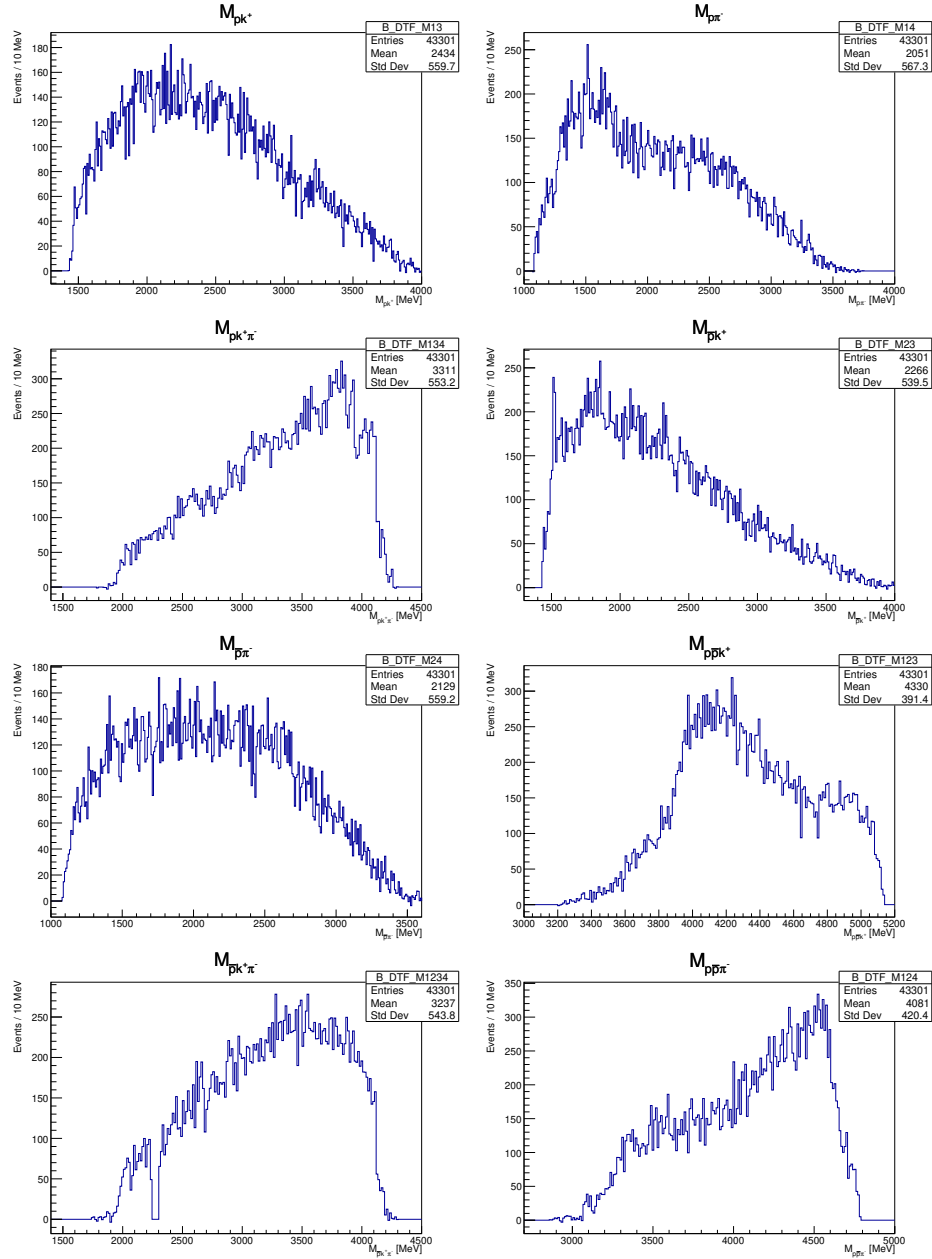


Figure 4.26: Distribution of all the invariant-mass combination of the final state particles reweighted with the sWeights.

where N_{toy} is the number of events on the corresponding bin of the generated histogram and N_{data} is the number of events on the corresponding bin of the data histogram. The pull distributions are shown in Figure 4.29. From the 2D pull plot 1D pull plot have been computed from the bin content, a gaussian fit have been

4.5. Mass fit models and extraction of the $B^0 \rightarrow \eta_c K^+ \pi^-$ candidates

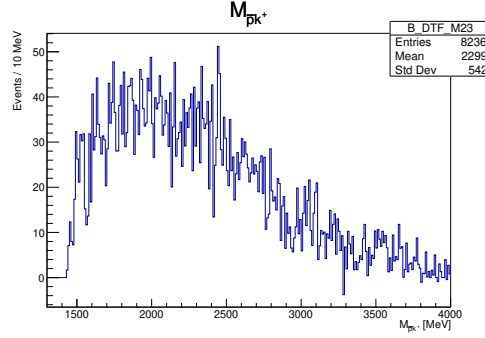


Figure 4.27: Distribution of the $m_{\bar{p}K^+}$ invariant-mass reweighted with the signal sWeights from the 2D fit described in subsection 4.5.3.

Parameter	Value [MeV]
Run 1	
μ_{B^0}	5283 ± 1
σ_{B^0}	15 ± 1
μ_{η_c}	2983 ± 1
Γ_{η_c}	34 ± 4
$\tau_{bkg}^{B^0}$	$(2.82 \pm 1.76) \cdot 10^{-3}$
τ_{bkg}^{NR}	$(-1.01 \pm 1.78) \cdot 10^{-3}$
τ_{bkg}^{comb}	$(-7.88 \pm 5.13) \cdot 10^{-3}$
N_{sig}	772 ± 70
N_{bkg}^{NR}	168 ± 69
N_{bkg}^{comb}	361 ± 34
Run 2	
μ_{B^0}	5281 ± 1
σ_{B^0}	15 ± 1
μ_{η_c}	2986 ± 1
Γ_{η_c}	31 ± 2
$\tau_{bkg}^{B^0}$	$(-5.74 \pm 8.13) \cdot 10^{-4}$
τ_{bkg}^{NR}	$(4.62 \pm 8.13) \cdot 10^{-4}$
τ_{bkg}^{comb}	$(-7.62 \pm 2.34) \cdot 10^{-3}$
N_{sig}	4266 ± 139
N_{bkg}^{NR}	823 ± 139
N_{bkg}^{comb}	1665 ± 68

Table 4.10: Parameters extracted from the 2D mass fit.

applied to them. The result of the fit are shown in Table 4.11. From the 2D fit sWeights are extracted, these are going to be used in the next section. In figure 4.30 the Dalitz plot computed using the sWeights is shown.

4.5. Mass fit models and extraction of the $B^0 \rightarrow \eta_c K^+ \pi^-$ candidates

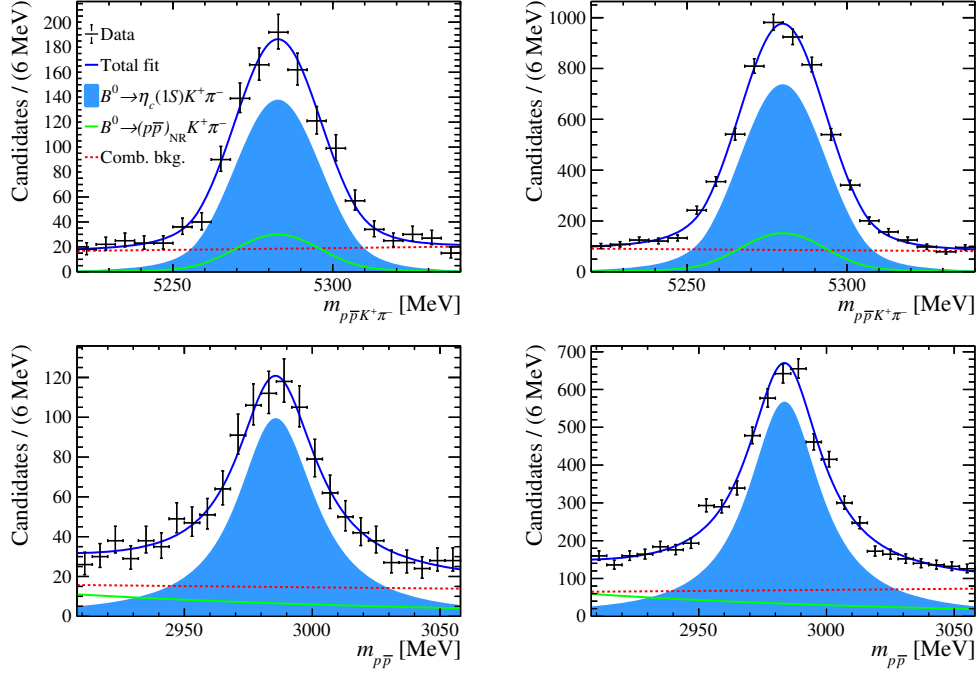


Figure 4.28: Results of the two-dimensional mass fit to the (left) $m_{p\bar{p}K^+\pi^-}$ and (right) $m_{p\bar{p}}$ distributions for the (top) Run 1 and (bottom) Run 2 datasets.

Parameter	Value
Run 1	
μ	-0.01 ± 0.28
σ	1.19 ± 0.2
Run 2	
μ	-0.04 ± 0.06
σ	0.95 ± 0.05

Table 4.11: Parameters extracted from the 1D pull plot.

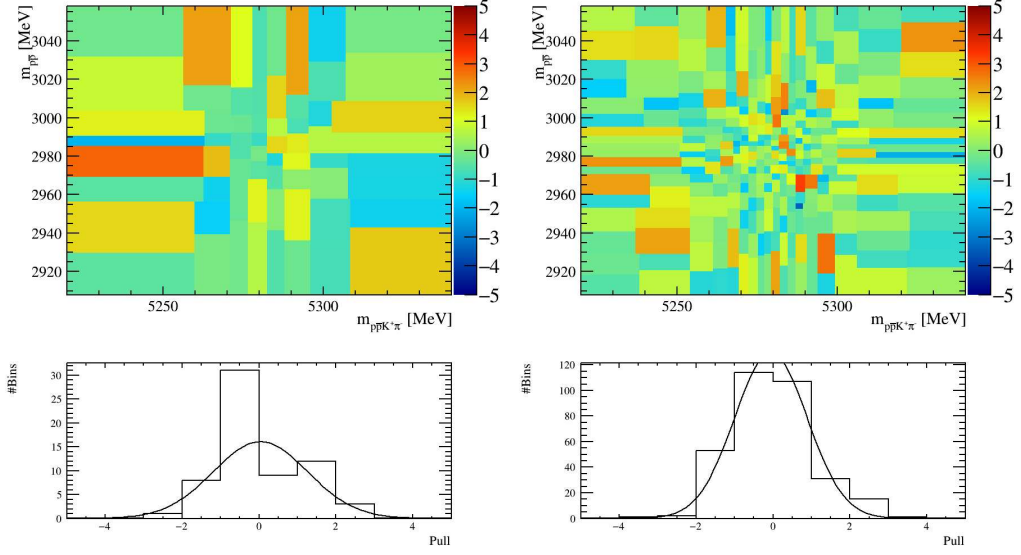


Figure 4.29: Run 1 (left) and Run 2 (right) 2D pull plot. On the bottom are shown the 1D pull plot computed from the bin contents of the respective 2D pull histogram.

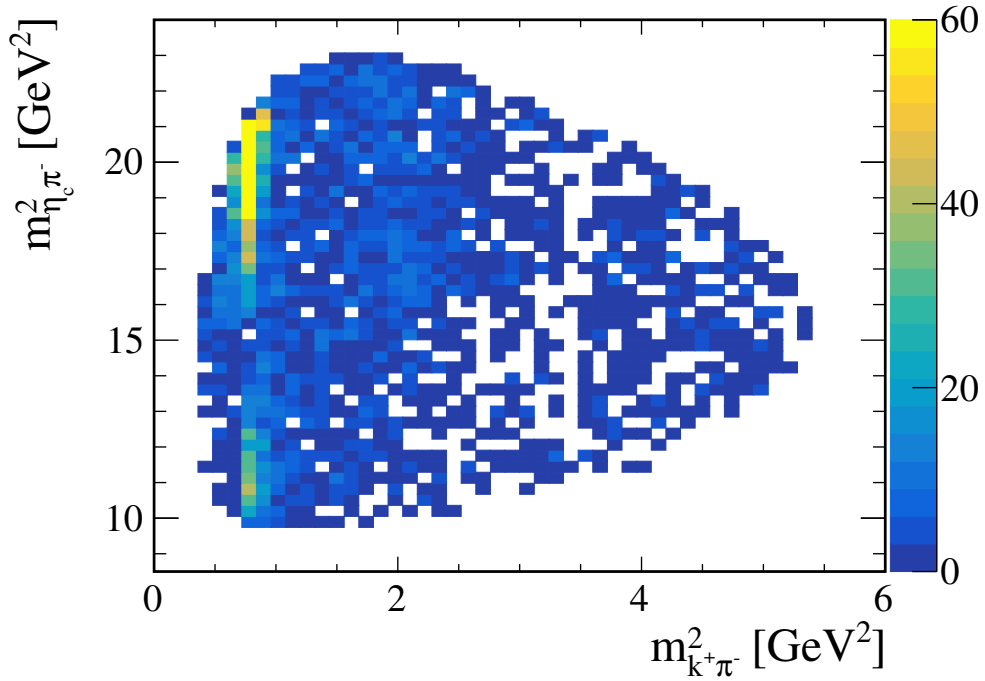


Figure 4.30: $B^0 \rightarrow \eta_c K^+ \pi^-$ Dalitz plot for the combined Run 1 and Run 2 sample.

4.6 $B^0 \rightarrow \eta_c K^+ \pi^-$ branching ratio measurement

The measurement of the branching ratio of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay is measured in order to also calculate the branching ratio of the various $B^0 \rightarrow \eta_c K^* (\rightarrow K^+ \pi^-)$ decays. The normalisation channel used is $B^0 \rightarrow J/\psi (\rightarrow p\bar{p}) K^+ \pi^-$. The branching ratio of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay can be calculated using the following:

$$\mathcal{B}(B^0 \rightarrow \eta_c K^+ \pi^-) = \mathcal{B}(B^0 \rightarrow J/\psi K^+ \pi^-) \frac{\mathcal{B}(J/\psi \rightarrow p\bar{p})}{\mathcal{B}(\eta_c \rightarrow p\bar{p})} \frac{N_{\eta_c}}{N_{J/\psi}} \frac{\epsilon_{J/\psi}}{\epsilon_{\eta_c}}, \quad (4.14)$$

where $\mathcal{B}(B^0 \rightarrow J/\psi K^+ \pi^-) = (1.15 \pm 0.05) \cdot 10^{-3}$, $\mathcal{B}(J/\psi \rightarrow p\bar{p}) = (2.12 \pm 0.029) \cdot 10^{-3}$ and $\mathcal{B}(\eta_c \rightarrow p\bar{p}) = (1.33 \pm 0.11) \cdot 10^{-3}$ are previously measured [36]. N_{η_c} and $N_{J/\psi}$ are the number of η_c and J/ψ candidates respectively. $\epsilon_{J/\psi}$ and ϵ_{η_c} the selection efficiencies of the signal and normalisation channel calculated from the MC. The Run 1 and Run 2 dataset are elaborated separately throughout the entire process.

4.6.1 η_c and J/ψ yields measurement

The strategy used in order to extract the yields is the same described in sec. 4.5.3. The fit is applied simultaneously to the $m_{p\bar{p}K^+\pi^-}$ and $m_{p\bar{p}}$ invariant-masses. The sample used is obtained after vetoing the regions $1835 < m_{K^+\pi^-} < 1880$ MeV, $2250 < m_{p\bar{p}K^+\pi^-} < 2305$ MeV and $1010 < m_{K\pi_{K\text{mass hypothesis}}} < 1028$ MeV. The yields obtained for the η_c candidates are the one reported in sec. 4.5.3 in Table 4.10.

For the $B^0 \rightarrow J/\psi (\rightarrow p\bar{p}) K^+ \pi^-$ 2D fit the components used are :

- $B^0 \rightarrow J/\psi (\rightarrow p\bar{p}) K^+ \pi^-$ signal ($B^0 \cdot J/\psi$)
- Combinatorial background component ($\text{exp1} \cdot \text{exp2}$)

Non resonant $B^0 \rightarrow p\bar{p} K^+ \pi^-$ and $J/\psi K^+ \pi^- \rightarrow p\bar{p} K^+ \pi^-$ components have been found negligible the fits. The $m_{p\bar{p}}$ is fitted in the range [3072, 3122] MeV. The fit result of the J/ψ are shown in figures 4.31, the extracted parameters are reported in Table 4.12. The ratio $\frac{N_{\eta_c}}{N_{J/\psi}}$ is calculated, corresponding to 0.355 ± 0.031 for Run 1 and 0.334 ± 0.012 for Run 2.

4.6.2 Efficiency calculations

The efficiency for the two channels are calculated by using the MC samples 11134003 and 11134010. The efficiency has been calculated as the ratio of the number of events at the end of the selection reported in the previous sections, after been truth matched, and the total number of events generated, reported in Table 4.1

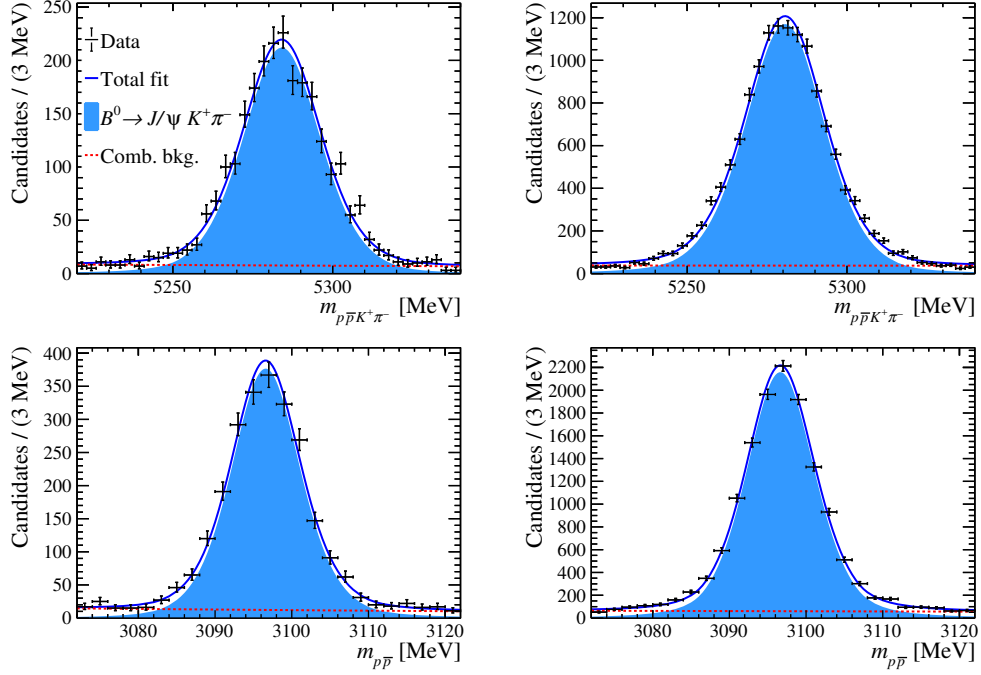


Figure 4.31: Results of the two-dimensional fit to the (left) $m_{p\bar{p}K^+\pi^-}$ and (right) $m_{p\bar{p}}$ distributions for the (top) Run 1 and (bottom) Run 2 datasets in the region of the J/ψ resonance.

and 4.2. Before applying the selection the MC samples are reweighted in order to correct for the B production kinematics and the number of tracks per event. The GBReweighter tool is trained on the data in order to obtain a reweighting scheme for the nominal MC samples. The variables used are:

- B_{P_T} ;
- B^0 pseudorapidity;
- Number of tracks in the event.

A reweighting of the Dalitz plot variables has also been performed in order to take into account for possible influences of the resonances to the efficiencies. Due to the limited statistics of the data samples, especially for Run 1, the resampled variables show some discrepancies. The effects of the reweight on the training variables are shown in Appendix O.

Parameter	Value [MeV]
Run 1	
μ_{B^0}	5284.2 ± 0.3
σ_{B^0}	14.7 ± 0.3
$\mu_{J/\psi}$	3096.2 ± 0.1
$\sigma_{J/\psi}$	5.0 ± 0.1
$\tau_{bkg}^{B^0}$	$(-1.7 \pm 2.0) \cdot 10^{-3}$
τ_{bkg}^{comb}	$(-7.1 \pm 4.0) \cdot 10^{-3}$
N_{sig}	2258 ± 50
N_{bkg}^{comb}	306 ± 24
Run 2	
μ_{B^0}	5281.0 ± 0.1
σ_{B^0}	14.9 ± 0.1
$\mu_{J/\psi}$	3096.73 ± 0.05
$\sigma_{J/\psi}$	5.03 ± 0.05
$\tau_{bkg}^{B^0}$	$(-2 \pm 9) \cdot 10^{-4}$
τ_{bkg}^{comb}	$(-2.5 \pm 2.0) \cdot 10^{-3}$
N_{sig}	12787 ± 119
N_{bkg}^{comb}	1496 ± 54

Table 4.12: Parameters extracted from the 2D mass fit for the candidates of the $B^0 \rightarrow J/\psi(\rightarrow p\bar{p})K^+\pi^-$ decay.

The efficiencies, and their ratio, have been calculated to be, for Run 1:

$$\epsilon_{\eta_c} = (7.49 \pm 0.03) \cdot 10^{-3}, \quad \epsilon_{J/\psi} = (7.08 \pm 0.02) \cdot 10^{-3} \rightarrow \frac{\epsilon_{J/\psi}}{\epsilon_{\eta_c}} = 0.945 \pm 0.004 \quad (4.15)$$

and for Run 2:

$$\epsilon_{\eta_c} = (1.357 \pm 0.002) \cdot 10^{-2}, \quad \epsilon_{J/\psi} = (1.287 \pm 0.002) \cdot 10^{-2} \rightarrow \frac{\epsilon_{J/\psi}}{\epsilon_{\eta_c}} = 0.948 \pm 0.002 \quad (4.16)$$

4.6.3 Systematics

The possible sources of systematic uncertainties taken into account are:

- Shape parameters;
- PDF for signal and background;
- Invariant-mass windows.

In the following subsections are described the strategies used to take into ac-

count these sources. The systematic uncertainties are reported at the end of this subsection.

Shape parameters

The shape parameter of the signal PDFs are fixed in the nominal fit. To take into account this possible source of systematic uncertainty new shape parameters can be generated, using the covariance matrix from the MC fit. New fits have been performed varying the shape parameters with respect to the distribution obtained. The yield ratio distributions obtained are reported in appendix M. The systematic uncertainty is taken as the difference between the nominal value and the value corresponding to $\pm\sigma$ of the yield ratio distributions.

Signal and background parameterisation

Different PDF are used to take into account possible sources of error from the choice of the PDFs chosen. The PDF used to describe the signal are Hypatia for the B^0 and J/ψ candidates and the convolution of a relativistic Breit-Wigner and a Crystal Ball function. The backgrounds have all been parameterised using RooExponential. New 2D fit have been performed, using as new PDF :

- B^0 and J/ψ : Crystal Ball function;
- η_c : RooVoigtian;
- Bkgs: RooPolynomial (1st order);

The fit results are shown in appendix M. New values of branching ratio have been calculated from the results of the 2D fits. The systematic errors associated to this source is calculated as the difference between the central values.

Mass windows

Different mass windows for the $m_{\bar{p}p}$ and $m_{\bar{p}pK^+\pi^-}$ variables have been used to perform the 2D fit. The new mass windows used for the η_c candidate is $2924 < m_{\bar{p}p} < 3044$ MeV, for the J/ψ candidate is $3082 < m_{\bar{p}p} < 3112$ MeV and for the B^0 candidate is $5235 < m_{\bar{p}pK^+\pi^-} < 5325$. The fit results are shown in appendix M. New efficiencies from the MC are also recalculated. The results are:

$$\epsilon_{\eta_c} = (3.74 \pm 0.02) \cdot 10^{-3}, \quad \epsilon_{J/\psi} = (3.54 \pm 0.02) \cdot 10^{-3} \rightarrow \frac{\epsilon_{J/\psi}}{\epsilon_{\eta_c}} = 0.945 \pm 0.006 \quad (4.17)$$

and for Run 2:

$$\epsilon_{\eta_c} = (6.79 \pm 0.02) \cdot 10^{-3}, \quad \epsilon_{J/\psi} = (6.44 \pm 0.02) \cdot 10^{-3} \rightarrow \frac{\epsilon_{J/\psi}}{\epsilon_{\eta_c}} = 0.948 \pm 0.003 \quad (4.18)$$

The yield obtained are used to calculate the new branching ratios. The systematic errors associated to this source is calculated as the difference between the central values.

Systematic results

Systematic source	Resulting branching ratio $\cdot 10^{-4}$
Run 1	
Shape parameters	± 0.05
PDFs	± 0.18
Mass windows	± 0.41
Run 2	
Shape parameters	± 0.05
PDFs	± 0.22
Mass windows	± 0.15

Table 4.13: Systematic uncertainties on the $B^0 \rightarrow \eta_c K^+ \pi^-$ branching ratios.

4.6.4 Results

Using the values calculated in the previous sections the two branching ratios are:

$$\text{Run 1 : } \mathcal{B}(B^0 \rightarrow \eta_c K^+ \pi^-) = (6.10 \pm 0.53 \pm 0.45 \pm 0.58) \cdot 10^{-4} \quad (4.19)$$

$$\text{Run 2 : } \mathcal{B}(B^0 \rightarrow \eta_c K^+ \pi^-) = (5.77 \pm 0.22 \pm 0.27 \pm 0.55) \cdot 10^{-4} \quad (4.20)$$

So, combining the measurements together as weighted sum:

$$\mathcal{B}(B^0 \rightarrow \eta_c K^+ \pi^-) = (5.82 \pm 0.20_{stat} \pm 0.23_{syst} \pm 0.55_{ext}) \cdot 10^{-4} \quad (4.21)$$

This results is in agreement with the state of art measurement $\mathcal{B}(B^0 \rightarrow \eta_c K^+ \pi^-) = (6.5 \pm 0.7) \cdot 10^{-4}$ [36]. The statistical uncertainty of the current analysis is improved by a factor $\sim \sqrt{2}$ with respect to the previous measurement, while the systematical uncertainties is higher, since new sources are considered.

4.7 Efficiency evaluation

In order to perform an amplitude fit of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay channel, the efficiency and resolution variations across the phase space of the decay must be taken into account. Variations in efficiency arise from detector effects impacting candidate reconstruction and the selection procedure. In the amplitude analysis context, the focus is not on the absolute efficiency value but on relative variations within the Dalitz Plot area. To establish the efficiency map and mitigate issues related to the curved DP boundaries MC sample is utilised requiring that the events were uniformly generated in the square Dalitz Plot coordinates. The SDP coordinates are defined as:

$$m' = \frac{1}{\pi} \arccos \left(2 \frac{m_{K\pi} - m_{K\pi}^{min}}{m_{K\pi}^{max} - m_{K\pi}^{min}} - 1 \right) \quad (4.22)$$

$$\theta' = \frac{1}{\pi} \theta_{K^*} \quad (4.23)$$

where θ_{K^*} is the helicity angle defined as the angle between the K^+ and the η_c in the $K^+ \pi^-$ rest frame, as shown in figure 4.32. The cosine of the helicity angle has been calculated as:

$$\cos(\theta_{K^*}) = \frac{m_K^2 + m_{\eta_c}^2 - m_{\eta_c K}^2 + 2(E_K)_{K\pi}(E_{\eta_c})_{K\pi}}{2 |\vec{p}| |\vec{q}|} \quad (4.24)$$

where $(E_K)_{K\pi}$ and $(E_{\eta_c})_{K\pi}$ are respectively the energy of the K^+ and the η_c in the $K^+ \pi^-$ rest frame, and $\vec{q}$ and $\vec{p}$ are the K^+ and η_c momenta in the $K^+ \pi^-$ rest frame, respectively. $m_{K\pi}^{min}$ and $m_{K\pi}^{max}$ are the phase space boundaries of the $K^+ \pi^-$ invariant-mass and their values are respectively 0.63 GeV and 2.3 GeV. Using the SDP coordinates, the ranges of the phase space are $0 < \theta' < 1$ and $0 < m' < 1$.

The efficiency distribution has been computed in the form of a 2D histogram in the θ' vs m' distribution. Using the expressions 4.22 and 4.23 one can build the distributions starting from the simulated proton, antiproton, kaon and pion momenta resulting by the fit to their B^0 parent vertex with its mass constrained to the PDG [66] value and its direction pointing to the primary vertex. The efficiency variations have been calculated by applying step by step the selection described

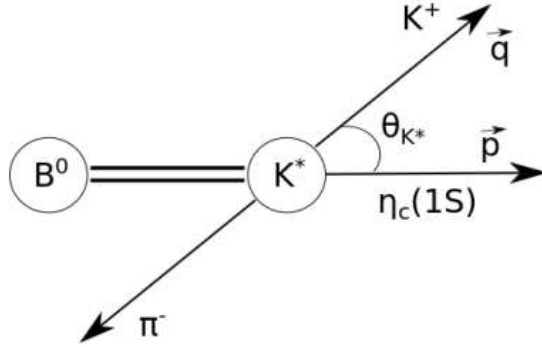


Figure 4.32: Helicity angle θ_{K^*} for the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay. It is defined as the angle between the K^+ momentum $\vec{q}$ and the η_c momentum $\vec{p}$ both evaluated in the rest frame of the K^* resonance. Note that the η_c is produced directly from the B-decay vertex.

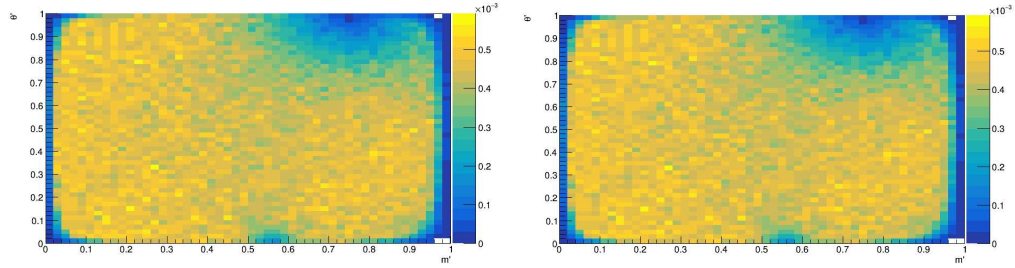


Figure 4.33: Efficiency of the trigger, stripping selection and preselection across the $B^0 \rightarrow \eta_c K^+ \pi^-$ Run 1 (left) and Run 2 (right) SDP space.

in the previous sections. In figure 4.33 are shown the θ' vs m' distributions for Run 1 and Run 2 from the MC after the trigger selection, stripping selection and preselection.

In figure 4.34 are shown the θ' vs m' distributions for Run 1 and Run 2 from the MC after the BDT selection.

In figure 4.35 are shown the θ' vs m' distributions for Run 1 and Run 2 from the MC after the veto of the Λ_c and ϕ . The D^0 veto is not implemented in the efficiency map because it is an explicit veto in the phase space and it will be taken into account during the amplitude fit.

The distribution shown in figure 4.36 are going to be used in the amplitude fit. In particular they are going to be passed to **Laura++** and undergo a smoothing procedure through a cubic spline interpolation. The efficiency drops at $m' = 0$ and at $m' = 1$ are related mainly to the stripping line and trigger cuts. The drops are

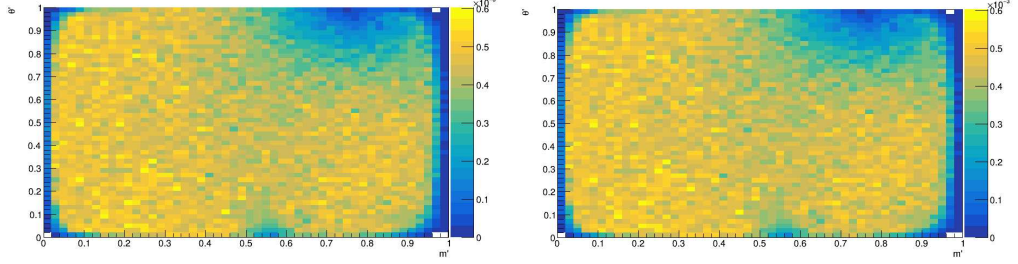


Figure 4.34: Efficiency of the BDT selection across the $B^0 \rightarrow \eta_c K^+ \pi^-$ Run 1 (left) and Run 2 (right) SDP space.

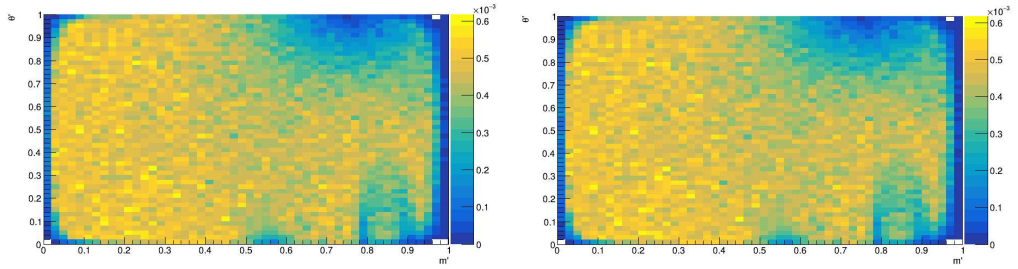


Figure 4.35: Efficiency of the veto of the Λ_c and ϕ across the $B^0 \rightarrow \eta_c K^+ \pi^-$ Run 1 (left) and Run 2 (right) SDP space.

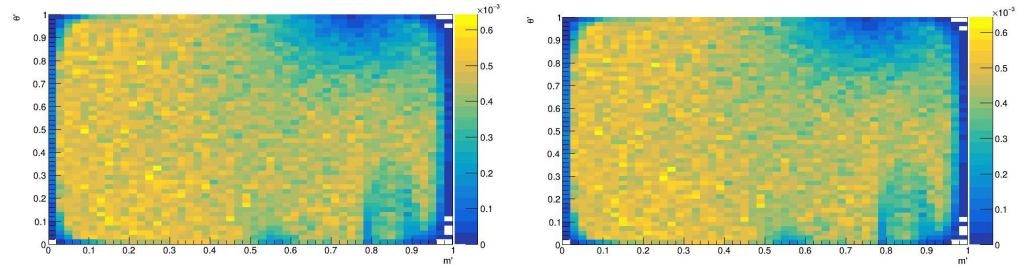


Figure 4.36: Efficiency after the elimination of the clones and multiple candidates and after the selection of the events in the range $5220 < m_{p\bar{p}K^+\pi^-} < 5340$ MeV and $2908 < m_{p\bar{p}} < 3058$ MeV across the $B^0 \rightarrow \eta_c K^+ \pi^-$ Run 1 (left) and Run 2 (right) SDP space.

also due to the neglect of the η_c width during the generation phase by EvtGen, that uses the PDG value of the η_c mass instead of the $m_{p\bar{p}}$ mass to compute m' and θ' values.

4.8 Dalitz Plot Analysis

Usually, in searches conducted through amplitude analysis, the model must include all known possible states to check whether the observed patterns in the invariant-mass distributions are actually reflections of already established states seen in other views. In this specific scenario, elaborated in Sec. 4.8.1, the $K^+\pi^-$ spectrum is abundant with well-established resonances. Following an introduction to the formalism utilised in this analysis, the Sec. 4.8.9 outlines the amplitude fit to the data samples. Within this chapter, an exclusive K^* model that incorporates only significant K^* contributions is introduced, the default model.

4.8.1 K^* contribution

In the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay, the $K^+ \pi^-$ system mass range is up to ~ 2.3 GeV. This means that all the neutral K^* resonances known to decay to $K^+ \pi^-$ should in principle be included in the model. They are listed in Tab. 4.14. Their mass, width and quantum numbers, taken by [66], are reported as well.

State	Mass [MeV]	Width [MeV]	J^P	$\mathcal{B}(K^{*0} \rightarrow K^+ \pi^-)$
$K^*(800)$	682 ± 29	547 ± 24	0^+	$\sim 100\%$
$K^*(892)$	895.55 ± 0.20	47.3 ± 0.5	1^-	$\sim 100\%$
$K^*(1410)$	1414 ± 15	232 ± 21	1^-	$(6.6 \pm 1.3)\%$
$K_0^*(1430)$	1425 ± 50	270 ± 80	0^+	$(93 \pm 10)\%$
$K_2^*(1430)$	1432.4 ± 1.3	109 ± 5	2^+	$(49.9 \pm 1.2)\%$
$K^*(1680)$	1717 ± 27	322 ± 110	1^-	$(38.7 \pm 2.5)\%$
$K_3^*(1780)$	1776 ± 7	159 ± 21	3^-	$(18.8 \pm 1.0)\%$
$K_0^*(1950)$	1945 ± 22	201 ± 90	0^+	$(52 \pm 14)\%$
$K_4^*(2045)$	2045 ± 9	198 ± 30	4^+	$(9.9 \pm 1.2)\%$
$B^0 \rightarrow \eta_c(1S) K^+ \pi^-$ phase space limit				
$K_5^*(2380)$	2382 ± 24	178 ± 50	5^-	$(6.1 \pm 1.2)\%$

Table 4.14: Known neutral K^* resonances seen in the $K^+ \pi^-$ final state. The $K_4^*(2045)$ width is such that it could contribute to $B^0 \rightarrow \eta_c K^+ \pi^-$.

4.8.2 Three-body decays and Dalitz Plot formalism

4.8.3 Isobar model

The isobar model describe the amplitude across the DP phase space as resulting from a coherent sum of resonant and non-resonant terms. As such the total

amplitude is given by:

$$\mathcal{A}(m_{ij}^2, m_{jk}^2) = \sum_{h=1}^N c_h \mathcal{F}_h(m_{ij}^2, m_{jk}^2) \quad (4.25)$$

where $\mathcal{F}_h(m_{ij}^2, m_{jk}^2)$ is the dynamical amplitude whose contribution is associated with the complex coefficient c_h (isobar coefficient). In this analysis $m_{ij}^2 = m_{K^+\pi^-}^2$ and $m_{jk}^2 = m_{\eta_c\pi^-}^2$. The dynamical amplitudes used are described in the following sections.

4.8.4 Dynamical amplitudes

For a resonance of spin J formed in the variable m_{ij}^2 the dynamical amplitude is given by:

$$\mathcal{F}(m_{ij}^2, m_{jk}^2) = T(m_{ij}^2) \times X_J(|\vec{p}|r_{BW}) \times X_J(|\vec{q}|r_{BW}) \times Z_J(\vec{p}, \vec{q}) \quad (4.26)$$

where $\vec{p}$ and $\vec{q}$ are the momenta of the bachelor particle and of one of the resonance bachelors, respectively, both evaluated in the rest frame of the resonance, $T(m_{ij}^2)$ is the resonance mass term, usually given by the Breit-Wigner amplitude. There are cases where the Breit-Wigner approximation is not valid, as described later. X is a Blatt-Weisskopf barrier form factor. $Z(\vec{p}, \vec{q})$ is the Zemach tensor describing the angular probability distribution.

Each amplitude is normalised such that the integral over the phase space is unity

$$\iint_{DP} |\mathcal{F}(m_{ij}^2, m_{jk}^2)|^2 dm_{ij}^2 dm_{jk}^2 = 1 \quad (4.27)$$

calculated using the Gauss-Legendre integration methods.

Angular distributions

Various methods can be employed to express the angular probability distribution term. In b -decays, the extensive phase space necessitates a well-behaved description of the resonance amplitude, particularly at masses considerably distinct from the pole mass. Ensuring transversality in the amplitudes is essential, and this is accomplished through the utilisation of the Zemach tensor formalism. The angular distribution terms are presented in Table 4.15. They are proportional to the Legendre polynomials $P_J(x)$, where x is the cosine of the helicity angle (the angle between $\vec{p}$ and $\vec{q}$).

J	$Z_J(\vec{p}, \vec{q})$
0	1
1	$-2\vec{p} \cdot \vec{q}$
2	$\frac{4}{3} [3(\vec{p} \cdot \vec{q})^2 - (\vec{p} \vec{q})^2]$
3	$-\frac{8}{5} [5(\vec{p} \cdot \vec{q})^3 - 3(\vec{p} \cdot \vec{q})(\vec{p} \vec{q})^2]$
4	$\frac{16}{35} [35(\vec{p} \cdot \vec{q})^4 - 30(\vec{p} \cdot \vec{q})^2(\vec{p} \vec{q})^2 + 3(\vec{p} \vec{q})^4]$
5	$-\frac{32}{63} [63(\vec{p} \cdot \vec{q})^5 - 70(\vec{p} \cdot \vec{q})^3(\vec{p} \vec{q})^2 + 15(\vec{p} \cdot \vec{q})(\vec{p} \vec{q})^4]$

Table 4.15: Angular distributions in the Zemach tensor formalism up to $J = 5$, where J is the spin of the resonance.

Blatt-Weisskopf barrier form factors

The Blatt-Weisskopf barrier factors in Equation 4.26 correspond to the decays of both the parent particle $X_J(|\vec{p}|r_{BW})$ and the resonance $X_J(|\vec{q}|r_{BW})$. These factors are dependent on the spin J of the decaying particle and are listed in Table 4.16, where $z = |\vec{p}|r_{BW}$. The parameter r_{BW} denotes the radius of the barrier, signifying the size of the decaying particle, set at a value of 4.0 GeV^{-1} in this analysis. The parameter z_0 represents the value of z when the invariant-mass equals the pole mass m_0 of the resonance. The Blatt-Weisskopf factors, also called orbital angular momentum barrier factors, take into account the difficulty in creating the orbital angular momentum J in the B^0 decay.

J	$X_J(z)$
0	1
1	$\sqrt{\frac{1+z_0^2}{1+z^2}}$
2	$\sqrt{\frac{z_0^4+3z_0^2+9}{z^4+3z^2+9}}$
3	$\sqrt{\frac{z_0^6+6z_0^4+45z_0^2+225}{z^6+6z^4+45z^2+225}}$
4	$\sqrt{\frac{z_0^8+10z_0^6+135z_0^4+1575z_0^2+11025}{z^8+10z^6+135z^4+1575z^2+11025}}$
5	$\sqrt{\frac{z_0^{10}+15z_0^8+315z_0^6+6300z_0^4+99225z_0^2+893025}{z^{10}+15z^8+315z^6+6300z^4+99225z^2+893025}}$

Table 4.16: Blatt-Weisskopf functions up to $J = 5$, where J is the decaying particle spin. $z = |\vec{p}|r_{BW}$, z_0 indicates z evaluated at the resonance peak mass.

Relativistic Breit-Wigner mass term

The relativistic Breit-Wigner (RBW) lineshape can be used for narrow and isolated resonances and takes the form:

$$T(m) = \frac{1}{m_0^2 - m^2 - im_0\Gamma(m)} \quad (4.28)$$

where the mass dependent width is given by:

$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0} \right)^{(2J+1)} \left(\frac{m_0}{m} \right) X_J^2(qr_{BW}) \quad (4.29)$$

where $q = |\vec{q}|$ and q_0 indicates the parameter evaluated at the pole mass m_0 . $X_J^2(qr_{BW})$ is the resonance Blatt-Weisskopf function, described previously. The RBW is used to parameterise the majority of resonances but in the presence of broad overlapping resonances the RBW is not a good description, since the coherent sum of overlapping resonances in the same partial wave violates unitarity.

LASS mass term

The amplitude parameterisations using RBW can lead to unitarity violation within the isobar model if there are overlapping resonances or if there is a significant interference with a non-resonant component. This is the case for the $K^+\pi^-$ S-wave, where the $K_0^*(1430)$ resonance interferes with a non-resonant S-wave term component. The LASS lineshape [295] is used to parameterise the $K^+\pi^-$ S-wave, that combines these two amplitudes:

$$T(m) = \frac{m}{q \cot \delta_B - iq} + e^{2i\delta_B} \frac{m_0 \Gamma_0 \frac{m_0}{q_0}}{m_0^2 - m^2 - im_0 \Gamma_0 \frac{q}{m} \frac{m_0}{q_0}} \quad (4.30)$$

where

$$\cot \delta_B = \frac{1}{aq} + \frac{rq}{2} \quad (4.31)$$

m_0 and Γ_0 are the pole mass and width of the $K_0^*(1430)$ and a and r are respectively the scattering length and the effective range. The parameters a and r depend on the production mechanism and hence depend on the decay under study. The first term in Eq. 4.30 is not well described at high masses and a cutoff term is introduced, in this analysis the cutoff applied is at 1.7 GeV. When the LASS is used together with higher mass $K\pi$ resonances, in particular the $K_0^*(1950)$, the $K\pi$ S-wave is well described by the LASS PDF, as shown in Ref. [296].

4.8.5 Isobar coefficients and fit fractions

Complex coefficients, which characterise the relative strength of components, are applied to multiply the dynamical amplitudes. The isobar coefficients are expressed using real and imaginary components. In order to facilitate comparison across various experiments with potential differences in normalisation, phase conventions, and amplitude formalism, fit fractions are presented alongside amplitude magnitudes. The fit fraction for a specific component, denoted as i , is determined

as the integral of the square decay amplitude i divided by the square coherent matrix element for the complete DP:

$$FF_i = \frac{\int \int_{DP} |c_i \mathcal{F}_i(m_{ij}^2, m_{jk}^2)|^2 dm_{ij}^2 dm_{jk}^2}{\int \int_{DP} |\mathcal{A}(m_{ij}^2, m_{jk}^2)|^2 dm_{ij}^2 dm_{jk}^2} \quad (4.32)$$

In general, the sum of the component fit fractions differs from unity since there are interference effects to be taken into account. The interference fit fractions are defined for $i < j$ as:

$$FF_i = \frac{\int \int_{DP} 2\text{Re}[c_i c_j^* \mathcal{F}_i(m_{ij}^2, m_{jk}^2) \mathcal{F}_j(m_{ij}^2, m_{jk}^2)^*] dm_{ij}^2 dm_{jk}^2}{\int \int_{DP} |\mathcal{A}(m_{ij}^2, m_{jk}^2)|^2 dm_{ij}^2 dm_{jk}^2} \quad (4.33)$$

4.8.6 Dalitz Plot fitting

The probability density for signal events across the DP, taking into account efficiency effects, can be written as:

$$\mathcal{P}_{sig}(m_{K^+\pi^-}^2, m_{\eta_c\pi^-}^2) = \frac{\epsilon(m_{K^+\pi^-}^2, m_{\eta_c\pi^-}^2) |A(m_{K^+\pi^-}^2, m_{\eta_c\pi^-}^2)|^2}{\int \int_{DP} \epsilon(m_{K^+\pi^-}^2, m_{\eta_c\pi^-}^2) |A(m_{K^+\pi^-}^2, m_{\eta_c\pi^-}^2)|^2 dm_{K^+\pi^-}^2 dm_{\eta_c\pi^-}^2} \quad (4.34)$$

where $\epsilon(m_{K^+\pi^-}^2, m_{\eta_c\pi^-}^2)$ is the efficiency parameterised as described in Sec. 4.7. The Dalitz plot analysis is performed using the unbinned maximum likelihood fit formalism. The likelihood is given by:

$$\mathcal{L} = e^{-N} \prod_i^{N_e} \left[\sum_k N_k \mathcal{P}_k^i(m_{K\pi,i}^2, m_{\eta_c\pi,i}^2) \right] \quad (4.35)$$

where $N = \sum_k N_k$, N_k is the yield found from the 2D fit performed in sec.4.5.3 for event category k (signal or background), N_e is the total number of candidates in the data sample, and $\mathcal{P}_k^i$ is the PDF for the category k for event i , which consists of the product of the DP PDF and any other PDFs that are used to discriminate between signal and background. Similar to how the signal efficiency is implemented, the distribution of background categories in the SDP is depicted through a 2D histogram with uniform binning. The PDF value is subsequently computed by dividing the interpolated number of background events by the total integrated area of the histogram. The function $-2\log \mathcal{L}$ is minimised in an unbinned fit to the data in order to extract all of the parameters.

4.8.7 Fit quality test procedures

To assess the quality of the fits, a comparison between actual data and simulated toy events generated by **Laura++** based on the fit model, efficiency map, and incorporating background probability density functions is performed. These plots display the $K^+\pi^-$ non-resonant component and the $K_0^*(1430)$ collectively as a single S-wave component, accounting for interference terms between these contributions. For this analysis, toy experiments with a number of event equal to $N_{data} \cdot 100$ are generated. The $m_{K^+\pi^-}^2$, $m_{\eta_c\pi^-}^2$, and $m_{\eta_c K^+}^2$ distributions are compared between data and toys, with Run 1 and Run 2 data combined. A two-dimensional χ^2 test between data and toys is also performed, as described below.

4.8.8 χ^2 test

Using an adaptive binning algorithm, data and toy distributions are compared in bins of the SDP phase space to determine the value of the χ^2/ndf , where $\text{ndf} = N_{\text{bins}} - N_{\text{par}} - 1$, with N_{par} being the number of free parameters in the fit. The binning is constructed to ensure at least approximately 20 events per bin. The square of the pulls is used as the χ^2 statistic, where the pull in a given bin is calculated as:

$$\frac{N_{data} - N_{toys}}{\sqrt{N_{data}}} \quad (4.36)$$

4.8.9 Fit to the joint Run 1 and Run 2 sample

The JFIT framework [297] is used to perform a joint fit keeping Run 1 and Run 2 data and efficiency maps separated. A specific open source implementation of JFIT has been developed within **Laura++**.

The joint fitting process employs a master-worker architecture, with the master coordinating the fit by combining likelihood function values from two distinct workers. These workers are specific for Run 1 and Run 2 data, and the objective is to maximise the likelihood simultaneously, derived as the product of likelihoods from the respective Run 1 and Run 2 datasets. The fit parameters and their values are shared by the two workers. This approach utilises the cFit method, since the employment of the sFit strategy leads to unstable fits.

In conventional Dalitz Plot analyses, the natural width of final state particles are negligible. However, the η_c deviates from this norm with a width of approximately 30 MeV. To accurately consider this, modifications to the **Laura++** code are performed. The default fitter typically relies on two pairs of squared invariant-masses

(m_{23}^2 and m_{13}^2) and, using these masses along with parent and child masses, computes all necessary kinematic variables event by event (e.g., m_{12}^2 , helicity angles, etc.). This adaptation involves dynamically incorporating the $m_{p\bar{p}}$ invariant-mass between cached variables on an event-by-event basis.

This modification ensures that, instead of relying on a hardcoded m_{η_c} value, the fitter utilises the $m_{p\bar{p}}$ value specific to the event under consideration for the computation of kinematic variables. In the generation of toys used for comparing data distributions with those resulting from the fit, initially a set of $m_{p\bar{p}}$ values is generated following the probability density functions used in the 2D fit $m_{p\bar{p}}$ projection (a relativistic Breit-Wigner convoluted with a double crystall-ball for the η_c signal and an exponential for the backgrounds). Subsequently, the code has been adjusted to account for these values during the event generation process. This approach guarantees consistency in phase space ranges between data and toys.

Input data

The input data to the amplitude fit are the events contained in the $m_{p\bar{p}K^+\pi^-}$ and $m_{p\bar{p}}$ ranges used for the 2D mass fit, that are the $5220 < m_{p\bar{p}K^+\pi^-} < 5340$ MeV and $2908 < m_{p\bar{p}} < 3058$ MeV. The D^0 veto used in the 2D mass fit is also applied within the amplitude analysis. This is handled by **Laura++** setting the signal efficiency to zero inside a vetoed region. Since the effect of explicit vetoes in the phase space is separately accounted for, the D^0 veto is not applied when constructing the efficiency histograms as already described in Sec. 4.7.

To mitigate issues arising from imperfect efficiency parameterisation at the DP boundaries, a 70 MeV veto is applied to both the lower and upper edges of the DP, effectively excluding regions where any of the invariant-masses, m_π , m_K , or $m_{K\pi}$, are within 70 MeV of their kinematic limits. The signal and backgrounds yields, resulting from the 2D mass fit, are scaled accordingly automatically by **Laura++**. In order to avoid steep variations in the efficiency parameterisations, this cut is not applied when building the efficiency histograms. This cut is also introduced in the fitter vetoing the borders of the DP.

Background parameterisation

The 2D mass fits to the Run 1 and Run 2 data are used to determine the sWeights corresponding to the combinatorial background and non-resonant background components to build the corresponding histograms, parameterised using the SDP variables. The background histograms are shown in figs. 4.37 for Run 1 and Run 2. These histograms are interpolated with a cubic spline before the amplitude fit.

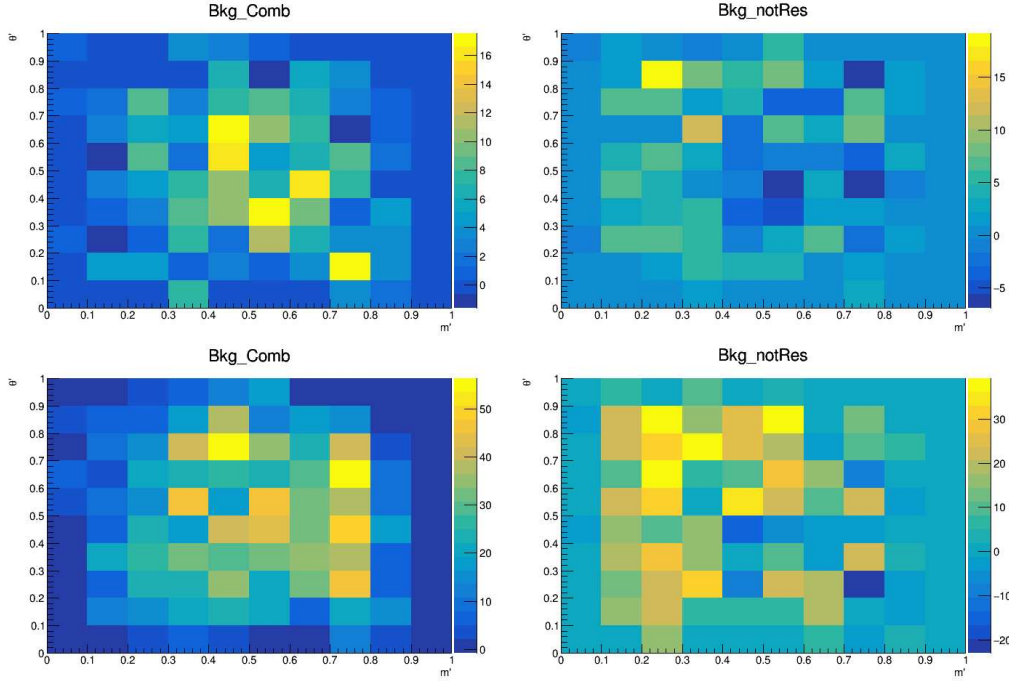


Figure 4.37: Background histograms for Run 1 (top) and Run 2 (bottom), left: combinatorial background; right: nonresonant background.

Default model

In the amplitude fit, both signal and background yields are fixed. The construction of the default model involves systematically incorporating well-established K^* states one by one until no significant improvements in the likelihood are observed. All the masses and widths of the K^* resonances are fixed to PDG values [66] in order to define the default model. The fit is repeated 200 times randomising the values of the isobar coefficients in order to check for local minima, common in DP fits, and find the absolute minima. In Table 4.17 the negative-log-likelihood (NLL) of different models, including every time a new K^* resonance, are shown. The $K_3^*(1780)$ and $K_4^*(2045)$ are not significant and are excluded from the default model (NLL improvement less than 5).

The parameterisation of the $K\pi$ S-wave employs the LASS PDF. The scattering length and effective range of the non-resonant component are left unconstrained. The mass and width of the resonant part are constrained to their PDG values [66]. Once the global minimum of the negative log-likelihood is reached, the four parameters of the LASS function are fixed to their best-fit values. Ensuring a higher stability and faster convergence of the fit. The fits are then repeated using these fixed parameters. The resonance used, with the function used to parameterise

Resonances used in the model	NLL	ΔNLL
$K^*(892)$, $K_0^*(1430)$	-40921.4	
Added $K_0^*(1950)$	-41026.1	-104.7
Added $K^*(1410)$	-41059	-32.9
Added $K_2^*(1430)$	-41074.8	-15.8
Added $K^*(1680)$	-41089.4	-15
Added $K_3^*(1780)$	-41089.4	0
Added $K_4^*(2045)$	-41089.4	0

Table 4.17: NLL values of the fit obtained using different K^* models.

them, are:

- $K^*(892)$ - relativistic Breit-Wigner
- $K^*(1410)$ - relativistic Breit-Wigner
- $K_0^*(1430)$ - LASS function
- $K_2^*(1430)$ - relativistic Breit-Wigner
- $K^*(1680)$ - relativistic Breit-Wigner
- $K_0^*(1950)$ - relativistic Breit-Wigner

As depicted in Figure 4.30, the $K^+\pi^-$ spectrum is predominantly influenced by $K^*(892)$. $K^*(892)$ serves as the reference amplitude, with its amplitude and phase fixed at 1.00 and 0.00, respectively. The isobar coefficients of additional contributions are expressed relative to those of the $K^*(892)$ meson. 200 fits using the default model previously described are performed. The absolute minima is found to be $NLL = -41104.3$. The resulting isobar coefficients are shown in Table 4.18. The full half-matrix of interference fit fractions is shown in Table 4.19. The LASS parameter are shown in Table 4.20. The errors of the fit fractions are calculated by generating 1000 toys and fitting them with same model. The standard deviations of each fit fraction is used as error on the fit fraction.

To check the fit quality, **Laura++** is used as generator of events: the $B^0 \rightarrow \eta_c K^+ \pi^-$ events are generated according to the model with the free parameters fixed to the fit output. Events are generated according to Run 1 and Run 2 efficiency maps and number of events. The toy events are generated as explained in Sec. 4.8.7. The three invariant-mass distribution are then normalised accordingly to the number of data event. The DP projections comparisons are shown in Figures 4.38. The pull distribution is shown in fig 4.39 and the χ^2/ndf computed as the square of the pulls is $425.286/(324 - 14 - 1) = 1.38$. The comparison of the first six unnormalised Legendre moments between data and toys is reported in Figure 4.40, 4.41 and 4.42.

Resonance	Amplitude	Phase
$K^*(892)$	1.00	0.00
$K^*(1410)$	0.30 ± 0.04	-0.15 ± 0.13
LASS	0.85 ± 0.03	2.92 ± 0.07
$K_2^*(1430)$	0.28 ± 0.03	-0.73 ± 0.10
$K^*(1680)$	0.21 ± 0.04	-2.93 ± 0.22
$K_0^*(1950)$	0.22 ± 0.03	-1.61 ± 0.29

Table 4.18: Isobar coefficients resulting from the amplitude fit using the default model.

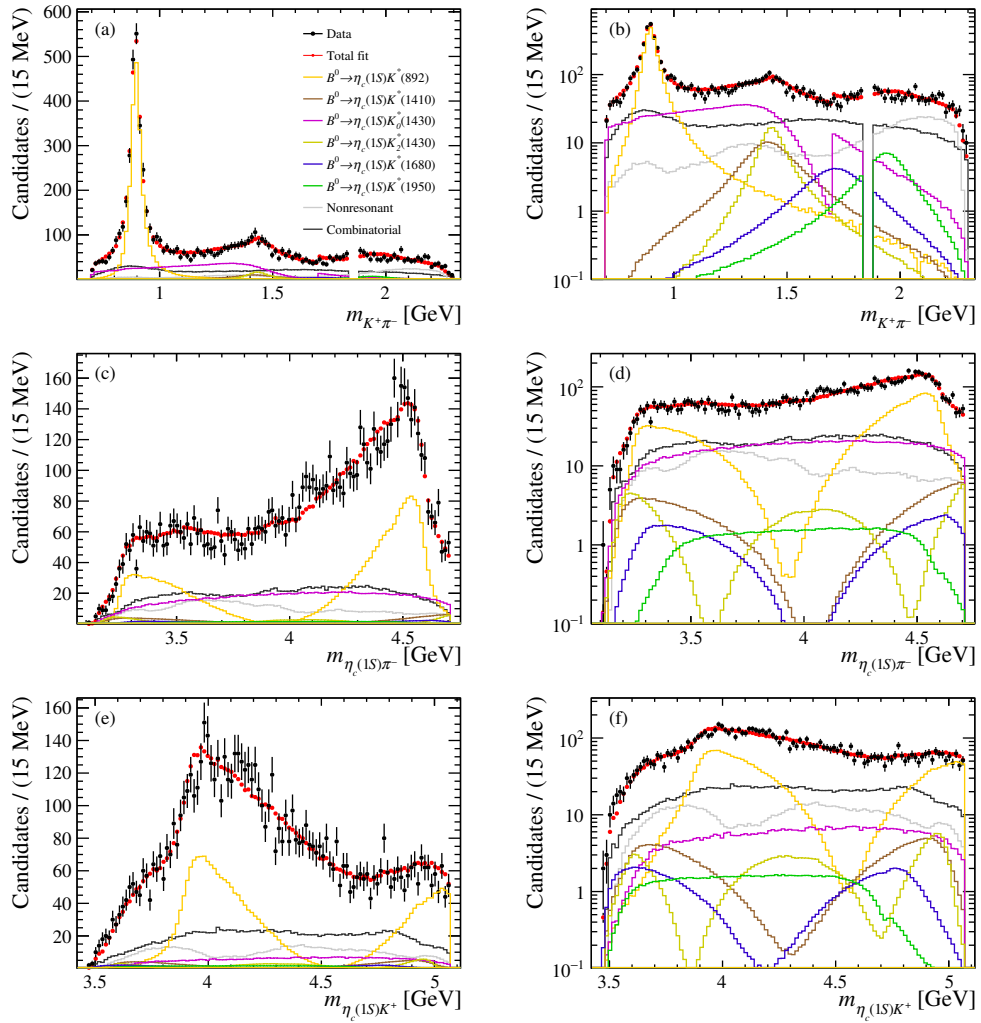


Figure 4.38: Projections of the data and amplitude fit using the default model with a $J^P = 1^-$ onto (a) $m_{K^+\pi^-}$, (c) $m_{\eta_c\pi^-}$ and (e) $m_{\eta_c K^+}$, with the same projections shown in (b), (d) and (f) on a logarithmic vertical-axis scale. The veto of $B^0 \rightarrow p\bar{p}\bar{D}^0$ decays is visible in plots (a) and (b).

	$K^*(892)$	$K^*(1410)$	LASS	$K_2^*(1430)$	$K^*(1680)$	$K_0^*(1950)$
$K^*(892)$	0.493 ± 0.012	-0.021 ± 0.018	0.0	0.0	0.008 ± 0.012	0.0
$K^*(1410)$		0.045 ± 0.014	0.0	0.0	-0.032 ± 0.015	0.0
LASS			0.356 ± 0.055	0.0	0.0	0.070 ± 0.103
$K_2^*(1430)$				0.037 ± 0.009	0.0	0.0
$K^*(1680)$					0.021 ± 0.009	0.0
$K_0^*(1950)$						0.024 ± 0.005

Table 4.19: Half-matrix of the fit fractions resulting from the fit using the default model.

Parameter	Value	Default value [298]
r	$4.04 \pm 1.05 \text{ GeV}^{-1}$	$2.07 \pm 0.10 \text{ GeV}^{-1}$
a	$4.34 \pm 0.80 \text{ GeV}^{-1}$	$3.32 \pm 0.34 \text{ GeV}^{-1}$
$m_{K_0^*(1430)}$	$1434 \pm 22 \text{ MeV}$	$1425 \pm 50 \text{ MeV}$
$\Gamma_{K_0^*(1430)}$	$410 \pm 44 \text{ MeV}$	$270 \pm 80 \text{ MeV}$

Table 4.20: Parameters of the LASS PDF resulting from the fit using the default model. In the third column are shown the values quoted by the LASS experiment for the a and r parameters and the mass and width of the $K_0^*(1430)$ reported in [66].

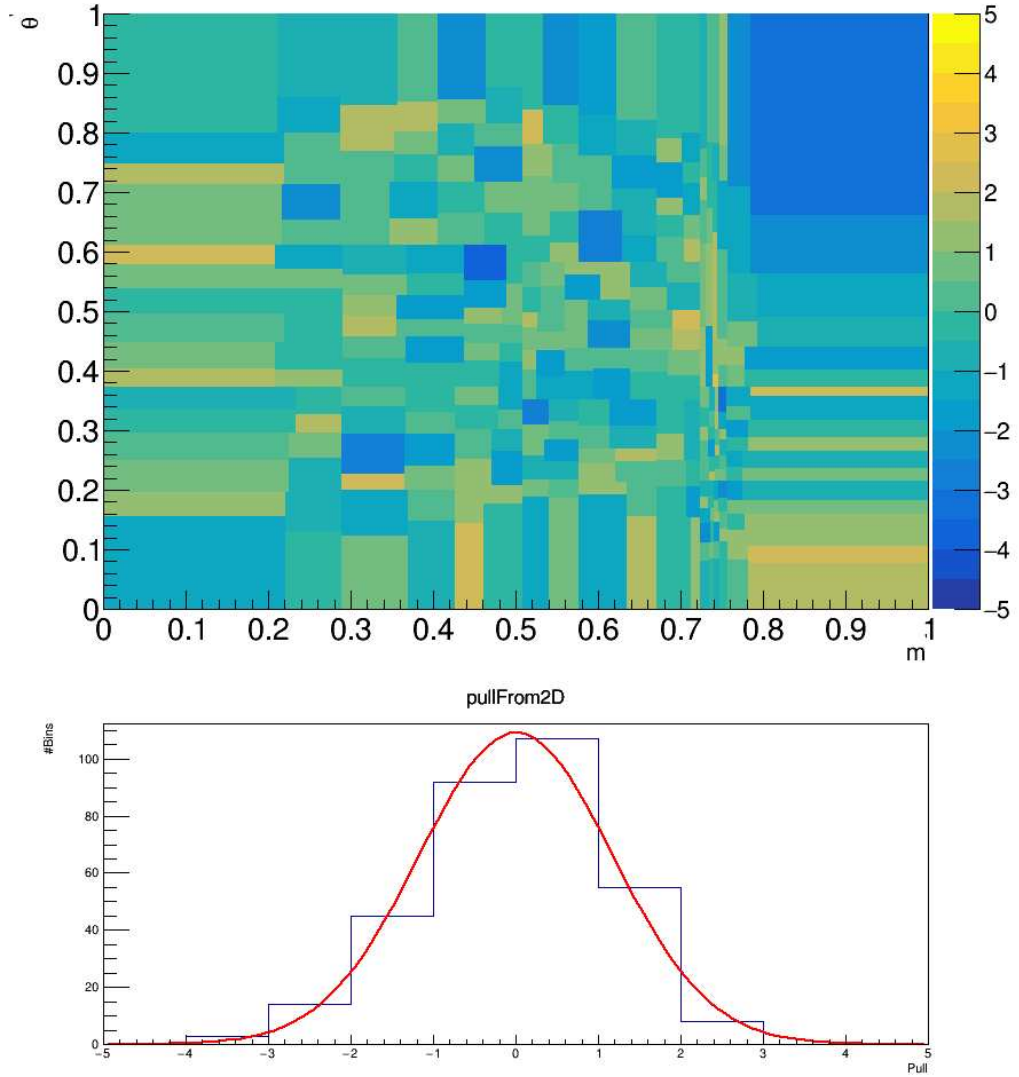


Figure 4.39: Top: 2D pull distribution of the fit with the default model . Bottom: 1D pull distribution computed from the bin contents of the 2D pull distribution. This distribution is fitted with a gaussian obtaining $\mu = 0.00 \pm 0.07$ and $\sigma = 1.17 \pm 0.05$.

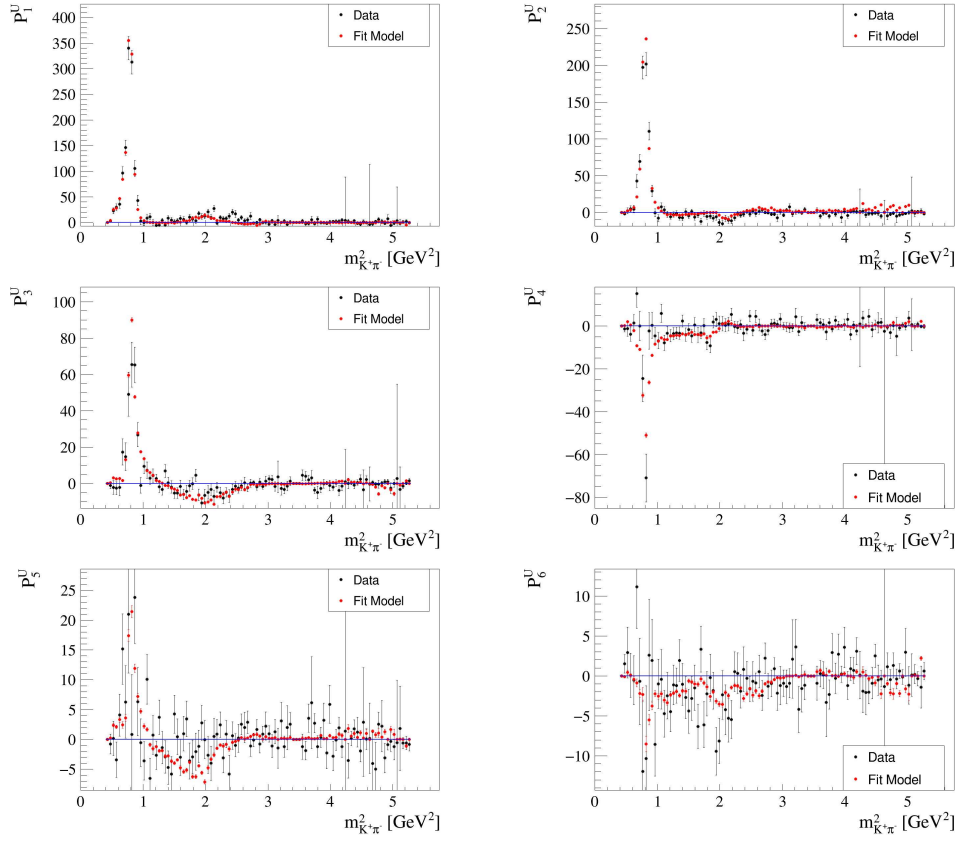


Figure 4.40: Comparison of the first six $K^+\pi^-$ unnormalised Legendre moments between background subtracted data (black) and the default model generated toys signal component (red).

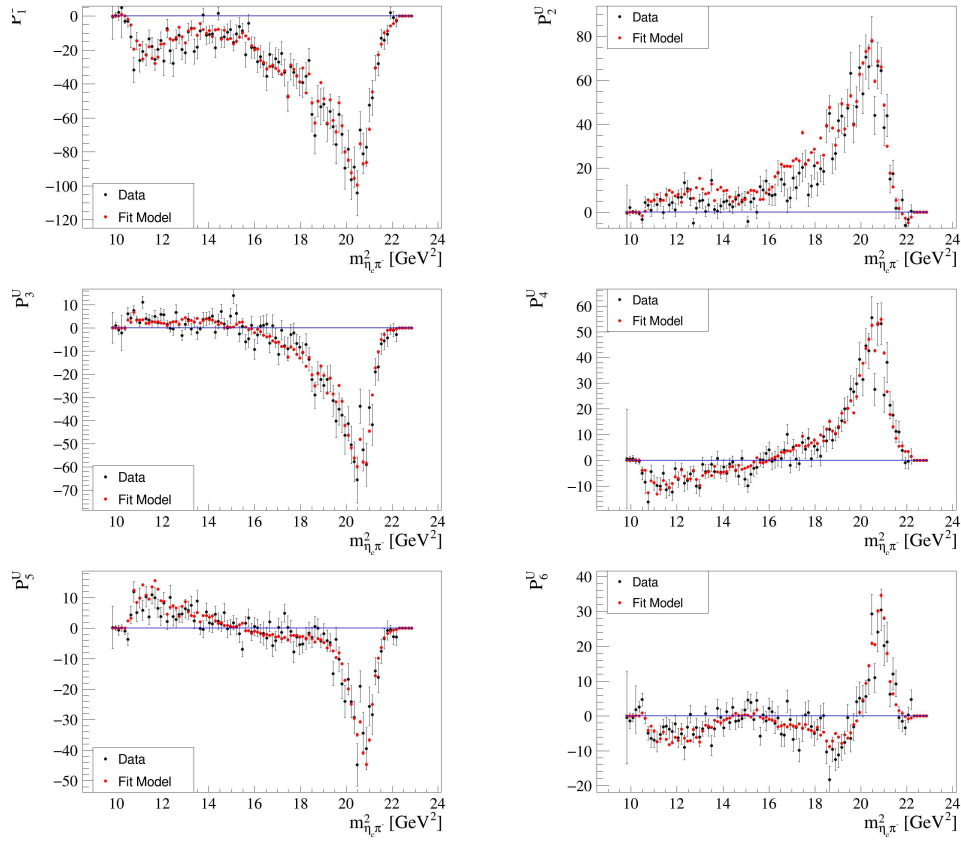


Figure 4.41: Comparison of the first six $\eta_c \pi^-$ unnormalised Legendre moments between background subtracted data (black) and the default model generated toys signal component (red).

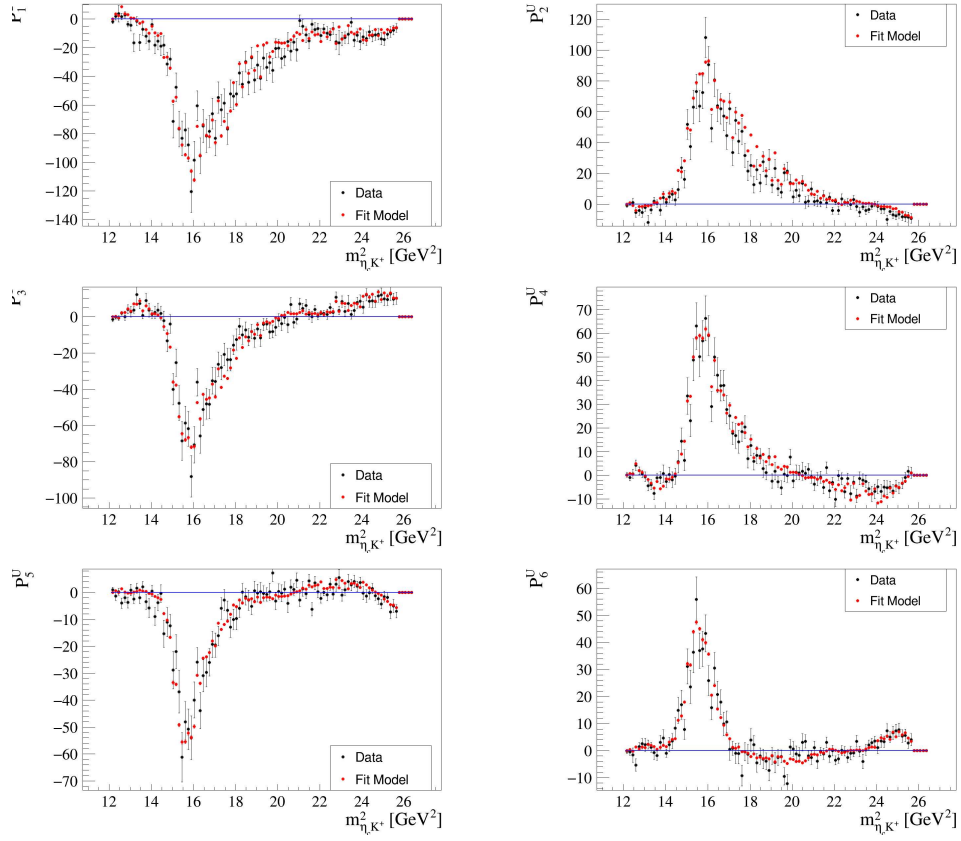


Figure 4.42: Comparison of the first six $\eta_c K^+$ unnormalised Legendre moments between background subtracted data (black) and the default model generated toys signal component (red).

4.9 Addition of exotic amplitudes and significance determination

An exotic contribution, $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$, is added to the default model with mass and width as floating parameter. Two different hypothesis of quantum numbers are tested, $J^P = 0^+$ and $J^P = 1^-$. The LASS parameter are managed as in the default model, scattering length and effective range are left unconstrained and mass and width are gaussian constrained and then fixed to results of the best-fit. This new model is called extended model.

4.9.1 Addition of a $J^P = 0^+$ $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component

The absolute minimum is found at $NLL = -41112.4$, a variation equal to $\Delta NLL = -8.1$. The LASS parameter found are reported in Table 4.21. The isobar coefficients and the mass and width of the $T_{c\bar{c}}$ are reported in tables 4.22 and 4.23, respectively. The full half-matrix of the fit fraction is reported in Table 4.24. The DP projections comparisons are shown in Figures 4.43. The pull distribution is shown in fig 4.44 and the χ^2/ndof computed as the square of the pulls is $418.526/(324 - 18 - 1) = 1.37$. The comparison of the first six unnormalised Legendre moments between data and toys is reported in Figure 4.45, 4.46 and 4.47.

Parameter	Value	Default value [298]
r	$3.88 \pm 0.92 \text{ GeV}^{-1}$	$2.07 \pm 0.10 \text{ GeV}^{-1}$
a	$4.45 \pm 0.79 \text{ GeV}^{-1}$	$3.32 \pm 0.34 \text{ GeV}^{-1}$
40 $m_{K_0^*(1430)}$	$1435 \pm 19 \text{ MeV}$	$1425 \pm 50 \text{ MeV}$
$\Gamma_{K_0^*(1430)}$	$385 \pm 44 \text{ MeV}$	$270 \pm 80 \text{ MeV}$

Table 4.21: Parameters of the LASS PDF resulting from the fit using the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 0^+$ added.. In the third column are shown the values quoted by the LASS experiment for the a and r parameters and the mass and width of the $K_0^*(1430)$ reported in [66].

Resonance	Amplitude	Phase
$K^*(892)$	1.00	0.00
$K^*(1410)$	0.29 ± 0.04	-0.97 ± 0.14
LASS	0.81 ± 0.03	2.91 ± 0.07
$K_2^*(1430)$	0.28 ± 0.03	-0.82 ± 0.11
$K^*(1680)$	0.24 ± 0.04	-2.85 ± 0.20
$K_0^*(1950)$	0.24 ± 0.03	-1.53 ± 0.27
$T_{c\bar{c}}(4100)^-$	0.10 ± 0.03	2.60 ± 0.39

Table 4.22: Isobar coefficients resulting from the amplitude fit using the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 0^+$ added.

Parameter	Value [MeV]
$M_{T_{c\bar{c}}(4100)^-}$	4067 ± 13
$\Gamma_{T_{c\bar{c}}(4100)^-}$	137 ± 78

Table 4.23: Mass and width of the $T_{c\bar{c}}(4100)^-$ resonance candidate resulting from the amplitude fit using the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 0^+$ added.

	$K^*(892)$	$K^*(1410)$	LASS	$K_2^*(1430)$	$K^*(1680)$	$K_0^*(1950)$	$T_{c\bar{c}}(4100)^-$
$K^*(892)$	0.491 ± 0.012	-0.018 ± 0.013	0	0	0.005 ± 0.010	0	0.008 ± 0.005
$K^*(1410)$		0.043 ± 0.014	0	0	-0.035 ± 0.015	0	0.002 ± 0.002
LASS			0.326 ± 0.035	0	0	0.073 ± 0.052	0.008 ± 0.008
$K_2^*(1430)$				0.037 ± 0.008	0	0	0.002 ± 0.001
$K^*(1680)$					0.024 ± 0.009	0	-0.002 ± 0.002
$K_0^*(1950)$						0.029 ± 0.007	-0.001 ± 0.005
$T_{c\bar{c}}(4100)^-$							0.006 ± 0.003

Table 4.24: Half-matrix of the fit fractions resulting from the fit using the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 0^+$ added.

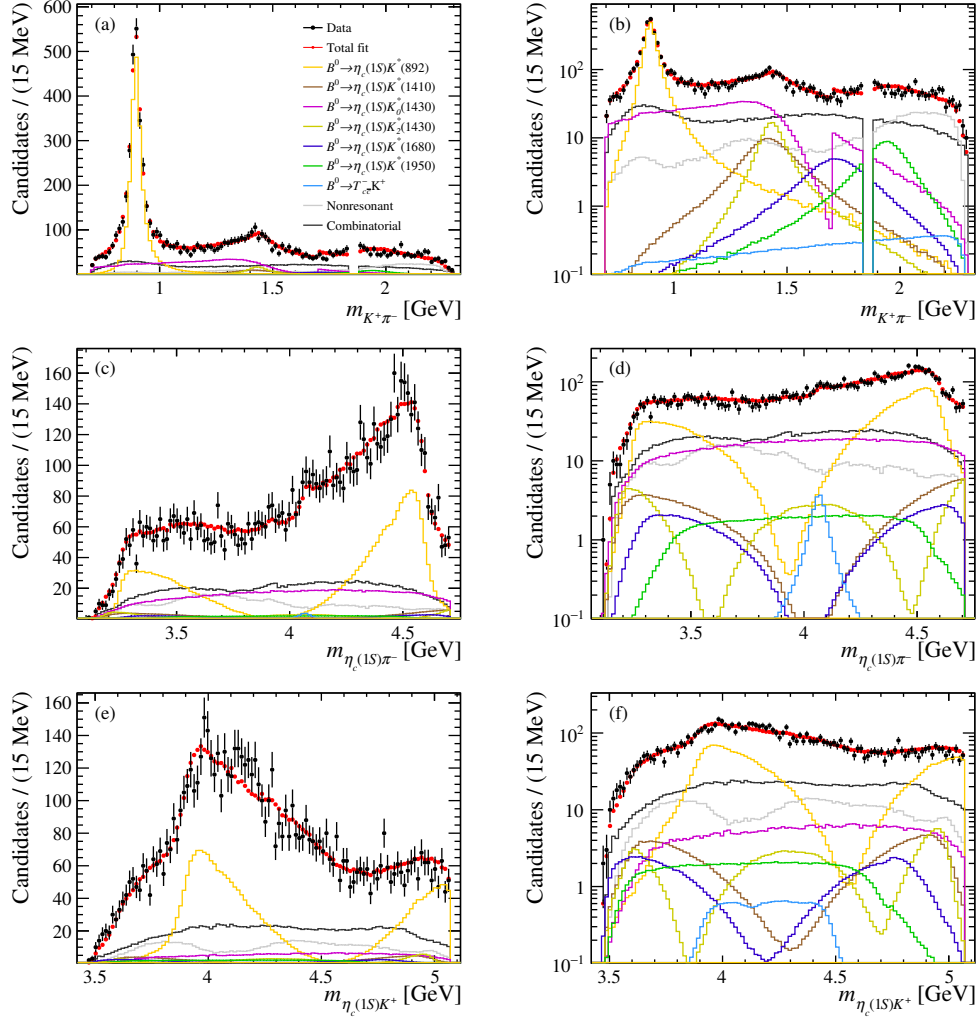


Figure 4.43: Projections of the data and amplitude fit using the extended model with a $J^P = 0^+$ onto (a) $m_{K^+\pi^-}$, (c) $m_{\eta_c\pi^-}$ and (e) $m_{\eta_c K^+}$, with the same projections shown in (b), (d) and (f) on a logarithmic vertical-axis scale. The veto of $B^0 \rightarrow p\bar{p}\bar{D}^0$ decays is visible in plots (a) and (b).

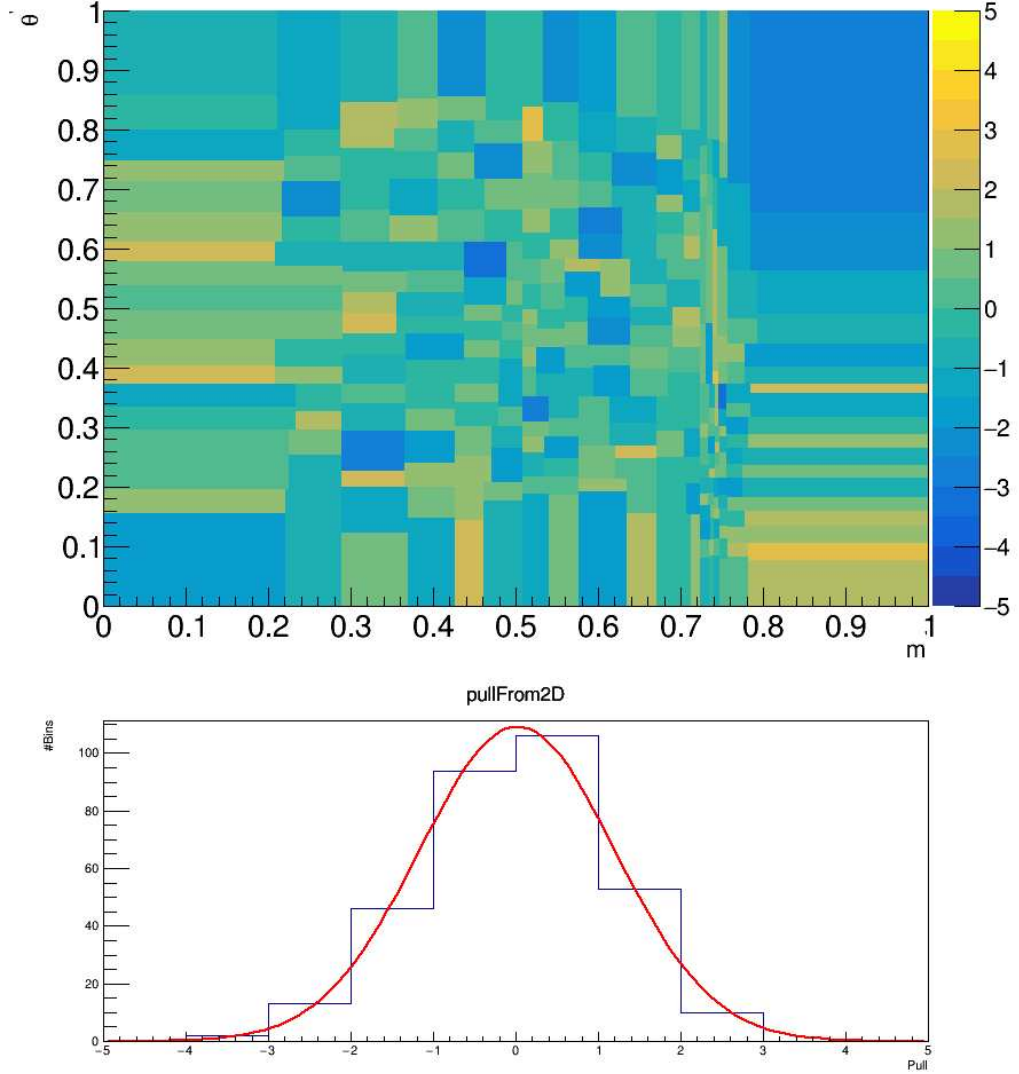


Figure 4.44: Top: 2D pull distribution of the fit with the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 0^+$ added. Bottom: 1D pull distribution computed from the bin contents of the 2D pull distribution. This distribution is fitted with a gaussian obtaining $\mu = 0.02 \pm 0.07$ and $\sigma = 1.18 \pm 0.05$.

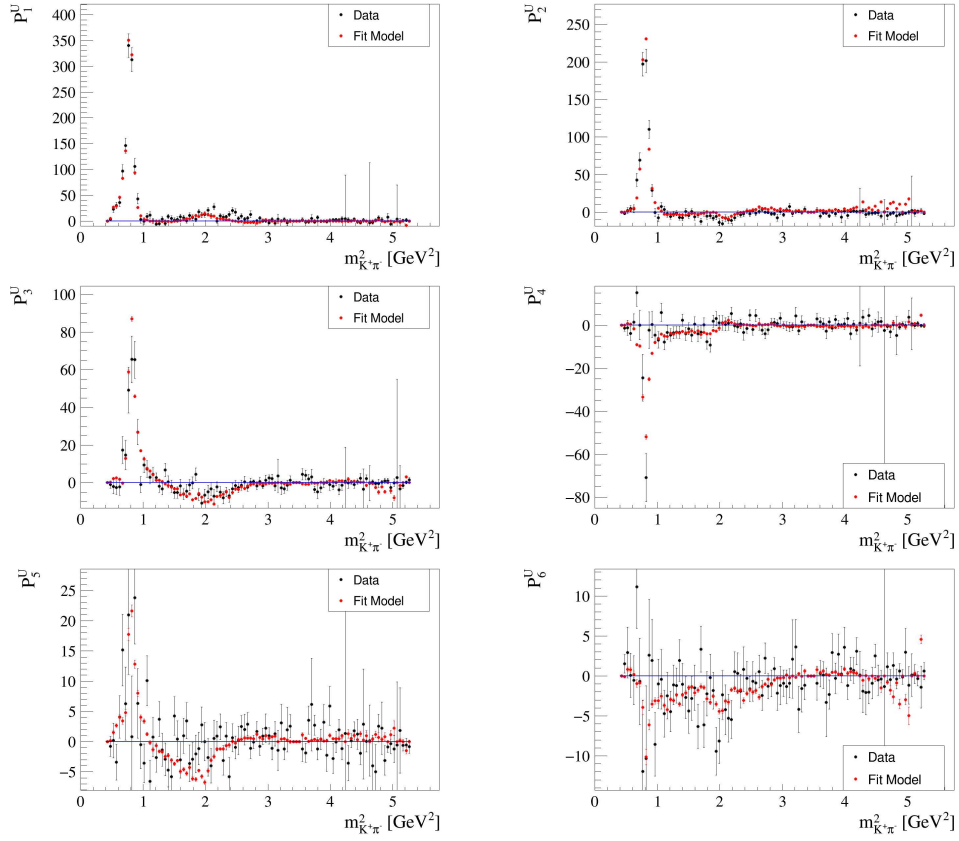


Figure 4.45: Comparison of the first six $K^+\pi^-$ unnormalised Legendre moments between background subtracted data (black) and the default model with an exotic $T_{cc}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 0^+$ added generated toys signal component (red).

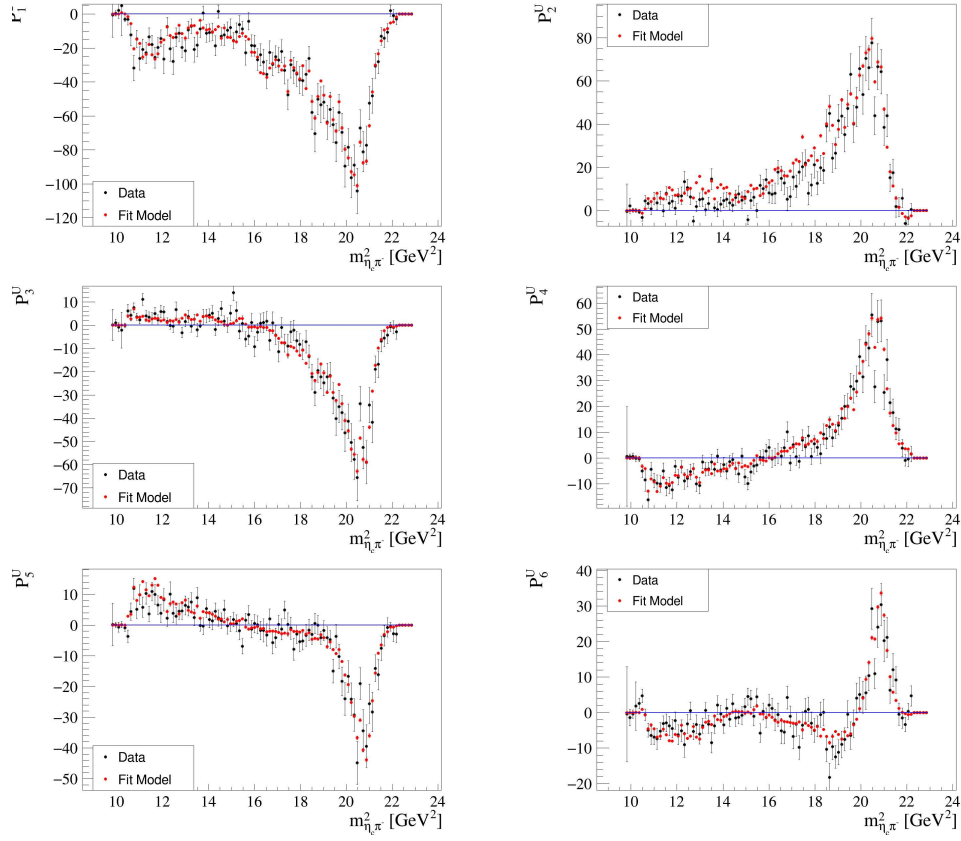


Figure 4.46: Comparison of the first six $\eta_c \pi^-$ unnormalised Legendre moments between background subtracted data (black) and the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 0^+$ added generated toys signal component (red).

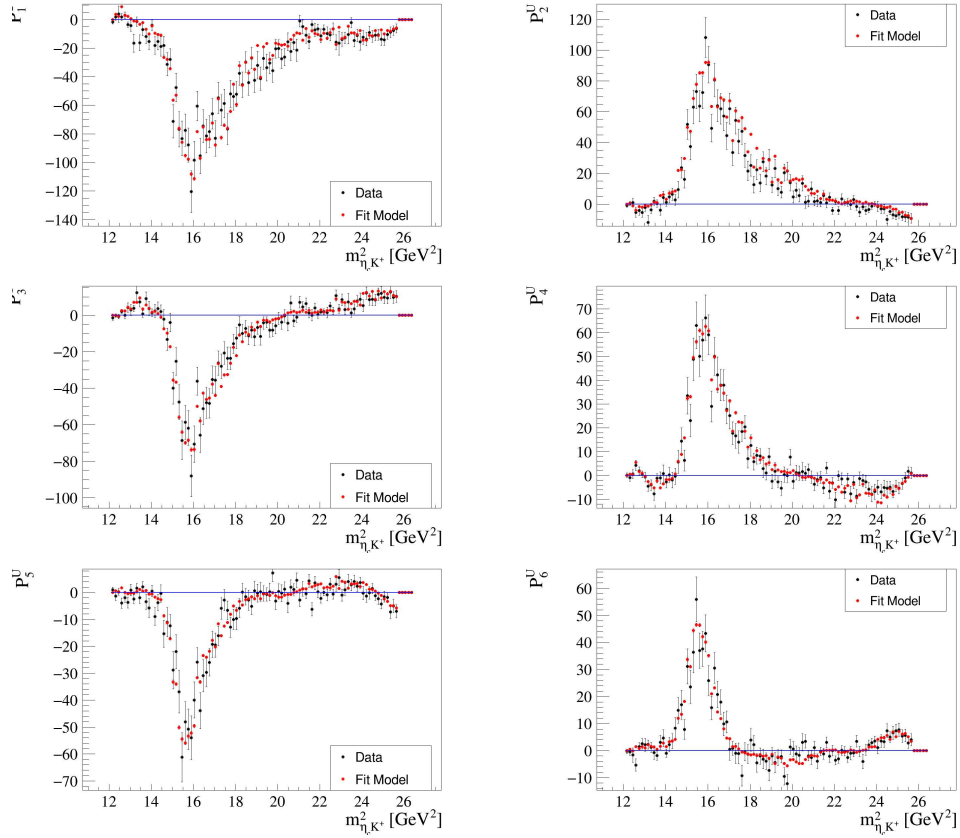


Figure 4.47: Comparison of the first six $\eta_c K^+$ unnormalised Legendre moments between background subtracted data (black) and the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 0^+$ added generated toys signal component (red).

4.9.2 Addition of a $J^P = 1^-$ $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component

The absolute minimum is found at $NLL = -41118.8$, a variation equal to $\Delta NLL = -14.5$. The LASS parameter found are reported in Table 4.25. The isobar coefficients and the mass and width of the $T_{c\bar{c}}$ are reported in tables 4.26 and 4.27, respectively. The full half-matrix of the fit fraction is reported in Table 4.28. The DP projections comparisons are shown in Figures 4.48. The pull distribution is shown in fig 4.49 and the χ^2/ndof computed as the square of the pulls is $400.694/(324 - 18 - 1) = 1.31$. The comparison of the first six unnormalised Legendre moments between data and toys is reported in Fig. 4.50, 4.52 and 4.51.

Parameter	Value	Default value [298]
r	$4.55 \pm 1.05 \text{ GeV}^{-1}$	$2.07 \pm 0.10 \text{ GeV}^{-1}$
a	$4.23 \pm 0.74 \text{ GeV}^{-1}$	$3.32 \pm 0.34 \text{ GeV}^{-1}$
$m_{K_0^*(1430)}$	$1434 \pm 19 \text{ MeV}$	$1425 \pm 50 \text{ MeV}$
$\Gamma_{K_0^*(1430)}$	$359 \pm 37 \text{ MeV}$	$270 \pm 80 \text{ MeV}$

Table 4.25: Parameters of the LASS PDF resulting from the fit using the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 1^-$ added.. In the third column are shown the values quoted by the LASS experiment for the a and r parameters and the mass and width of the $K_0^*(1430)$ reported in [66].

Resonance	Amplitude	Phase
$K^*(892)$	1.00	0.00
$K^*(1410)$	0.28 ± 0.03	-0.04 ± 0.15
LASS	0.80 ± 0.03	2.94 ± 0.07
$K_2^*(1430)$	0.28 ± 0.03	-0.86 ± 0.11
$K^*(1680)$	0.18 ± 0.05	-2.82 ± 0.23
$K_0^*(1950)$	0.21 ± 0.03	-0.92 ± 0.35
$T_{c\bar{c}}(4100)^-$	0.15 ± 0.03	-3.14 ± 0.58

Table 4.26: Isobar coefficients resulting from the amplitude fit using the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 1^-$ added.

Parameter	Value [MeV]
$M_{T_{c\bar{c}}(4100)^-}$	4106 ± 23
$\Gamma_{T_{c\bar{c}}(4100)^-}$	514 ± 166

Table 4.27: Mass and width of the $T_{c\bar{c}}(4100)^-$ resonance candidate resulting from the amplitude fit using the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 1^-$ added.

	$K^*(892)$	$K^*(1410)$	LASS	$K_2^*(1430)$	$K^*(1680)$	$K_0^*(1950)$	$T_{c\bar{c}}(4100)^-$
$K^*(892)$	0.491 ± 0.013	-0.014 ± 0.011	0	0	0.003 ± 0.009	0	0.011 ± 0.007
$K^*(1410)$		0.039 ± 0.009	0	0	-0.026 ± 0.012	0	-0.007 ± 0.003
LASS			0.316 ± 0.050	0.0	0	0.051 ± 0.057	0.034 ± 0.016
$K_2^*(1430)$				0.040 ± 0.009	0	0	0.001 ± 0.002
$K^*(1680)$					0.015 ± 0.009	0	0.005 ± 0.003
$K_0^*(1950)$						0.022 ± 0.005	0.007 ± 0.002
$T_{c\bar{c}}(4100)^-$							0.011 ± 0.005

Table 4.28: Half-matrix of the fit fractions resulting from the fit using the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 1^-$ added.

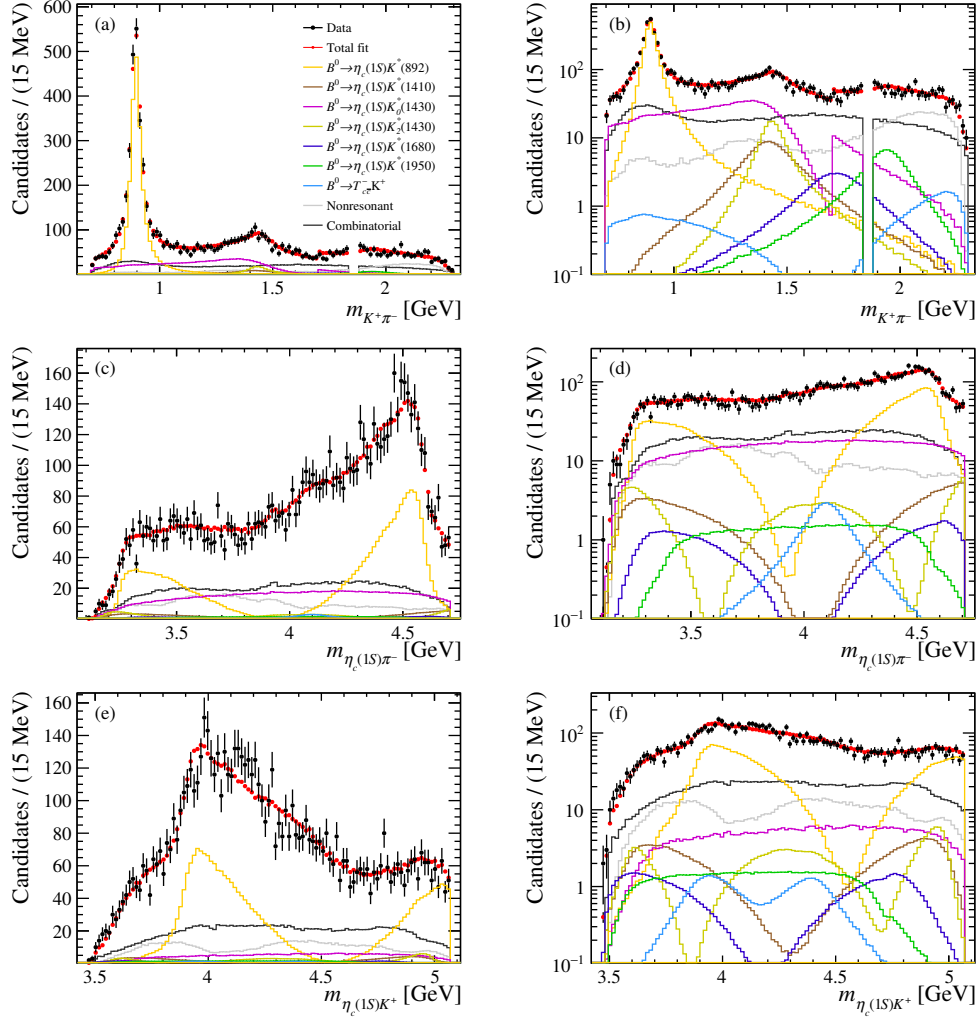


Figure 4.48: Projections of the data and amplitude fit using the extended model with a $J^P = 1^-$ onto (a) $m_{K^+\pi^-}$, (c) $m_{\eta_c\pi^-}$ and (e) $m_{\eta_c K^+}$, with the same projections shown in (b), (d) and (f) on a logarithmic vertical-axis scale. The veto of $B^0 \rightarrow p\bar{p}\bar{D}^0$ decays is visible in plots (a) and (b).

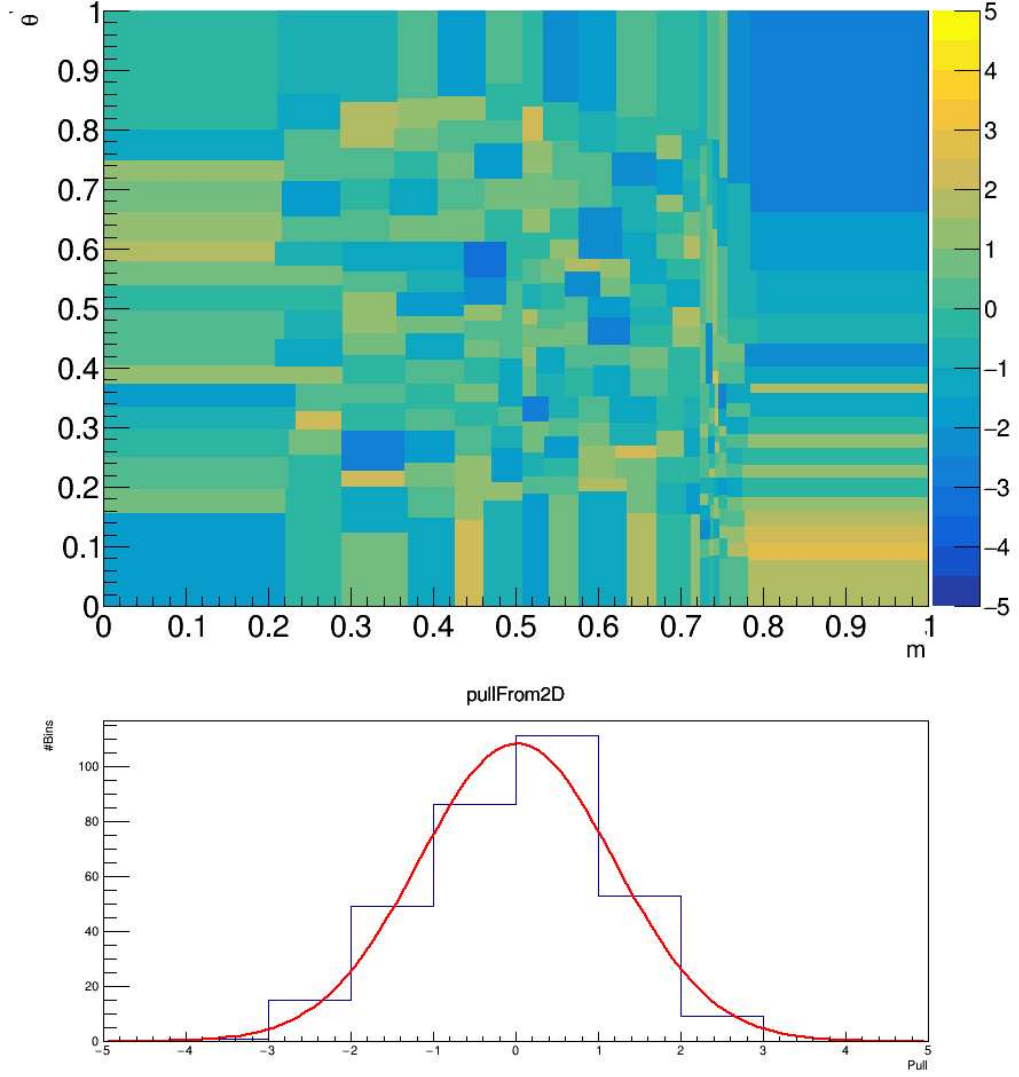


Figure 4.49: Top: 2D pull distribution of the fit with the default model with an exotic $T_{c\bar{c}}(4100)^- \rightarrow \eta_c \pi^-$ component $J^P = 1^-$ added. Bottom: 1D pull distribution computed from the bin contents of the 2D pull distribution. This distribution is fitted with a gaussian obtaining $\mu = 0.02 \pm 0.07$ and $\sigma = 1.12 \pm 0.05$.

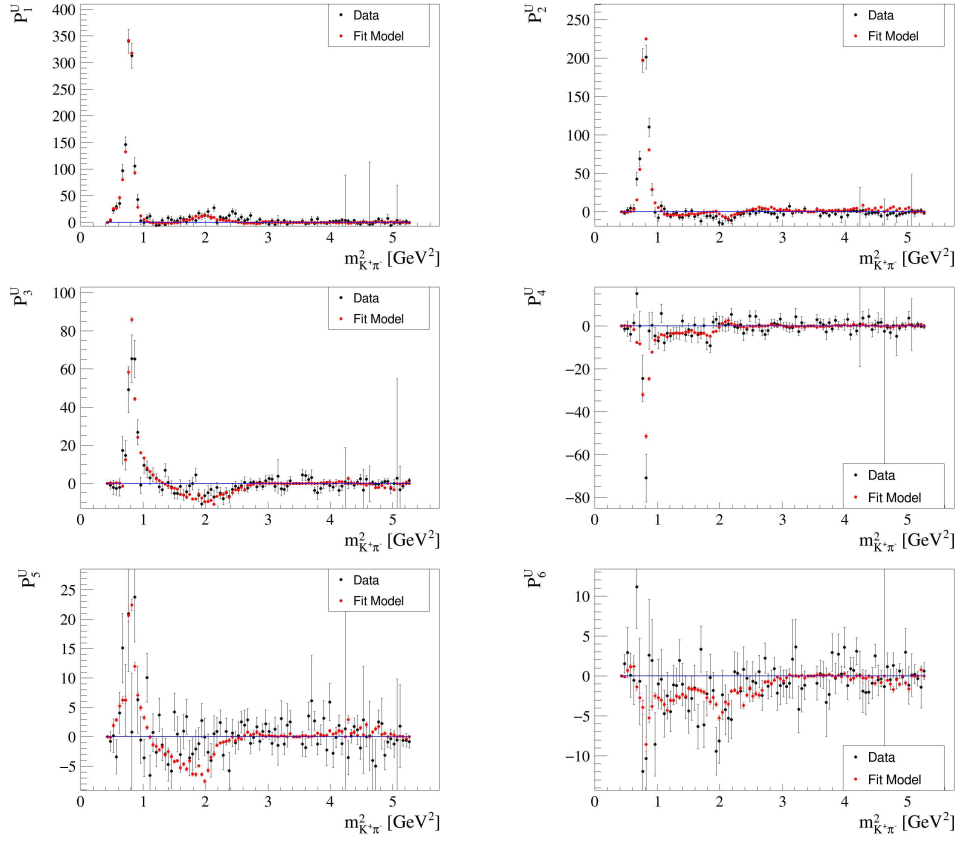


Figure 4.50: Comparison of the first six $K^+\pi^-$ unnormalised Legendre moments between background subtracted data (black) and the default model with an exotic $Z_c^- \rightarrow \eta_c \pi^-$ component $J^P = 1^-$ added generated toys signal component (red).

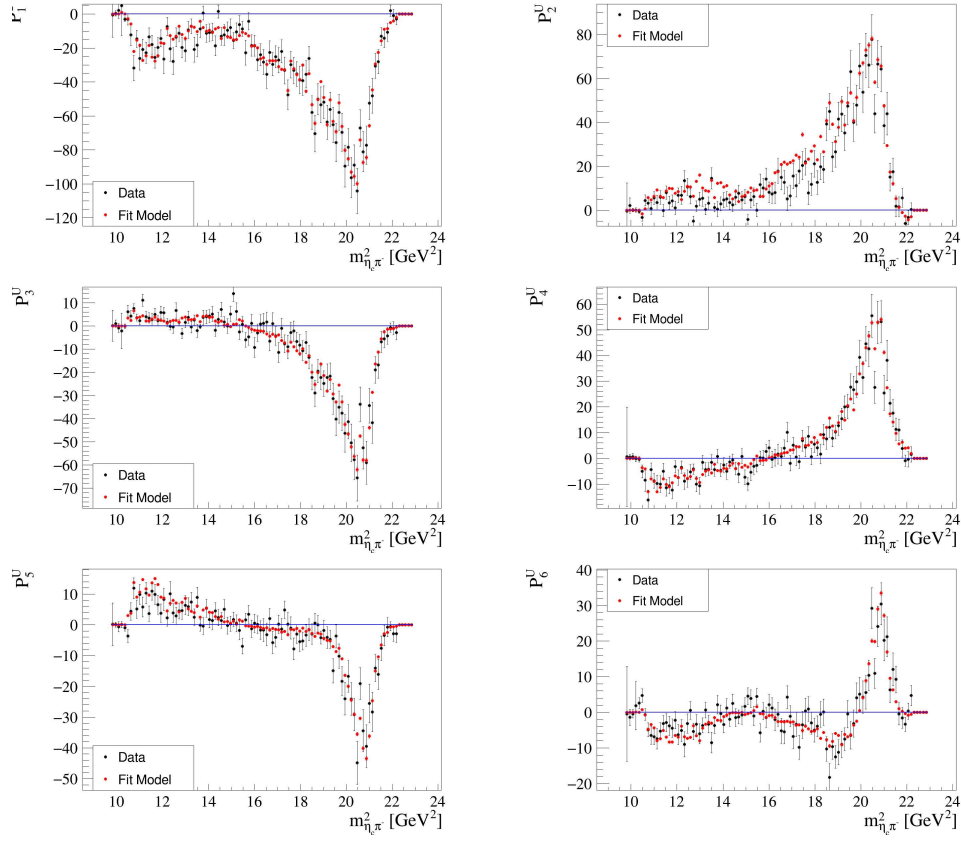


Figure 4.51: Comparison of the first six $\eta_c \pi^-$ unnormalised Legendre moments between background subtracted data (black) and the default model with an exotic $Z_c^- \rightarrow \eta_c \pi^-$ component $J^P = 1^-$ added generated toys signal component (red).

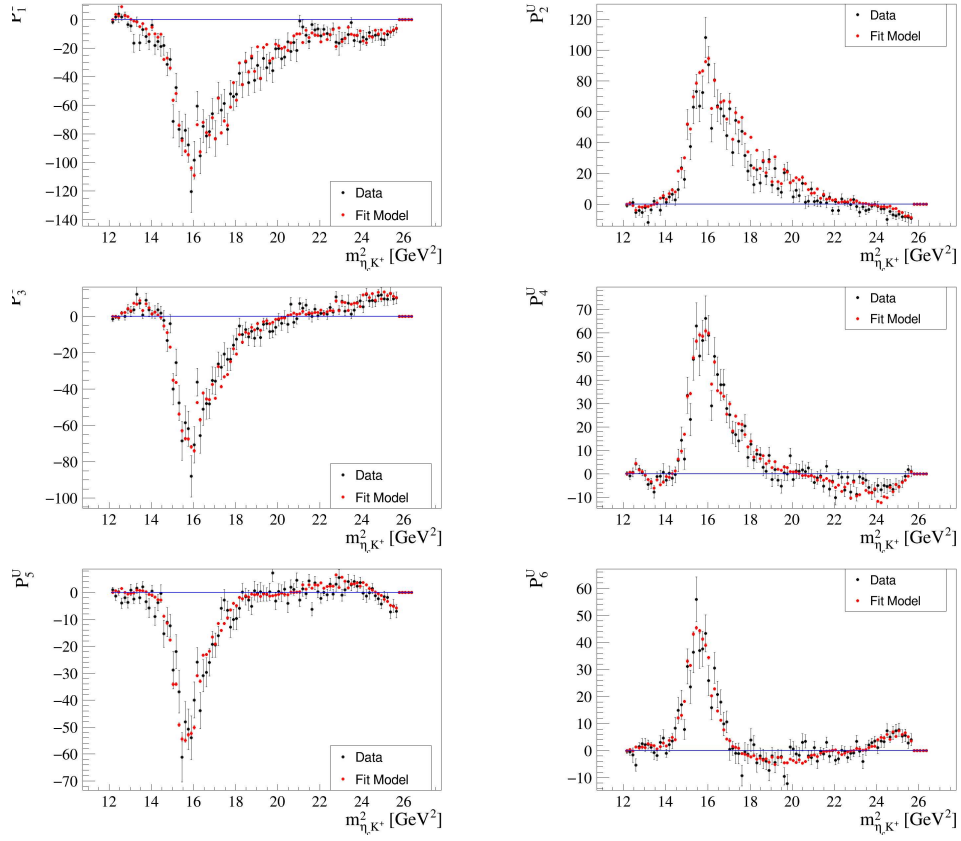


Figure 4.52: Comparison of the first six $\eta_c K^+$ unnormalised Legendre moments between background subtracted data (black) and the default model with an exotic $Z_c^- \rightarrow \eta_c \pi^-$ component $J^P = 1^-$ added generated toys signal component (red).

4.9.3 Significances determination

In order to calculate the sensitivity to a $T_{c\bar{c}}(4100)^-$ state in the data, discrimination between fits with and without the exotic resonance is of fundamental importance. Two hypothesis are defined, the first, where the $T_{c\bar{c}}(4100)^-$ does not exist (null hypothesis H_0), and the second, where the $T_{c\bar{c}}(4100)^-$ does exist (alternative hypothesis H_1). The probability to reject a true null hypothesis is called p -value: the lower the p -value the more significant the $T_{c\bar{c}}(4100)^-$ is, whatever statistical test is used. The choice of the statistical test is driven by the need to have a low probability of accepting a false null hypothesis.

The test used in this analysis to distinguish between H_0 and H_1 is the profile likelihood ratio test. The profile likelihood ratio is defined as

$$\lambda = -2 \log \frac{\mathcal{L}(H_0)}{\mathcal{L}(H_1)} = \Delta(-2 \log \mathcal{L}) \quad (4.37)$$

To calculate the p -value for the profile likelihood ratio value obtained on data, the PDF of λ must be known under the null hypothesis. Wilks' theorem states that, under some regularity conditions, λ asymptotically follows a χ^2 distribution with a number of degrees of freedom equals to the difference of number of parameters between the null hypothesis and the alternative hypothesis.

The Wilks' theorem regularity conditions needed are:

- The models must be nested.
- The parameter specified by H_1 must not be on the boundary of their possible values under H_0 .
- All of the parameters must have values under H_0 .

If the Wilks' theorem is applicable, the significance can be expressed in standard deviations using:

$$n_\sigma(\lambda) = \sqrt{2} \text{ TMath::ErfcInverse(TMath::Prob}(\lambda, \text{ ndof})) \quad (4.38)$$

Wilks' theorem is correct only asymptotically but usually provides a good approximation for finite data samples. To check if Wilks' theorem is applicable, toy experiments must be performed under the null hypothesis. Fitting them with both the hypotheses allows us to obtain the λ distribution. The distribution can be fitted with a χ^2 allowing to extrapolate the number of degree of freedom. In this case the null hypothesis H_0 is the no- $T_{c\bar{c}}(4100)^-$ hypothesis (i.e. the default model), while the alternative hypothesis H_1 is the extended model. The procedure is repeated

for the $J^P = 0^+$ and $J^P = 1^-$ assignments. In order for H_0 to be nested in H_1 , as required by the regularity conditions, the H_0 model is built adding the $T_{c\bar{c}}(4100)^-$ component used in the H_1 model (same mass, width and quantum numbers) but fixing its isobar amplitude and phase to zero (if the phase is not constrained it becomes undefined when the magnitude is fixed to zero affecting the regularity of the likelihood around the point of constraint). The number of free parameters is 4. The Wilks' theorem states that λ follow a χ^2 distribution with a number of degree of freedom equal to the number of additional parameters ($\Delta n_{\text{par}} = 4$) resulting from the addition of the $T_{c\bar{c}}(4100)^-$ component. When the mass and width are floating as in this case, the Wilks' theorem does not hold even in the asymptotic case, since the mass and width are undefined under the null hypothesis, preventing the likelihood regularity requirement to be satisfied. This effect is known as look-elsewhere effect [233, 176]. Other LHCb amplitude analyses found out that the χ^2 distribution still provides a good approximation of the λ distribution but with an effective number of degrees of freedom that is $\simeq 2\Delta n_{\text{par}}$ [233, 176, 299]. To determine the λ distribution, 1000 toy experiments are generated according to the default model, each toy experiment having the same statistics as in the data sample. The isobar coefficients used for the generation are the ones reported in Table 4.18. These toys are fitted with both the H_0 and H_1 hypotheses. The NLL obtained for each toy are then used to calculate $\lambda = \Delta(-2\ln\mathcal{L})$. The H_1 model is used setting the starting values of the isobar coefficients to the generated values. The starting values of the isobar coefficients of the H_0 model are randomised. The λ distribution is then fitted using a χ^2 distribution leaving floating the number of degrees of freedom. As shown in the following sections, the λ distribution is well approximated by a χ^2 distribution. To determine the significance of rejecting one quantum number assignment in favor of another, toys are generated according to the null hypothesis and fitted using both the null and alternative hypotheses. In this case the models corresponding to the null and alternative hypotheses differ for the quantum numbers assigned to the $T_{c\bar{c}}(4100)^-$. The null hypothesis is given by the disfavoured quantum numbers when fitting data, in this case the $J^P = 0^+$ hypothesis.

4.9.4 Significance calculation of the $J^P = 0^+$ hypothesis

The generated pseudoexperiments are fitted with the default model, adding to the $T_{c\bar{c}}(4100)^-$ with mass and width reported in Table 4.23, quantum numbers $J^P = 0^+$ and fixing its isobar coefficients to 0.0, and with the extended model with free mass and width and with the $J^P = 0^+$ assignment. The λ distribution obtained is shown in Figure 4.53. The fit with a χ^2 PDF with floating ndof gives $\text{ndof} = 7.10$. Using the NLL values obtained in the amplitude fit to data and the

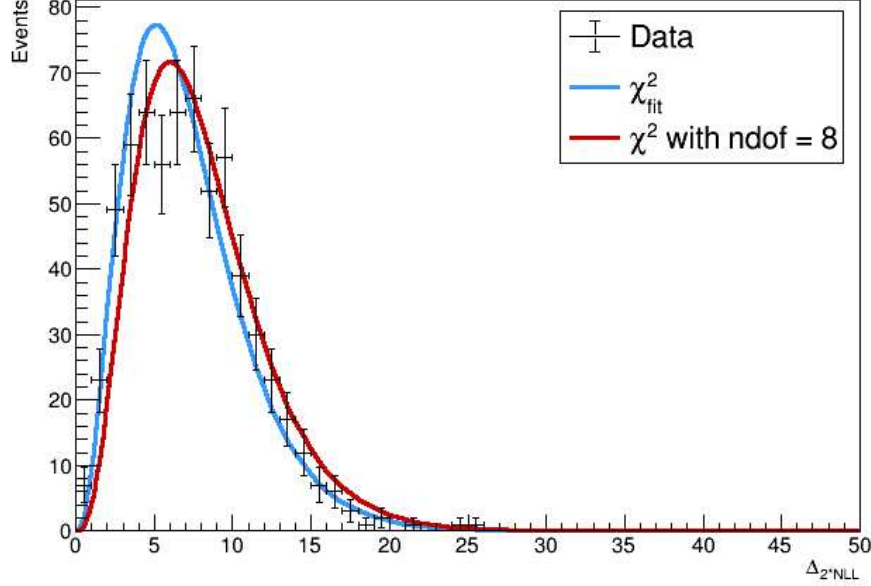


Figure 4.53: Distribution of $\lambda = \Delta(-2\ln\mathcal{L})$ for about 1000 toys generated according to the amplitude fit to data without the $T_{c\bar{c}}(4100)^-$ contribution, each fit without and with the $J^P = 0^+ T_{c\bar{c}}(4100)^-$ with floating mass and width. The fit with a χ^2 PDF with floating ndof gives ndof = 7.10 (blue curve). The red curve shows the χ^2 PDF with ndof = 8.

eq. 4.38, the significance of the $J^P = 0^+$ component is 2.27σ . Using ndof = 8, the significance is 2.05σ .

4.9.5 Significance calculation of the $J^P = 1^-$ hypothesis

The generated pseudoexperiments are fitted with the default model, adding to the $T_{c\bar{c}}(4100)^-$ with mass and width reported in Table 4.27, quantum numbers $J^P = 1^-$ and fixing its isobar coefficients to 0.0, and with the extended model with free mass and width and with the $J^P = 1^-$ assignment. The λ distribution obtained is shown in Figure 4.54. The fit with a χ^2 PDF with floating ndof gives ndof = 7.35. Using the NLL values obtained in the amplitude fit to data and the eq. 4.38, the significance of the $J^P = 1^-$ component is 3.8σ . Using ndof = 8, the significance is 3.6σ .

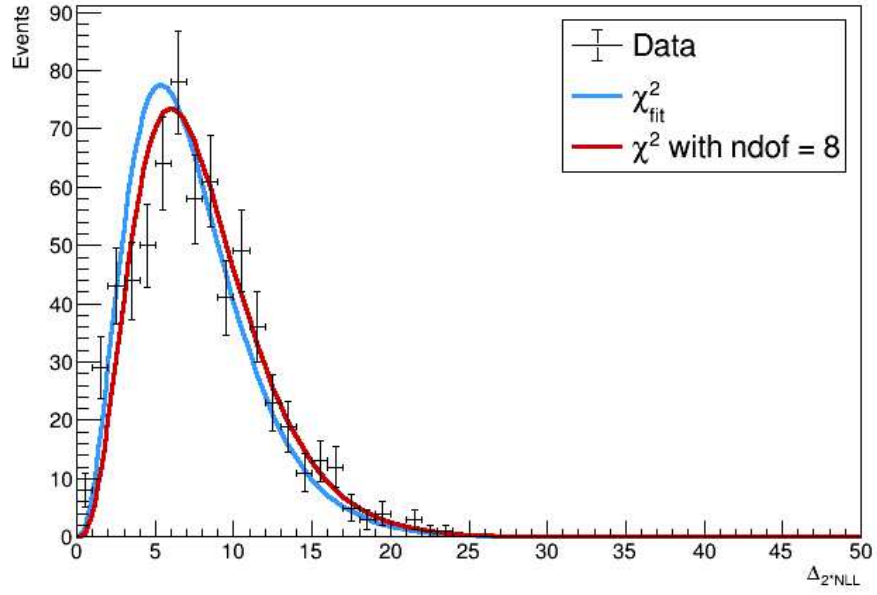


Figure 4.54: Distribution of $\lambda = \Delta(-2\ln\mathcal{L})$ for about 1000 toys generated according to the amplitude fit to data without the $T_{c\bar{c}}(4100)^-$ contribution, each fit without and with the $J^P = 1^- T_{c\bar{c}}(4100)^-$ with floating mass and width. The fit with a χ^2 PDF with floating ndof gives ndof = 7.35 (blue curve). The red curve shows the χ^2 PDF with ndof = 8.

4.9.6 Spin analysis

1000 toys are generated according to the extended model, with $J^P = 0^+$ quantum numbers hypothesis assigned to the exotic amplitude, i.e. the disfavoured hypothesis in data. The toys are generated with the same statistics as in the data sample. These toys are fitted with both the $J^P = 0^+$ and $J^P = 1^-$ hypothesis. The $\lambda = 2((- \ln \mathcal{L}(J^P = 0^+) - (- \ln \mathcal{L}(J^P = 1^-)))$ distribution is shown in Figure 4.55. This distribution is fitted with a gaussian obtaining $\mu = -18.3 \pm 0.8$ and $\sigma = 9.9 \pm 0.3$. The λ obtained from the amplitude fit results to data is $\lambda = \Delta(-2 \ln \mathcal{L}) = 12.8$, the rejection of the $J^P = 0^+$ hypothesis with respect to the $J^P = 1^-$ hypothesis is 3.2σ .

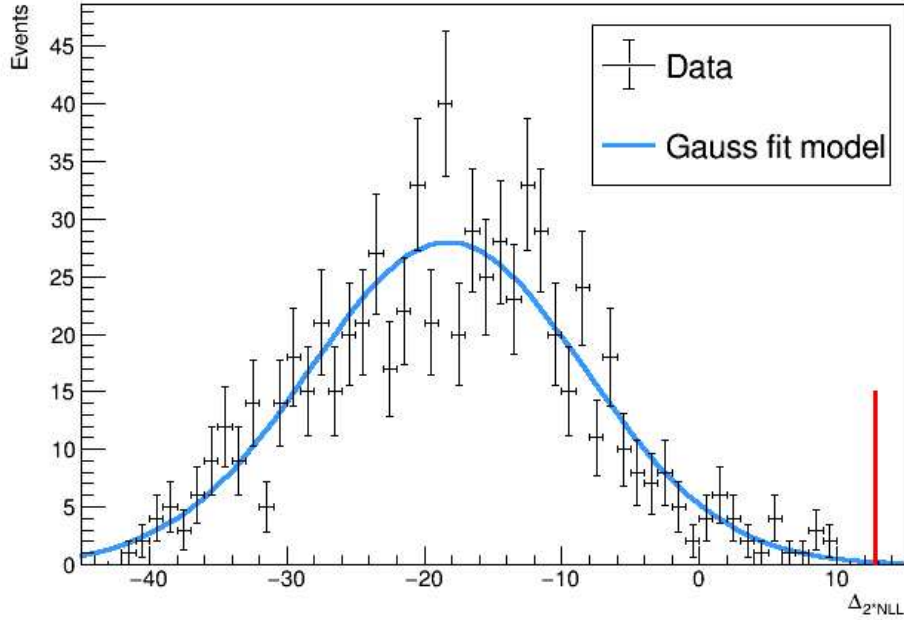


Figure 4.55: Distribution of $\lambda = 2((- \ln \mathcal{L}(J^P = 0^+) - (- \ln \mathcal{L}(J^P = 1^-)))$ for about 1000 toys generated according to the amplitude fit to data with the $T_{c\bar{c}}(4100)^-$ set to the disfavoured $J^P = 0^+$ (blue points) fitted with the disfavoured $J^P = 0^+$ and favoured $J^P = 1^-$ spin hypotheses. The red line is the λ value obtained from data.

4.10 Systematics uncertainties

The systematic uncertainties are evaluated on the fit fractions resulting from the amplitude fit giving the best NLL value, corresponding to the default model and the $K^* + T_{c\bar{c}}(4100)^-$ model with $J^P = 1^-$ state. The procedure to determine the corresponding systematic uncertainty is described in more details in the following.

The list of systematic uncertainties that have been carried out is the following:

- background parameterisation
- fixed signal and background yields
- amplitude fit bias
- efficiency parameterisation
- η_c mass value used to build the DP kinematics
- fixed resonance parameters
- $K_0^*(1430)$ parameterisation
- addition of $T_{c\bar{c}}(4100)^-$ candidate (considered only for the default model)

To probe the sensitivity of the significance of the $T_{c\bar{c}}(4100)^-$ state to systematic effects listed above the same procedure described in sec. 4.9.3 is used as general method. The minimum NLL is extracted for the fits with the default model and the $K^* + T_{c\bar{c}}(4100)^-$ model, the λ value is calculated and using eq. 4.38 the significance is extracted.

4.10.1 Background parameterisation

The strategy to check for this source of systematic uncertainties is to randomly extract, for each bin of the background maps, a number from the gaussian distribution having the mean equal to the bin content and the standard deviation equal to the corresponding bin uncertainty. In case of negative bin content two methods have been tested, using the negative value as mean of the gaussian or using 0. The results have been found compatible in the two cases so the maps generated by setting the mean to the negative value are used. The procedure to calculate the error on the fit fractions is repeated 1000 times in order to create new bkg maps and for each map the amplitude fit is repeated. For each fit fraction, a gaussian fit is applied to the distribution obtained, and its standard deviation is taken as the systematic uncertainty assigned to the background parameterisation. To calculate the significance 1000 background maps are calculated, for each map the amplitude fit is repeated to the corresponding toy experiment generated, as described

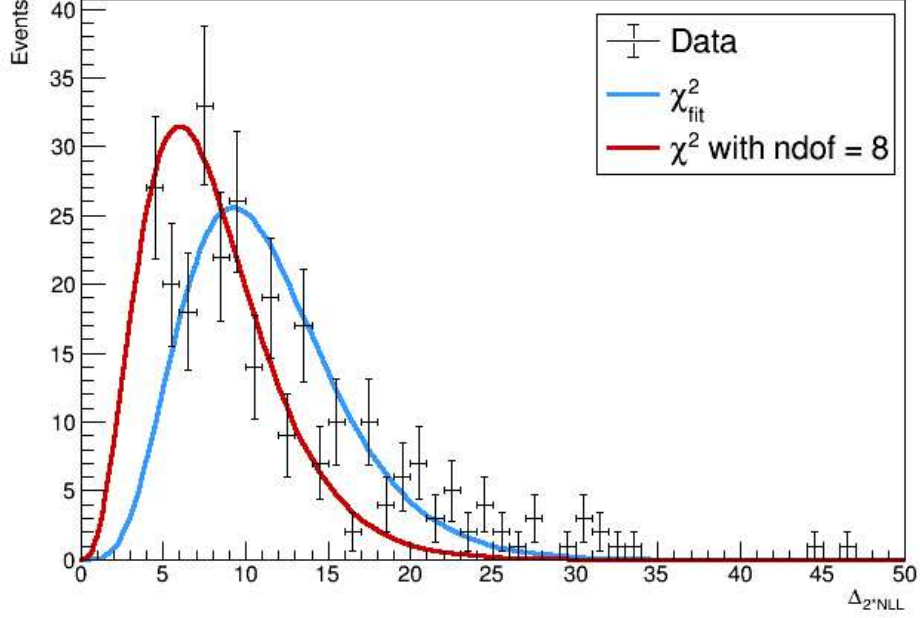


Figure 4.56: Distribution of $\lambda = \Delta(-2\ln\mathcal{L})$ for the toys generated according to the amplitude fit to data without the $T_{c\bar{c}}(4100)^-$ contribution, each fit without and with the $J^P = 1^- T_{c\bar{c}}(4100)^-$ with floating mass and width and with the background maps varied as described before. The fit with a χ^2 PDF with floating ndof gives ndof = 10.5 (blue curve). The red curve shows the χ^2 PDF with ndof = 8.

in sec. 4.9.3. The NLL are used to calculate λ . The λ distribution is fitted using a χ^2 distribution leaving floating the number of degrees of freedom, as shown in Figure 4.56. The strategy was to use the new number of degree of freedom to calculate the significance of the candidate once the background parameterisation is taken into account. Since the fit results is not good and the χ^2 distribution do not correctly describe the distribution, the significance is calculated using the p-value. The p-value is calculated as the number of events with a λ above the value obtained in the data, $\lambda_{data} = 29$, divided by the total number of entries in the of figure 4.56, i.e. the number of fit that converge. The p-value found is $\frac{10}{691} = 0.0145$. Using eq. 4.38 the significance is found to be of 2.44σ . So the candidate is not significant once the background systematic is taken into account. Systematic errors on the mass and width of the $T_{c\bar{c}}(4100)^-$ have not been calculated since the candidate is found not significant once systematics uncertainties are taken into account.

4.10.2 Fixed signal and background yields

The signal and background yields used in the amplitude fit are fixed to the values obtained from the 2D mass fit. Their uncertainties are propagated to the amplitude fit by gaussian constraining the yields within their statistical uncertainties.

4.10.3 Amplitude fit bias

A systematic uncertainty related to the fit bias is determined. 1000 pseudoexperiments are generated with the same statistics as in the data sample and fitted with the $K^* + T_{c\bar{c}}(4100)^-$ model. A Gaussian fit is applied to each fit fractions distribution in order to obtain its mean p_{fitted} and the error on the mean δ_p . The systematic uncertainty associated to the bias introduced by the amplitude fit procedure is determined as the square root of the mean squared error, given by

$$\sigma = \sqrt{(p_{fitted} - p_{true})^2 + \delta_p^2} \quad (4.39)$$

p_{true} is the value of the fit fraction used in the generation.

4.10.4 Efficiency parameterisation

The same strategy used for the systematic uncertainty associated to the background parameterisation is considered. Another systematic uncertainty is associated to the cut applied to the DP borders because of concerns about the parameterisation of the efficiencies in those regions. The amplitude fit is repeated removing these cuts.

4.10.5 η_c mass value used to build the DP kinematics

The kinematics variables used in the amplitude fit are computed starting from the B^0 , K^+ , π^- and η_c mass values. In the nominal amplitude fit the η_c mass values returned by the 2D fit are used, reported in tab 4.10. To estimate the effect of the η_c width the amplitude fit is repeated varying the η_c mass value. The fit is repeated two times, once using the $m + \Gamma/2$ value and once using the $m - \Gamma/2$ value. In case of same sign variations the highest one is considered.

4.10.6 Fixed resonance parameters

The masses and widths of the K^* resonances (except the LASS parameters) and the Blatt-Weisskopf barrier factor radii are fixed in the nominal amplitude fit. The uncertainties associated to the fixed masses and widths are determined varying simultaneously and randomly each parameter within their error, as reported in [66]

and repeating the amplitude fit 100 times. The systematics due to the fixed Blatt-Weisskopf barrier factor radii for the K^* and the $T_{c\bar{c}}(4100)^-$ resonances are determined changing the nominal value of 4 GeV^{-1} to 3 GeV^{-1} and 5 GeV^{-1} . The assigned systematics to the amplitude fit model fixed parameters are the combination in quadrature of all the contributions.

4.10.7 $K_0^*(1430)$ parameterisation

The $K_0^*(1430)$ resonance have been parameterised using the LASS PDF in the final model, in order to take into account the strong interferes with a non-resonant S-wave term component, as described in section 4.8.4.

To determine the systematic uncertainty associated to the choice of the LASS PDF, the fit is repeated replacing the LASS shape with a RBW for the $K_0^*(1430)$, adding the $K^*(800)$ amplitude parameterised with a RBW and a non-resonant S-wave $K^+\pi^-$ amplitude as a uniform amplitude.

4.10.8 Addition of $T_{c\bar{c}}(4100)^-$ candidate

The model $K^* + T_{c\bar{c}}(4100)^-$ is used to consider a possible $T_{c\bar{c}}(4100)^-$ candidate as source of systematic. The systematic associated is calculated as the difference between the nominal values and the values obtained from the fit under study.

4.10.9 Summary of systematic uncertainties

The systematic errors calculated for the fit fractions are reported in Table 18, 19 and 20. The summary of all systematic uncertainties is reported in Table 21. The single results are shown in Table 4.29, 4.30 and 4.31. The summary of all this systematic uncertainties is reported in Table 4.32. The significance of the $T_{c\bar{c}}(4100)^-$ candidate found for every systematic source studied are reported in Table 4.33. The systematic uncertainties of the fit fractions using the $K^* + T_{c\bar{c}}(4100)^-$ model are reported in appendix N.

Component	Nominal value	Fixed yields	Fit bias	Efficiency	η_c
$K^*(892)$	0.493 ± 0.012	-0.001	± 0.006	± 0.004	-0.008
$K^*(1410)$	0.045 ± 0.014	-0.010	± 0.003	± 0.003	-0.015
LASS	0.356 ± 0.055	+0.004	± 0.029	± 0.013	-0.023
$K_2^*(1430)$	0.037 ± 0.009	-0.001	± 0.002	± 0.004	+0.002
$K^*(1680)$	0.021 ± 0.009	+0.001	± 0.004	± 0.003	$^{+0.001}_{-0.002}$
$K_0^*(1950)$	0.024 ± 0.005	-0.002	± 0.022	± 0.003	+0.006
$K^*(892) - K^*(1410)$	-0.021 ± 0.018	+0.005	± 0.007	± 0.004	+0.001
$K^*(892) - K^*(1680)$	0.008 ± 0.012	+0.008	± 0.005	± 0.006	+0.013
$K^*(1410) - K^*(1680)$	-0.023 ± 0.015	+0.011	± 0.004	± 0.052	+0.019
LASS- $K_0^*(1950)$	0.070 ± 0.103	-0.015	± 0.052	± 0.010	-0.005

Table 4.29: Systematic uncertainties associated to the fit fractions calculated using the default model. The nominal value from the amplitude fit is reported in the second column with its statistical uncertainty. Components labelled like $K^*(892) - K^*(1410)$ stand for the interference fit fraction between the two resonances.

Component	Fixed parameters	Addition of $T_{c\bar{c}}(4100)^-$ candidate
$K^*(892)$	+0.002	-0.009
$K^*(1410)$	+0.021	+0.013
LASS	+0.030	+0.006
$K_2^*(1430)$	+0.003	-0.003
$K^*(1680)$	+0.003	+0.001
$K_0^*(1950)$	+0.007	-0.005
$K^*(892) - K^*(1410)$	+0.002	-0.008
$K^*(892) - K^*(1680)$	+0.021	+0.005
$K^*(1410) - K^*(1680)$	+0.022	-0.007
LASS- $K_0^*(1950)$	+0.004	+0.018

Table 4.30: Systematic uncertainties associated to the fit fractions calculated using the default model. The nominal value from the amplitude fit is reported in the second column with its statistical uncertainty. Components labelled like $K^*(892) - K^*(1410)$ stand for the interference fit fraction between the two resonances.

Component	Background parameterisation	$K_0^*(1430)$	Efficiency veto
$K^*(892)$	± 0.031	-0.004	+0.021
$K^*(1410)$	± 0.014	-0.021	+0.015
LASS	± 0.033	+0.033	+0.008
$K_2^*(1430)$	± 0.012	-0.001	-0.012
$K^*(1680)$	± 0.011	-0.001	-0.011
$K_0^*(1950)$	± 0.009	+0.009	+0.002
$K^*(892) - K^*(1410)$	± 0.013	+0.027	-0.029
$K^*(892) - K^*(1680)$	± 0.022	-0.028	-0.028
$K^*(1410) - K^*(1680)$	± 0.021	+0.005	+0.051
LASS- $K_0^*(1950)$	± 0.059	-0.017	-0.018

Table 4.31: Systematic uncertainties associated to the fit fractions calculated using the default model. The nominal value from the amplitude fit is reported in the second column with its statistical uncertainty. Components labelled like $K^*(892) - K^*(1410)$ stand for the interference fit fraction between the two resonances.

Component	Nominal value	Total
$K^*(892)$	0.493 ± 0.012	$+0.038$ -0.034
$K^*(1410)$	0.045 ± 0.014	$+0.032$ -0.031
LASS	0.356 ± 0.055	$+0.065$ -0.051
$K_2^*(1430)$	0.037 ± 0.009	$+0.013$ -0.020
$K^*(1680)$	0.021 ± 0.009	$+0.013$ -0.017
$K_0^*(1950)$	0.024 ± 0.005	$+0.027$ -0.024
$K^*(892) - K^*(1410)$	-0.021 ± 0.018	$+0.031$ -0.033
$K^*(892) - K^*(1680)$	0.008 ± 0.012	$+0.037$ -0.045
$K^*(1410) - K^*(1680)$	-0.032 ± 0.015	$+0.065$ -0.023
LASS- $K_0^*(1950)$	0.070 ± 0.103	$+0.079$ -0.086

Table 4.32: Summary of systematic uncertainties associated to the fit fractions calculated using the default model. The nominal value from the amplitude fit is reported in the second column with its statistical uncertainty. Components labelled like $K^*(892) - K^*(1410)$ stand for the interference fit fraction between the two resonances.

Source	Significance
fixed yields	3.2σ
bkg parameterisation	2.4σ
m_{η_c}	3.0σ
fixed parameters	3.2σ

Table 4.33: Significance of the $T_{c\bar{c}}(4100)^-$ candidate due to the systematic variations. Only the largest variation are reported.

4.11 Results

The model with the addition of a $T_{c\bar{c}}(4100)^-$ candidate is found with a significance lower than 3σ when taking into account the systematic uncertainties. In particular, the description of the backgrounds lacks sufficient precision to determine whether the peak observed in the $[4, 4.2]$ GeV region of the $\eta_c\pi^-$ invariant-mass distribution is a background fluctuation or an exotic candidate. Therefore the $T_{c\bar{c}}(4100)^-$ candidate is found to be not significant, this result is in contrast with the result of the previous analysis performed by LHCb [173]. The disagreement between the two measurements points toward the possibility that the result of [173] was a statistical fluctuation, as the present analysis uses a dataset twice as large and an improved candidate selection. The properties of the candidate, found considering only statistical uncertainties are in agreement with the candidate reported in [173]. A measurement of the branching ratio of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay have been performed, the result is $\mathcal{B}(B^0 \rightarrow \eta_c K^+ \pi^-) = (5.82 \pm 0.20_{stat} \pm 0.23_{syst} \pm 0.55_{ext}) \cdot 10^{-4}$. The branching fraction of the $(B^0 \rightarrow \eta_c K^*)$ decays, have been calculated starting from the measured branching ratio of $\mathcal{B}(B^0 \rightarrow \eta_c K^+ \pi^-)$ and the fit fraction obtained from the amplitude fit performed with the default model reported in Table 4.34. The results are shown in Table 4.35. The results of this analysis, using a data sample size that is approximately doubled with respect to the previous study, supersede those obtained in Ref. [173]. The results of this analysis, at the time of submission of this thesis, are submitted to EPJC and can be found in Ref. [300].

	$K^*(892)$	$K^*(1410)$	$K_0^*(1430)$	LASS NR	$K_2^*(1430)$	$K^*(1680)$	$K_0^*(1950)$
$K^*(892)$	0.493 ± 0.012	-0.021 ± 0.018	0.0	0.0	0.0	0.008 ± 0.012	0.0
$K^*(1410)$		0.045 ± 0.014	0.0	0.0	0.0	-0.032 ± 0.015	0.0
$K_0(1430)$			0.308 ± 0.047	-0.108 ± 0.013	0.0	0.0	0.070 ± 0.103
LASS NR				0.154 ± 0.024	0.0	0.0	0.0
$K_2^*(1430)$					0.037 ± 0.009	0.0	0.0
$K^*(1680)$						0.021 ± 0.009	0.0
$K_0^*(1950)$							0.024 ± 0.005

Table 4.34: Half-matrix of the fit fractions resulting from the fit using the default model, the $K_0(1430)$ and the $K - \pi S - wave$ components have been separated.

Decay	Branching ratio (10^{-5})
$B^0 \rightarrow \eta_c K^*(892)(\rightarrow K^+ \pi^-)$	$28.69 \pm 1.21 \pm 1.13^{+2.21}_{-1.98} \pm 2.71$
$B^0 \rightarrow \eta_c K^*(1410)(\rightarrow K^+ \pi^-)$	$2.62 \pm 0.82 \pm 0.10^{+1.86}_{-1.80} \pm 0.25$
$B^0 \rightarrow \eta_c K_0^*(1430)(\rightarrow K^+ \pi^-)$	$17.92 \pm 2.80 \pm 0.71^{+3.26}_{-2.56} \pm 1.69$
$B^0 \rightarrow \eta_c K^+ \pi^- (S - Wave)$	$8.96 \pm 1.42 \pm 0.35^{+1.63}_{-1.28} \pm 0.84$
$B^0 \rightarrow \eta_c K_2^*(1430)(\rightarrow K^+ \pi^-)$	$2.15 \pm 0.53 \pm 0.08^{+0.76}_{-1.16} \pm 0.20$
$B^0 \rightarrow \eta_c K^*(1680)(\rightarrow K^+ \pi^-)$	$1.22 \pm 0.52 \pm 0.05^{+0.76}_{-0.99} \pm 0.12$
$B^0 \rightarrow \eta_c K_0^*(1950)(\rightarrow K^+ \pi^-)$	$1.40 \pm 0.29 \pm 0.06^{+1.57}_{-1.40} \pm 0.13$

Table 4.35: Results for branching fractions obtained multiplying the fit fractions of Table 4.19 by $\mathcal{B}(B^0 \rightarrow \eta_c K^+ \pi^-)$. The uncertainties are statistical, systematics coming from $\mathcal{B}(B^0 \rightarrow \eta_c K^+ \pi^-)$, fit fraction systematics and PDG uncertainties.

Chapter 5

Charged-hadron identification performance in Run 3

Charged-hadron identification is a critical aspect of flavour physics experiments, addressing challenges such as reducing combinatorial background and distinguishing final states of otherwise identical topology. High-performance charged-particle identification is therefore an essential component of the LHCb flavour physics program in Run 3 (2022-2026). In this context, the development of robust calibration tools is central to optimising the performance of the upgraded LHCb detector and maximising its physics potential.

Sec. 5.1 briefly describes the LHCb Upgrade I. A brief introduction on charged-hadron identification is provided in Sec. 5.2. The strategy used to extract the efficiencies of charged-hadron identification is detailed in Sec. 5.3, while Sec. 5.4 presents the early Run 3 performance. Finally, Sec. 5.5 describes the development of a tool for real-time monitoring of the RICH detectors charged-hadron identification performance.

5.1 The LHCb Upgrade I

Although LHCb achieved excellent performance in Runs 1 and 2, the precision of key flavour-physics measurements remained limited by statistics [301]. Table 5.1 report the expected errors on key measurements of the LHCb experiment. Collecting larger datasets to significantly improve the uncertainty on several observables was not feasible with the original experiment design, as the hardware-based L0 trigger, relying on simple transverse-momentum cuts, saturated at higher luminosities and became inefficient, particularly for hadronic final states as can be seen

in Figure 5.1. To exploit the higher luminosities of Run 3 ($\mathcal{L} = 2 \cdot 10^{33} \text{cm}^{-2} \text{s}^{-1}$) and beyond, LHCb required the removal of this bottleneck and the adoption of a fully software-based trigger, capable of performing selections using full event reconstruction at the online stage. This, together with the need to cope with higher data rates, pile-up, and increased luminosity, made a comprehensive detector and readout upgrade essential.

Observable	Run 1 and 2 (up to 9 fb ⁻¹)	Upgrade I (23 fb ⁻¹) (50 fb ⁻¹)	
CKM tests			
γ ($B \rightarrow DK$, etc.)	2.8°	1.3°	0.8°
ϕ_s ($B_s^0 \rightarrow J/\psi \phi$)	20 mrad	12 mrad	8 mrad
$ V_{ub} / V_{cb} $ ($\Lambda_b^0 \rightarrow p \mu^- \bar{\nu}_\mu$, etc.)	6%	3%	2%
Charm			
ΔA_{CP} ($D^0 \rightarrow K^+ K^-, \pi^+ \pi^-$)	29×10^{-5}	13×10^{-5}	8×10^{-5}
A_Γ ($D^0 \rightarrow K^+ K^-, \pi^+ \pi^-$)	11×10^{-5}	5×10^{-5}	3.2×10^{-5}
Δx ($D^0 \rightarrow K_s^0 \pi^+ \pi^-$)	18×10^{-5}	6.3×10^{-5}	4.1×10^{-5}
Rare Decays			
$\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-)/\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)$	69%	41%	27%
$A_T^{(2)}$ ($B^0 \rightarrow K^{*0} e^+ e^-$)	0.10	0.060	0.043
A_T^{lm} ($B^0 \rightarrow K^{*0} e^+ e^-$)	0.10	0.060	0.043
$S_{\phi\gamma}$ ($B_s^0 \rightarrow \phi \gamma$)	0.32	0.093	0.062
α_γ ($\Lambda_b^0 \rightarrow \Lambda \gamma$)	$^{+0.17}_{-0.29}$	0.148	0.097

Table 5.1: Uncertainties in key LHCb physics quantities, derived from Run 1 and Run 2 data, are compared with projections for the upgrade I, evaluated using a Run 3 data sample of 23 fb^{-1} and assuming systematic uncertainties are not the limiting factor.

The VELO was redesigned to handle higher luminosity and a 40 MHz trigger-less readout while maintaining high track-finding efficiency within CPU limits. Its optimised layout, similar to the previous design, consists of 52 identical pixel-based modules arranged perpendicular to the beamline to cover the full LHCb pseudorapidity range. Each module integrates silicon pixel detectors, cooling, power, and readout systems. Only the vacuum vessel and motion services were retained from the old version, while the RF boxes were fully redesigned to reduce material and inner radius.

The upstream tracker (UT) replaces the TT stations used in Run 1 and 2, with several improvements. Two stations with four silicon planes each, designed with a hole for the beam pipe with the staves now incorporate active cooling and support both sides of the sensors for full vertical coverage. Z-staggered and horizontally overlapping staves improve acceptance, especially near the beam pipe, reducing gaps compared to the TT.

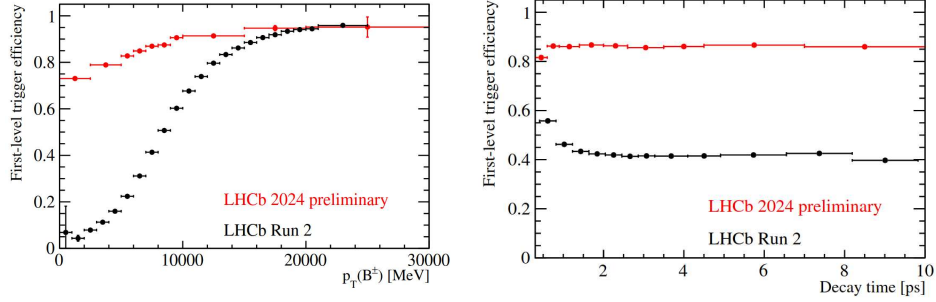


Figure 5.1: First-level trigger efficiency as a function of the $B^\pm$ transverse momentum (left) and decay time (right). Efficiencies are computed using the TIS/TOS method [302]. In Run 2, the trigger included a hardware stage based on calorimeter and muon information, followed by software; in Run 3 it is fully software-based, leading to improved performance.

The SciFi Tracker replaces the downstream tracking performed by the IT and OT in Run 1 and 2. SciFi uses plastic scintillating fibres (250 μm diameter) instead of the silicon strips and drift tubes technologies used by the IT and OT, respectively. 12 detection planes across three stations (T1–T3) in $xuvx$ configuration provide higher granularity. Modules feature honeycomb-carbon fibre sandwiches with mirrors and SiPM readout, allowing precise light collection and reduced dead zones.

The active material of the calorimeters remains unchanged to minimise modifications in the calorimeter system, but front-end and readout electronics are re-designed for 40 MHz operation. The SPD and PS subdetectors are removed due to their reduced role in the full-software trigger.

The M1 station of the Muon system, used in Run 1 and 2 for the hardware L0 trigger, is removed. The remaining stations (M2–M5) retain MWPC detectors with iron absorbers but feature completely upgraded readout electronics for 40 MHz operation. The front-end electronics remain radiation-hard and unchanged.

The HLT1 trigger is fully software-based and optimised for GPUs. It performs a partial event reconstruction using tracking detectors, the electromagnetic calorimeter, and muon chambers to select interesting events, reducing the LHC $\sim 25 - 30$ MHz collision rate (depending on the filling scheme) to about 1 MHz. Selected events are temporarily buffered while real-time detector alignment and calibration occur. HLT1 does not use RICH information; this is deferred to HLT2. GPU-optimised algorithms allow parallel reconstruction of thousands of events, with each event around 100 kB, efficiently handled within GPU memory (~ 10 GB).

HLT2 receives events from the buffer after HLT1 selection. It performs full event reconstruction, incorporating RICH data and reconstructing all tracks. Using over a thousand lines tailored for specific physics signals, HLT2 applies inclusive and exclusive selections, reducing the rate to ~ 100 kHz. Calibration and alignment constants are updated in real time to optimise selections. The Turbo persistence model enables flexible data storage, from minimal objects involved in the trigger decision to full event data, including raw banks for calibration streams.

5.1.1 The RICH upgrade

The overall design and concept of the RICH system from LHCb Run 1 and 2 have been largely preserved. However, key upgrades were required to handle the higher design luminosity while maintaining performance comparable to previous runs [303, 37]. To support a 40 MHz readout rate, the photon detection chain in both RICH1 and RICH2 was completely overhauled, as the previous hybrid photon detectors [304] had front-end electronics bump-bonded in the vacuum tube limited to a 1 MHz output.

With a fivefold increase in instantaneous luminosity, RICH1 optics were redesigned to reduce peak occupancy, the fraction of detected photons per detector channel, which is critical for maintaining optimal pattern recognition performance. Experience from Run 1 and 2 shows that occupancy should remain below 30%. To achieve this, the radius of the spherical mirrors was increased from 2.7 m to 3.7 m, moving the focal plane backward and enlarging the Cherenkov ring size. Photodetectors are now positioned deeper inside the magnetic shielding. The upgraded photon detection system was optimised for the highly non-uniform occupancy expected: roughly 30% in central RICH1, decreasing to 5% in peripheral RICH2. Hit rates are highest for Cherenkov photons from numerous tracks at high pseudorapidity. Detector geometry and channel granularity were optimized to balance mechanical constraints, readout channel counts, and cost.

HPDs were replaced with multi-anode photomultiplier tubes (MaPMTs) paired with new FE electronics. Selected MaPMTs feature an 8×8 anode matrix. RICH1 and central RICH2 use 1-inch MaPMTs with $2.88 \times 2.88 \text{ mm}^2$ pixels for high-occupancy regions, while outer RICH2 employs 2-inch MaPMTs with $6 \times 6 \text{ mm}^2$. The coarser granularity in peripheral regions reduces the number of MaPMTs and readout channels without significantly affecting performance. Figure 5.2 shows installed MaPMTs. FE boards host custom CLARO ASICs, each with 8 channels, an analog pulse-shaping amplifier, and a binary discriminator for single-photon detection. Amplified and digitised signals are sent to an FPGA, synchronized with the LHC clock, and transmitted via GBT links to the Event-Builder farm [305].



Figure 5.2: The MaPMTs selected for the upgraded RICH detectors with the 2-inch model on the left and the 1-inch model on the right.

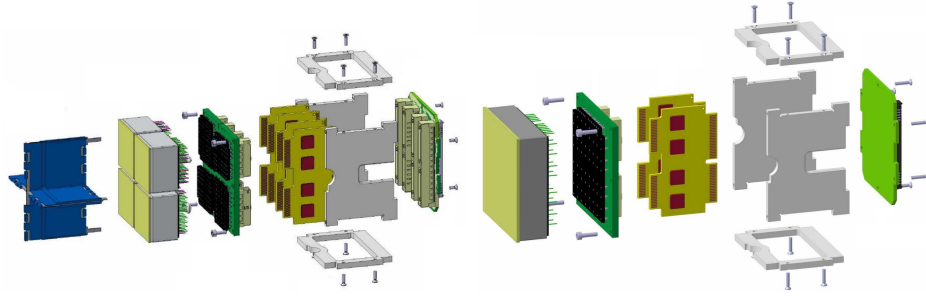


Figure 5.3: (Left) Exploded view of R-type elementary cell and (right) H-type elementary cell

Photon detectors and CLARO chips are organised into elementary cells (ECs), with two types for the different MaPMT models: R-type (EC-R) and H-type (EC-H) (Figure 5.3 [306]). EC-R reads out four 1-inch MaPMTs and includes resistive dividers for biasing dynodes. In high-occupancy areas, dedicated supply lines power the last two dynodes to prevent nonlinear gain effects. Magnetic shielding reduces stray fields by a factor of ~ 20 . Signals are routed to FE boards, then through a backboard to photon detector module digital boards (PDMDBs), shown in Figure 5.4 [306]. Thermal management is achieved via aluminum casings and copper layers, with temperature monitoring. RICH1 uses 472 EC-Rs, central RICH2 192 EC-Rs, and peripheral RICH2 384 EC-Hs. Four ECs and one or two PDMDBs form a photon detector module (PDM), the core of RICH columns. PDMDBs transmit digitised signals to the LHCb readout system, integrating with the Experiment Control System (ECS). FPGA-based logic allows flexible data capture, formatting, and interface between front-end ASICs and GBT modules.

Prompt Cherenkov photons arrive nearly simultaneously at the detection plane,

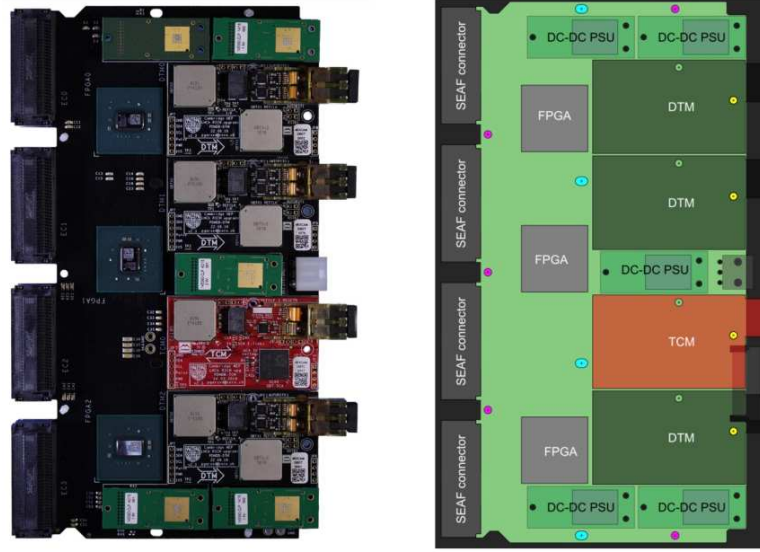


Figure 5.4: (Left) Picture of a fully populated PDMDB-R board. (Right) Schematic view of the PDMDB-R board main components. The PDMDB-H differs by having one less FPGA and DTM with respect to the PDMDB-R.

enabling FE time gates to filter out-of-time background while admitting valid photon signals. Simulations of RICH1 show a ~ 2 ns signal peak, defining the minimum FE gate width. Considering CLARO time walk, channel variations, MaPMT transit spread, and digital sampling, a 6.25 ns gate is used to maximise the signal over noise ratio. This gating effectively reduces background from beam interactions and sensor noise, improving particle identification [307].

5.2 Charged-hadron identification in flavour physics

A central objective of charged-hadron identification in flavour-physics experiments is the suppression of combinatorial background. Many interesting decay modes of hadrons containing b and c quarks involve complex multibody final states. At hadron colliders such as the LHC, pions dominate the charged-particle yield. Since the heavy-flavour decays of interest typically produce kaons, pions, and protons, correctly selecting the relevant charged hadrons during invariant-mass reconstruction is essential to reduce combinatorial contamination.

Another important role of particle identification is to distinguish between final states that share the same topology. A clear example is provided by two-body hadronic decays of B mesons, $B \rightarrow h^+ h^-$, where h denotes a generic charged hadron [308]. Relying only on kinematic and vertex-based cuts results in summing

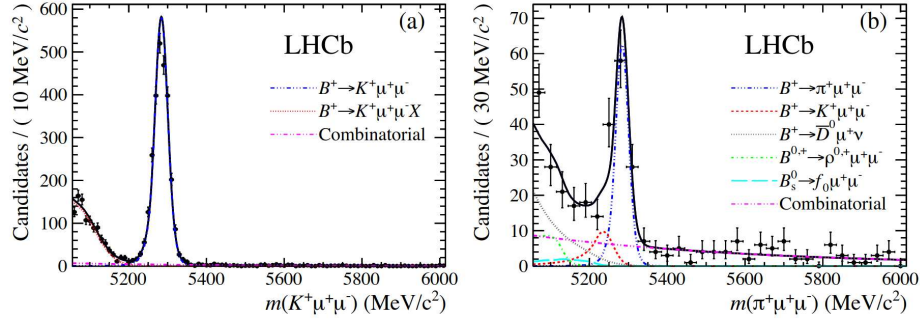


Figure 5.5: The effect of applying hadron PID selections to suppress the dominant kaon mode (a) to measure the CKM suppressed pion mode (b), in rare B decays. Here 82% of pion events are retained, suppressing the kaons by a factor of 80 relative to the pions, leaving a clear peak from the pion mode.

over all such decays, despite their differing CP asymmetries. Precision studies of CP violation therefore require separation of the individual contributions (e.g. $B^0 \rightarrow \pi^+\pi^-$, $B_s^0 \rightarrow K^+K^-$). This separation is made possible by the high-efficiency hadron identification provided by the RICH detectors.

PID also plays a key role in flavour tagging [309]. Heavy-flavoured hadrons are typically produced in pairs, and studies of CP violation or neutral meson mixing depend on determining the initial flavour state. One powerful tagging method exploits the identification of charged kaons from the $b \rightarrow c \rightarrow s$ decay chain of the companion particle. Such kaons, as well as those produced in b fragmentation when a B_s^0 is formed, have relatively low momentum, averaging around 10 GeV. Extending PID performance down to a few GeV thus provides a substantial gain in tagging efficiency.

The benefits of PID vary depending on the kinematics of the signal channels and the structure of background processes. A notable example comes from rare decay analyses, such as the separation of $B^+ \rightarrow K^+\mu^+\mu^-$ and $B^+ \rightarrow \pi^+\mu^+\mu^-$ decays shown in Figure 5.5 [310]. The pion channel is suppressed by a factor of roughly 25 relative to the kaon channel, due to CKM matrix element ratios. By applying PID, the kaon mode is suppressed by a factor of 80 while retaining 82% of the pion candidates, enabling a clear pion signal peak, something unattainable without effective hadron identification.

The momentum distribution of decay products in two-body b -hadron decays peaks around 50 GeV. Efficient reconstruction of these channels requires PID performance up to at least 100 GeV. The lower threshold of about 2 GeV is set by the need to reconstruct tracks from high-multiplicity B decays and by the fact that

particles below this momentum fail to cross the 4 Tm dipole field of the LHCb spectrometer.

5.3 Efficiency calculation strategy

For each track, the RICH reconstruction algorithm, running at the second stage of the software trigger and using tracking information as input, assigns a mass hypothesis. The procedure begins by computing a global log-likelihood value based on the photon-detector hits and the expected hit patterns from all tracks traversing the Cherenkov radiators [311]. The absolute minimum of this likelihood is found by iteratively updating the particle hypothesis of each track. Then, while keeping all other tracks fixed at their configuration corresponding to the minimum likelihood, a delta-log-likelihood (DLL) value is calculated for each track as the difference between the log-likelihood of a given mass hypothesis and that of the pion hypothesis.

The performance of charged-hadron identification at LHCb is evaluated using data-driven methods with control samples of particles identified through purely kinematic selections, thereby avoiding any bias from RICH information. Kaon and pion samples are obtained from the decay chain $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$, while proton samples come from $\Lambda \rightarrow p\pi^-$ decays.

RICH performance is typically quantified in terms of the identification (ID) efficiency versus the misidentification (misID) rate. The efficiency of a particular PID requirement for a given hadron species (pion, kaon, or proton) is extracted via a maximum-likelihood fit to the m -mass distribution of the parent particle. This fit separates the signal from residual background. By performing the fit both with and without the PID requirement applied to the probe particle, the efficiency is determined as the ratio of the signal yields in the two fits. By scanning the PID requirement from loose to tight selections, performance curves are produced that map the trade-off between identification efficiency (e.g. correctly tagging kaons) and misidentification probability (e.g. pions misidentified as kaons). The uncertainties shown in all figures are statistical only and are highly correlated across points on the same curve.

5.4 Early Run 3 performance

Studies of charged hadron identification in Run 3 has been performed using data collected in 2024 [312]. Two complementary strategies were employed to extract these quantities. In one approach, statistical weights derived from mass fits were

applied to subtract background contributions prior to studying the response of the likelihood variables. This method, closely connected to physics analyses, provides a calibration that can be directly transferred to physics channels of interest. In the other approach, signal yields were determined from independent fits performed before and after each selection requirement. The resulting ratios provide a straightforward measure of efficiency or misidentification, serving primarily as a tool for detector monitoring. Although the methodologies differ in their scope and intended use, they give results that are broadly consistent. Small variations can be attributed to differences in selections, fit strategies, and trigger choices, but the overall picture is coherent. The comparison between the two approaches shows the expected trade-off between efficiency and misidentification probability, while more detailed studies reveal the dependence of the identification performance on kinematic properties and detector conditions.

These early results mark an important milestone, as they establish the first measurements of charged hadron identification with the upgraded detectors in Run 3 at the nominal luminosity of $\mathcal{L} = 2 \cdot 10^{33} \text{cm}^{-2} \text{s}^{-1}$. The agreement between methods, together with the observed trends in momentum and occupancy, provides strong evidence that the new system is functioning reliably and is on track to meet the demands of the experimental programme. Figure 5.6 shows the particle identification and misidentification efficiencies as a function of momentum derived from the sWeight method. Figure 5.7 shows the particle identification performance as a function of the detector occupancy.

5.5 Quasi-online charged-hadron identification performance in Run 3

The ability to monitor charged-hadron identification performance in real time is vital for ensuring the reliability and efficiency of the RICH detector during Run 3. While offline calibration procedures ensure long-term stability and optimal PID performance, they are limited in their ability to provide immediate feedback on detector conditions. The development of an online tool to evaluate PID performance in real time is therefore of strategic importance. Such a framework would enable continuous monitoring of key performance indicators, allowing for the prompt identification of inefficiencies, detector misconfigurations, or degradation in the detector response. Moreover, it would provide rapid validation of interventions and configuration changes applied to the detector, thereby reducing latency between operational adjustments and their physics impact. Real-time performance assessment thus constitutes an essential component of a robust calibration strategy, ensuring the stability of PID performance.

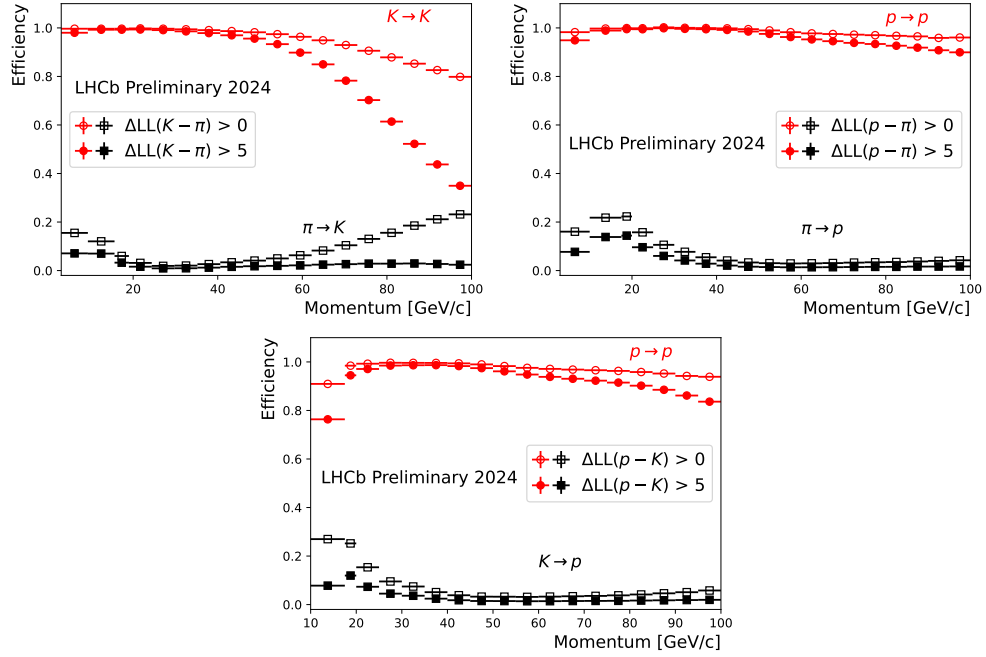


Figure 5.6: (Top left) Kaon identification efficiencies and pion misidentification probabilities against momentum, with curves for selection requirements of $DLL_K > 0$ and 5. (Top right) Proton identification efficiencies and pion misidentification probabilities against momentum, with curves for selection requirements of $DLL_p > 0$ and 5. (Bottom) Proton identification efficiencies and kaon misidentification probabilities against momentum, with curves for selection requirements of $DLL_p > 0$ and 5 for protons and $DLL_p - DLL_K > 0$ and 5 for kaons.

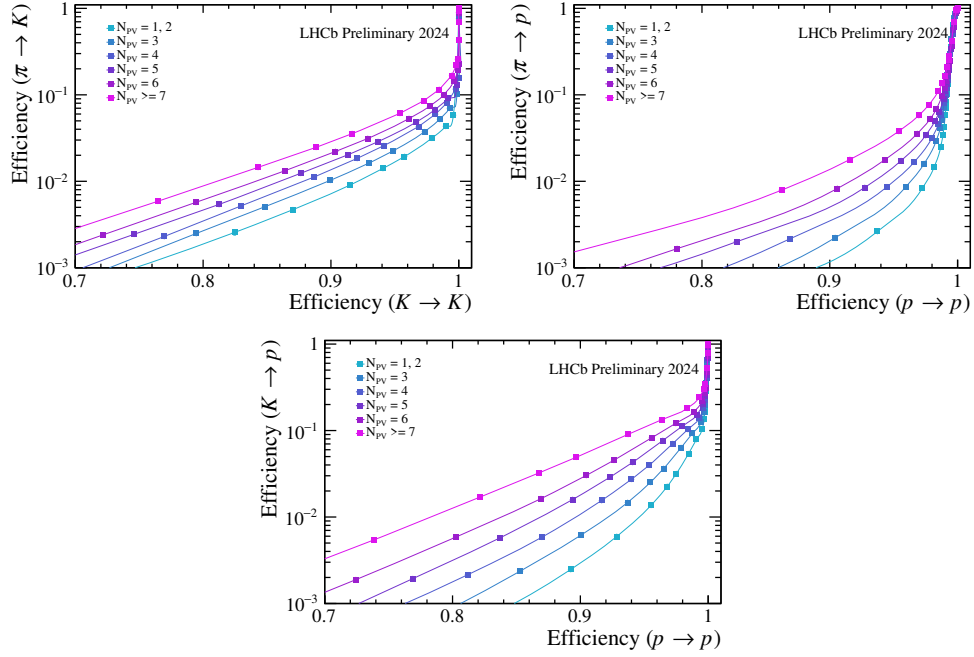


Figure 5.7: (Top left) Kaon identification efficiency against pion misidentification probability for fill 9986, in different bins of number of primary vertices. The average momenta of the kaons and pions are both 35 GeV. (Top right) Proton identification efficiency against pion misidentification probability for fill 9986, in different bins of number of primary vertices. The average momenta of the protons and pions are 29 GeV and 35 GeV, respectively. (Bottom) Proton identification efficiency against kaon misidentification probability for fill 9986, in different bins of number of primary vertices. The average momenta of the protons and kaons are 29 GeV and 35 GeV, respectively. The average number of primary vertices is 4.7.

5.5.1 Data infrastructure

HLT1 does not employ information from the RICH for particle identification while HLT2 executes a full event reconstruction, reconstructing all tracks and incorporating information from the RICH detectors, so it can also be used to compute data that serves as monitoring data for the RICH detectors. Specific lines have been developed in HLT2 to select the $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$ and $\Lambda \rightarrow p\pi^-$ decays without using the information from the RICH detectors to provide unbiased samples. Tables 5.2 and 5.3 report the selection implemented in the HLT2 lines. The data can be further divided into bin of momentum or pseudorapidity following the binning scheme reported in Table 5.4, corresponding to six categories. Each of these bins is subsequently divided into eleven PID bins, corresponding to the ranges greater than -1000 that is used as a reference for no PID cuts, and, above and below $-10, -5, 0, 5,$ and 10 . Histograms selected with $\text{PIDX} < Y$ are referred to as fail histograms, whereas those with $\text{PIDX} > Y$ are referred to as pass histograms. Two-dimensional histograms are saved for the data passing the $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$ line, with $m_{K^-\pi^+}$ on the x -axis and $m_{K^-\pi^+\pi^+} - m_{K^-\pi^+}$ on the y -axis. One-dimensional histograms are saved for the data passing the $\Lambda \rightarrow p\pi^-$ line, corresponding to the $m_{p\pi^-}$ invariant-mass. Figure 5.8 shows the histograms resulting from the Hlt2CalibMon_DstToD0Pi and Hlt2CalibMon_L0ToPPi_LL lines, integrated over the full phase space and with $\text{PIDX} > -1000$. These histograms are calculated for each run of LHCb.

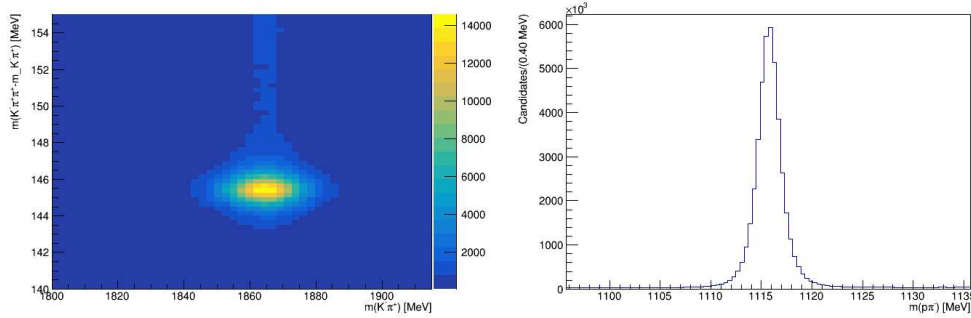


Figure 5.8: Histogram from the (left) Hlt2CalibMon_DstToD0Pi and (right) Hlt2CalibMon_L0ToPPi_LL lines, integrated over the full phase space and with $\text{PIDX} > -1000$.

Variable	Requirement
K^- and π^+	
p_T	> 250 MeV
p	> 2 GeV
χ^2_{IP}	> 16
χ^2/ndf	< 3
Track Ghost Prob	< 0.4
$K^- \pi^+$ system	
$ m_{K^- \pi^+} - m_{D^0}^{PDG} $	< 95 MeV
p_T	> 1.5 GeV
DOCA	> 0.08 mm
D^0	
m_{D^0}	$\in [1800, 1915]$ MeV
$ m_{K^- \rightarrow \pi^-} - m_{D^0}^{PDG} $	< 25 MeV
χ^2/ndf	< 10
DIRA _{PV}	> 0.99997
χ^2_{FD}	< 49
$ m_{K^- \rightarrow \pi^-, \pi^+ \rightarrow K^+} - m_{D^0}^{PDG} $	< 25 MeV
NumChild($p_T > 1$ GeV)	> 1
π_s^+	
p_T	> 100 MeV
p	> 1 GeV
χ^2/ndf	< 3
Track Ghost Prob	< 0.4
$D^0 \pi_s^+$ system	
$ m_{D^0 \pi_s^+} - m_{D^{*+}}^{PDG} $	$\in [135, 175]$ MeV
DOCA	> 0.20 mm
D^{*+}	
$ m_{D^{*+}} - m_{D^{*+}}^{PDG} $	$\in [140, 155]$ MeV
χ^2/ndf	< 10

Table 5.2: Definition of the HLT2 line Hlt2CalibMon_DstToD0Pi used for selecting candidates from the $D^{*+} \rightarrow D^0(\rightarrow K^- \pi^+) \pi^+$ decay channel. With π_s is represented the pion decaying from the D^{*+} while π represents the one from the D^0 decay.

5.5.2 Strategy and workflow

To calculate the efficiency for each PID cut, the pass and fail method is used:

$$\epsilon = \frac{N_{\text{pass}}}{N_{\text{pass}} + N_{\text{fail}}}, \quad (5.1)$$

5.5. Quasi-online charged-hadron identification performance in Run 3

Variable	Requirement
p	
p_T	$< 1 \text{ TeV}$
p	$> 2 \text{ GeV}$
χ^2_{IP}	> 36
χ^2/ndf	< 4
Track Ghost Prob	< 0.4
π^-	
χ^2_{IP}	> 36
χ^2/ndf	< 4
Track Ghost Prob	< 0.4
$p\pi^-$ system	
$ m_{p\pi^-} - m_{\Lambda}^{PDG} $	$< 50 \text{ MeV}$
Λ	
$ m_{\Lambda} - m_{\Lambda}^{PDG} $	$< 20 \text{ MeV}$
$ m_{p \rightarrow \pi^+} - m_{\Lambda}^{PDG} $	$< 20 \text{ MeV}$
χ^2/ndf	< 30
χ^2_{IP}	> 50
τ	$> 2 \text{ ps}$

Table 5.3: Definition of the HLT2 line Hlt2CalibMon_L0ToPPi_LL used for selecting candidates from the $\Lambda \rightarrow p\pi^-$ decay channel.

Binned variable	η	p [GeV]
Integrated	$\in [2.0, 4.9]$	$\in [p_{min}, 150]$
η	$\in [2.0, 3.5]$	$\in [p_{min}, 150]$
	$\in [3.5, 4.9]$	
p	$\in [2.0, 4.9]$	$\in [p_{min}, 30]$
		$\in [30, 60]$
		$\in [60, 100]$

Table 5.4: Binning schemes used for quasi-online PID analysis, with ranges specified for pseudorapidity (η) and momentum (p). The binning over a variable is obtained by requiring that both particles under study fall within the same interval of that variable. p_{min} is defined as 3 GeV for the $p\pi$ and $K\pi$ comparison and 10 GeV for the pK one.

where N_{pass} denotes the number of signal events in the pass histogram, and N_{fail} denotes the number of signal events in the fail histogram, both corresponding to the same PID cut. The yields are extracted from fit to the histograms, as described in the following.

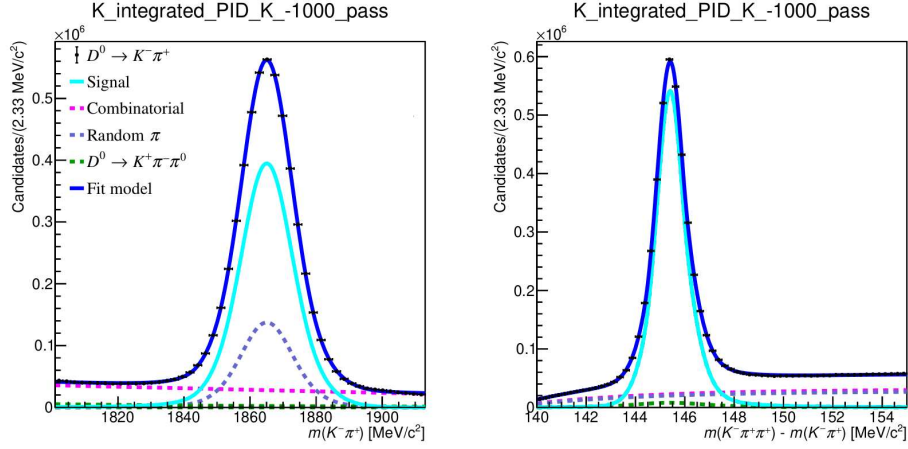


Figure 5.9: Projections of the two-dimensional mass fit to the (left) $m_{K^-\pi^+}$ and (right) $m_{K^-\pi^+\pi^+} - m_{K^-\pi^+}$ variables from the integrated histogram with $\text{PIDK} > -1000$. The signal yield is 3563678 ± 3256 corresponding to a recorded luminosity of $7561550 \mu\text{b}^{-1}$

Two fit models are used to parametrise the $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$ and $\Lambda \rightarrow p\pi^-$ decays. The $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$ model has to take into account the shape of the 2D distribution. It has four components, one to take into account the full combinatorial background, one to describe a non-resonant background where a random pion is coupled to a D^0 meson, one component to describe the $D^0 \rightarrow K^+\pi^-\pi^0$ decay, where the π^0 is not reconstructed and a random π^+ is selected and finally the signal component. A typical fit result is shown in Figure 5.9.

The $\Lambda \rightarrow p\pi^-$ model comprises three components, a full combinatorial background component, one component to account for ghost track contributions and the signal component. A typical fit result is shown in Figure 5.10.

The fitting strategy is identical for both models and the fit are performed for the six categories described in Table 5.4. In the initial fit, the parameters are set to their expected values, and the first histogram fitted is the one with $\text{PIDX} > -1000$. Subsequently, the pass and fail histograms for the $\text{PIDX} > -10$ cut are fitted, using the parameter values obtained from the previous fit as initialisation. This procedure is then iterated for progressively tighter cuts until the final histograms are fitted. Throughout the process, the parameters of the pass and fail histograms are treated independently. Once all the fit of one categories are performed the efficiencies are calculated using Eq. 5.1. Figure 5.11, 5.12 and 5.13 shows the ROC curves calculated from a single run of 1 hour. The ROC curves are used to check if the efficiency of the RICH detectors is stable during the data taking. Another tool, illustrated in Figure 5.14 for the $K\pi$ system, display the efficiency and the

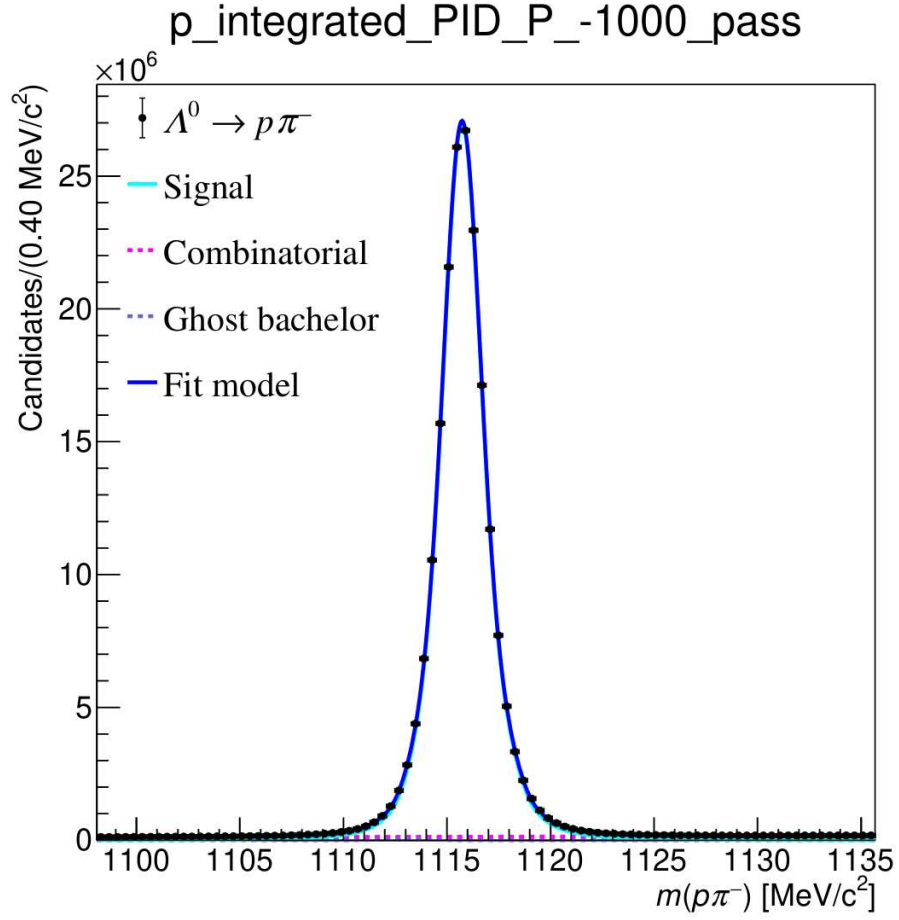


Figure 5.10: Result of the fit to the $m_{p\pi^-}$ variable from the integrated histogram with $\text{PIDP} > -1000$. The signal yield is 194585253 ± 25612 corresponding to a recorded luminosity of $7561550 \mu\text{b}^{-1}$

corresponding misidentification efficiency for a fixed PID cut, set to 5. The figure shows an overall uniform behavior with two notable exceptions. The first, between run numbers 327000 and 327200, displays a significant decrease in efficiency together with a smaller but still noticeable reduction in misidentification efficiency. This effect was due to a change in the RICH detector reconstruction algorithm. The second, between run numbers 327700 and 327730, shows a clear improvement in efficiency along with a lower misidentification rate. These runs correspond to a very long LHC fill, during which the luminosity decreased; the resulting lower occupancy had a strong impact on the detector performance. Even though the calculated efficiencies using this online method are in agreement with each other and their behaviour is stable along the data taking, they have been found to be not in agreement with the one measured with the offline method. For the $K\pi$ system the two measurements are in good agreement while for the pK and $p\pi$ ones a difference of up to 10% is observed. These discrepancies are currently under investigation. Discrepancies in low level variables (χ^2/ndf for example) have been found between the offline and online sample. These differences are likely due to variations in how the variables are defined, since both datasets are taken directly from the HTL2 output.

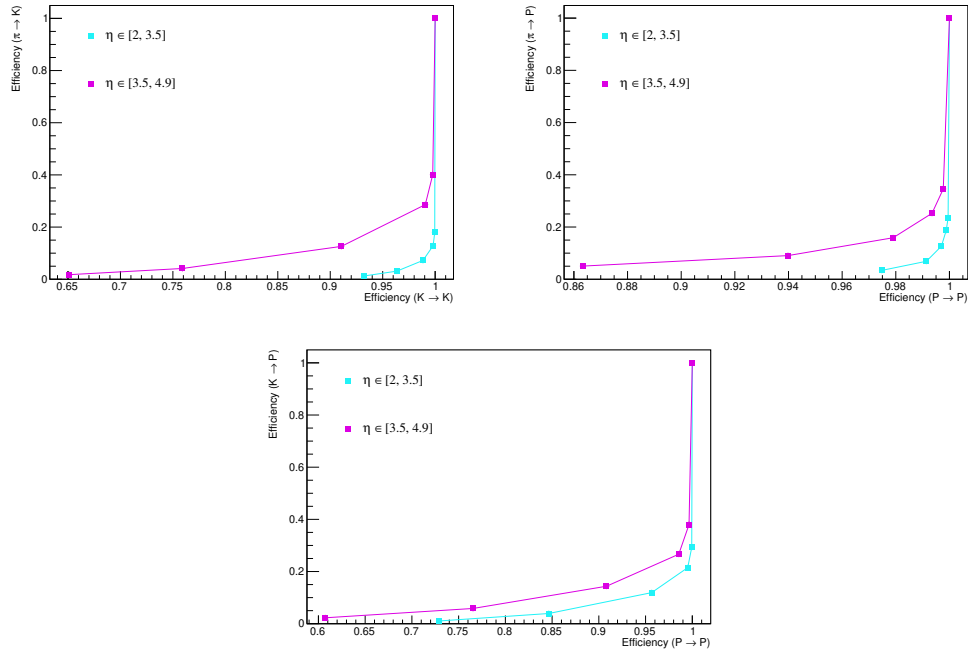


Figure 5.11: Efficiency of selecting (top left) kaons, (top right) protons, (bottom) pions, with the associate leakage from misidentifying (top left) pions, (top right) pions, (bottom) kaons in the η bins defined in Table 5.4 from run 328855.

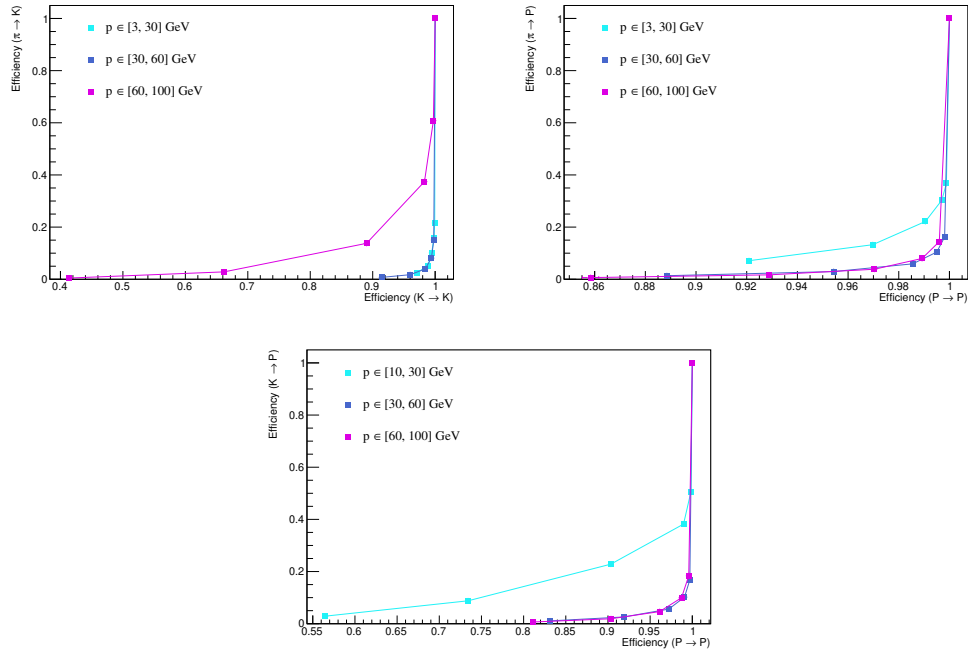


Figure 5.12: Efficiency of selecting (top left) kaons, (top right) protons, (bottom) protons, with the associate leakage from misidentifying (top left) pions, (top right) pions, (bottom) kaons in the p bins defined in Table 5.4 from run 328855.

5.5. Quasi-online charged-hadron identification performance in Run 3

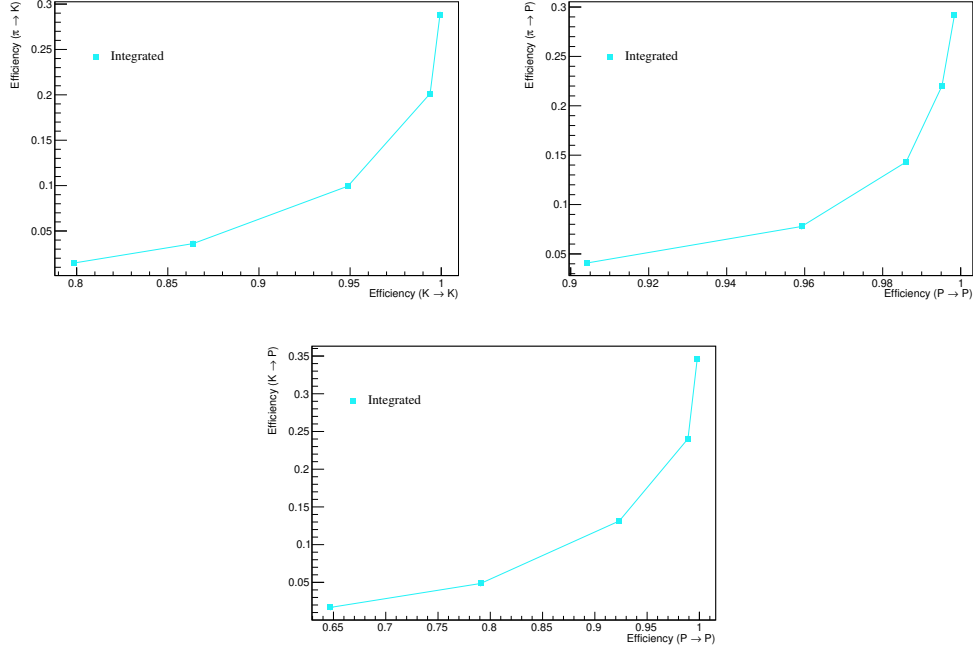


Figure 5.13: Efficiency of selecting (top left) kaons, (top right) protons, (bottom) protons, with the associate leakage from misidentifying (top left) pions, (top right) pions, (bottom) kaons in the integrated bin defined in Table 5.4 from run 328855.

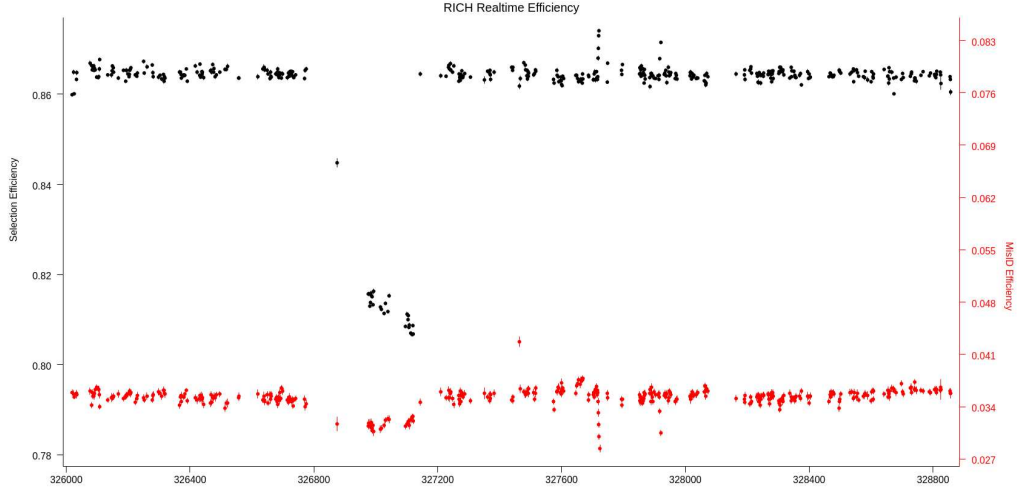


Figure 5.14: Trend of the (black) efficiency of selecting kaons and (red) associated misidentification of pion for $PIDK > 5$ in the integrated bin as a function of the LHCb run number. The figure reports data collected by LHCb from June 2025 to September 2025.

Chapter 6

The LHCb Upgrade II

The LHCb Upgrade II is designed to take maximum advantage of the High-Luminosity LHC (HL-LHC) environment. Operating at an instantaneous luminosity of $1 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ [313], starting with Run 5 in 2036. This enhancement will enable the collection of an unprecedented dataset of up to 300 fb^{-1} , positioning LHCb at the lead of a wide range of flavour physics studies [314].

The physics program of LHCb includes searches for new physics in CP violation phenomena, addressing the still-unresolved matter–antimatter asymmetry of the universe; precision studies in the charm sector; and the observation of rare decays, including forbidden tree-level flavour changing neutral current transitions. Other highlights include stringent tests of Lepton Flavour Universality, the exploration of exotic hadronic states, and numerous additional probes, where even small deviations from the Standard Model may signal new particles or interactions.

The central objective of LHCb Upgrade II is to achieve world-leading precision measurements [314] showed in Table 6.1, enabled by its vast datasets and advanced detector and data-processing capabilities. ATLAS and CMS will also contribute significantly to heavy-flavor studies at the HL-LHC with very specific decay modes involving muons in the final state.

In Sec. 6.1 a brief description of the proposed upgrade of the LHCb apparatus is presented. In Sec 6.2 descriptions of the scenarios, expected performance and detector candidates for the RICH upgrade are presented.

6.1. The Upgrade II of the LHCb apparatus

Observable	Run 1 and 2 (up to 9 fb ⁻¹)	Upgrade I (23 fb ⁻¹)	Upgrade I (50 fb ⁻¹)	Upgrade II (300 fb ⁻¹)
CKM tests				
γ ($B \rightarrow DK$, etc.)	2.8°	1.3°	0.8°	0.3°
ϕ_s ($B_s^0 \rightarrow J/\psi \phi$)	20 mrad	12 mrad	8 mrad	3 mrad
$ V_{ub} / V_{cb} $ ($\Lambda_b^0 \rightarrow p\mu^-\bar{\nu}_\mu$, etc.)	6%	3%	2%	1%
Charm				
ΔA_{CP} ($D^0 \rightarrow K^+K^-, \pi^+\pi^-$)	29×10^{-5}	13×10^{-5}	8×10^{-5}	3.3×10^{-5}
A_Γ ($D^0 \rightarrow K^+K^-, \pi^+\pi^-$)	11×10^{-5}	5×10^{-5}	3.2×10^{-5}	1.2×10^{-5}
Δx ($D^0 \rightarrow K_s^0\pi^+\pi^-$)	18×10^{-5}	6.3×10^{-5}	4.1×10^{-5}	1.6×10^{-5}
Rare Decays				
$\mathcal{B}(B^0 \rightarrow \mu^+\mu^-)/\mathcal{B}(B_s^0 \rightarrow \mu^+\mu^-)$	69%	41%	27%	11%
$S_{\mu\mu}$ ($B_s^0 \rightarrow \mu^+\mu^-$)	—	—	—	0.2
$A_T^{(2)}$ ($B^0 \rightarrow K^{*0}e^+e^-$)	0.10	0.060	0.043	0.016
A_T^{Im} ($B^0 \rightarrow K^{*0}e^+e^-$)	0.10	0.060	0.043	0.016
$S_{\phi\gamma}$ ($B_s^0 \rightarrow \phi\gamma$)	0.32	0.093	0.062	0.025
α_γ ($\Lambda_b^0 \rightarrow \Lambda\gamma$)	$^{+0.17}_{-0.29}$	0.148	0.097	0.038

Table 6.1: Uncertainties in key LHCb physics quantities, derived from Run 1 and Run 2 data, are compared with projections for the upgrades, evaluated using a Run 3 data sample of 23 fb⁻¹ and assuming systematic uncertainties are not the limiting factor. Upgrade II achieves up to a $\sim 90\%$ reduction in uncertainties compared to Run 1 and Run 2

6.1 The Upgrade II of the LHCb apparatus

The primary objective of Upgrade II of the LHCb apparatus [315] is to sustain and even enhance the detector performance. Trying to achieve this, several experimental challenges arise, given the extremely high luminosity foreseen, corresponding to roughly 40 pp interactions per bunch crossing. This results in much higher particle multiplicities (~ 2000 charged particles per crossing) and event rates compared with those in Upgrade I. Ongoing efforts are focused on developing new technologies for the detector. Beyond increasing granularity and reducing the overall material budget, a key innovation of Upgrade II will be the integration of precise timing information in combination with spatial hit reconstruction. This capability is crucial for suppressing combinatorial background in the very high pile-up environment expected.

In the following the description of the planned upgrades of each LHCb subdetector is presented, with the exception of the RICH system. The latter will be discussed in detail in the next section. Figure 6.1 shows the scheme of the LHCb detector according to the Upgrade II provisions.

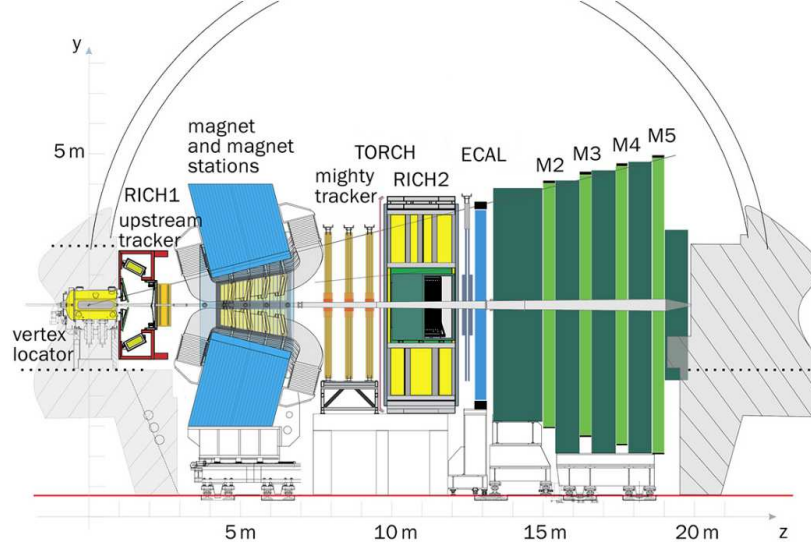


Figure 6.1: Scheme of the LHCb apparatus according to the Upgrade II design. The detector layout is shown in the zy plane. All detector components are indicated. The presence of the new tracking stations inside the magnet is evident, as are the additions of the TORCH system and the redesigned ECAL.

6.1.1 The LHCb tracking system upgrade

Vertex reconstruction and separation become particularly challenging at high luminosity, as the average distance between vertices decreases from the current 4.2 mm to about 1.5 mm. For Upgrade II, two design scenarios have been proposed, both aiming to maintain performance at the level of the current detector, with the same sensor layout along the z axis, but with a complete redesign of the mechanics and material budget.

- Scenario 1: The innermost radius is kept at the current value of 5.1 mm, and the average hit resolution remains unchanged. However, due to radiation damage in the sensors closest to the interaction point, regular replacements will be required.
- Scenario 2: The inner radius is increased to 12.5 mm, using improved sensors with hit resolutions better than 8 μm . This configuration would match the cluster occupancies of the Upgrade I VELO, while avoiding the need for sensor replacements.

In both cases, precise timing information is essential. Without it, track reconstruction efficiency degrades severely at higher occupancies. The key requirement

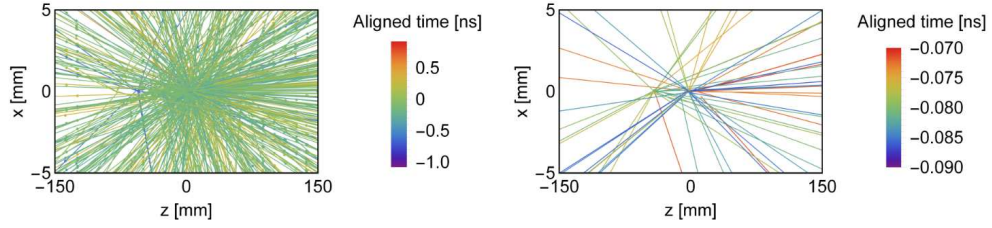


Figure 6.2: Comparison between the simulated tracks scatter plot from one bunch crossing of 42 colliding protons in a timestamp of 1 ns (left), with the one from the same bunch crossing in a timestamp of 20 ps (right). The comparison emphasizes the importance of the timing information added to the VELO hits, to not reduce the particles tracks reconstruction efficiency at the future high luminosity era.

is a per-track timestamp resolution of 20 ps or better (Figure 6.2), which can be achieved with individual hit timing resolutions of about 50 ps, thanks to multiple measurements per track [316]. The two scenarios described represent design choices intended mainly to guide the development of VELO. In practice, intermediate operating points between these extremes can be selected, enabling the detector to meet performance requirements while optimising aspects such as material budget, sensor longevity, and overall occupancy.

The current upstream and downstream trackers require significant upgrades to meet the demands of the future HL-LHC era. Their present designs cannot handle the expected particle rates. A highly promising technology under development is the Depleted Monolithic Active Pixel Sensors (DMAPS), foreseen for the upstream tracking system (UT) and the innermost region of the downstream tracker, referred to as the Mighty Tracker (MT) in Upgrade II. This technology, already adopted by other experiments [317, 318], enables readout pixels as small as $30 \times 30 \mu\text{m}^2$ with a time resolution of about 5 ns per pixel. Additionally, alternative sensor technologies are being explored to achieve time resolutions on the order of 100 ps, which would enable full tracking-time capabilities at reasonable costs. The proposed UT system layout consists of four planes of DMAPS, arranged in 12 staves with alternating orientations. For the MT system, a hybrid design has been suggested: three stations with two innermost layers of DMAPS pixel sensors close to the beam pipe, where high granularity is crucial, complemented by four layers of scintillating fibre (SciFi) detectors in the outer regions. To meet Upgrade II requirements, substantial progress is also needed in the development of scintillating fibres and SiPM photomultipliers, particularly to mitigate radiation damage, enhance light yield, and suppress noise [319]. A scheme of the proposed trackers is shown in Figure 6.3. A possible improvement in LHCb Upgrade II is the planned

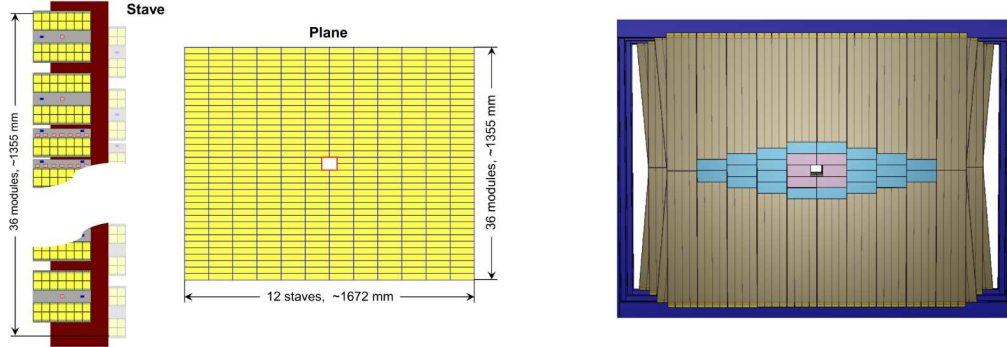


Figure 6.3: Layout of a pixel sensor plane planned for the UT tracker in Upgrade II, including details of the sensor arrangement. Each UT plane will consist of 12 staves, alternately populated on both sides with 36 modules, forming a fully active area (Left). Schematic of the downstream tracker stations for Upgrade II. Three stations are planned, each featuring two layers of DMAPS pixel sensors in the innermost regions (pink and light blue) and four SciFi detection layers in the outer regions (Right).

installation of Magnet Stations (MS) inside the dipole magnet, as shown in Figure 6.4. These tracking stations would measure both the position and direction of particles interacting with the magnet side walls. This addition extends tracking capabilities to particles with momenta below 5 GeV normally leaving signals only in the VELO and UT detectors due to magnetic deflection. The design foresees four panels, each containing four layers of triangular scintillating bars with embedded wavelength shifters. Positioned outside the primary detector acceptance, the MS introduces no extra material for long tracks.

6.1.2 The LHCb PID system upgrade

The Time Of internally Reflected Cherenkov light (TORCH) system will improve the PID at low momentum (2–10 GeV) introducing pK separation below 10 GeV. The TORCH system consists of 18 modules of innovative time-of-flight (ToF) detectors. Each module features a 10 mm thick quartz radiator plate that generates Cherenkov photons from charged particle tracks. These photons propagate via total internal reflection to a cylindrically mirrored surface and are then focused onto pixelated photon detectors [320] as shown in Figure 6.5. TORCH is specifically designed to distinguish protons from kaons using ToF measurements from the pp interaction point. The system is expected to achieve a timing resolution of about 15 ps, taking into account several factors: the positions of the incident photons on the detectors, their propagation directions, the time distribution of pixel signals

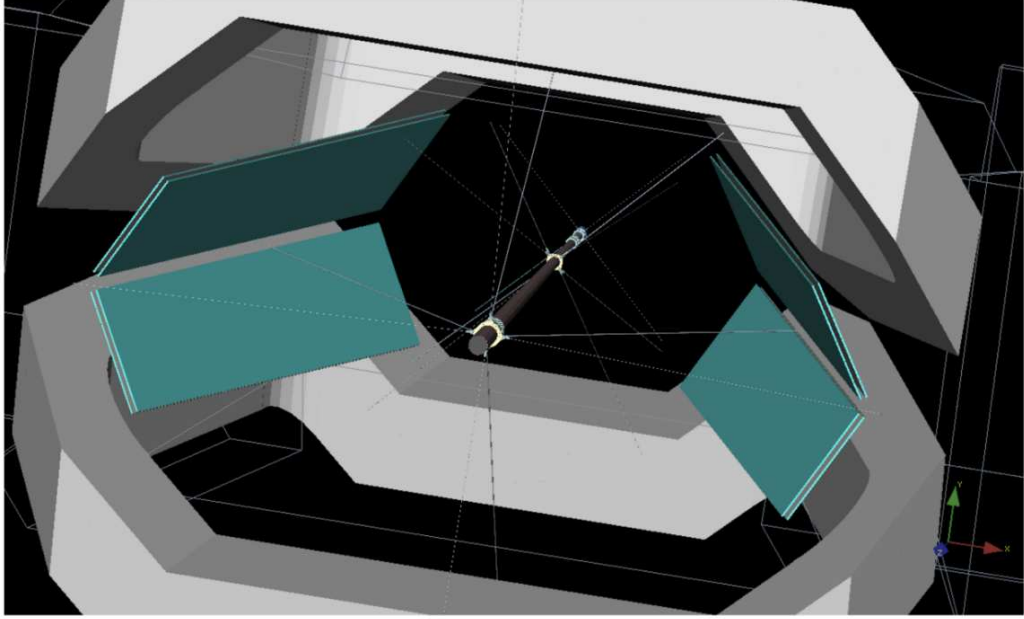


Figure 6.4: Layout of the Magnet Stations (cyan panels) in the LHCb dipole magnet.

influenced by photon paths within the radiator [321, 322], and track information provided by the tracking system.

Regarding the ECAL, a finer cell granularity is required for Upgrade II to handle overlapping electromagnetic showers, which can degrade particle energy resolution and reconstruction efficiency. This efficiency can be maintained at levels comparable to the current detector by reducing the Molière radius, which also decreases the lateral size of the showers. Such improvement is essential for distinguishing the two photons from π^0 decays in heavy-flavour events. Several technological options are being considered. The most promising approach combines the current ECAL technology in the outer regions with a new technology in the central region. This central region would employ a tungsten/lead absorber with an active medium of longitudinal crystal fibres that serve both as scintillators and as light guides to the PMTs. Polystyrene plastic fibres are also suitable for the intermediate region. This design, known as the Spaghetti Calorimeter (SpaCal) [323], does not require wavelength-shifting fibres and is compatible with the higher radiation doses expected in Run 5 and beyond. The SpaCal design offers flexibility in defining cell size according to the fibre readout area, improving spatial resolution. However, accurate timing information on particle arrival is also required. Studies indicate that both the current ECAL technology (outer region) and the SpaCal system

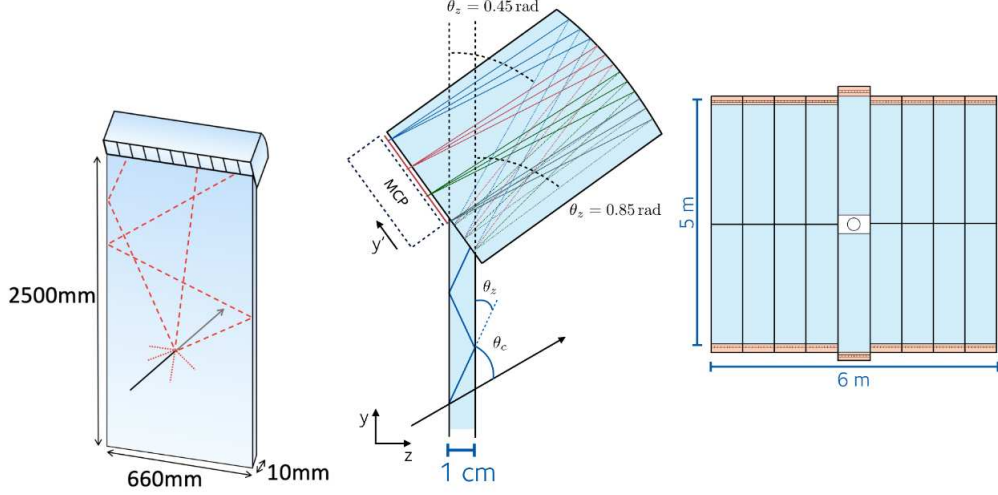


Figure 6.5: Schematics of the TORCH detector design. (Left) a single TORCH module, (middle) the focusing block, and (right) the 18-module TORCH detector proposed for LHCb.

(central region) provide excellent light generation and transport, enabling timing resolutions of up to 15 ps at high energy. High occupancy in the central region motivates splitting the SpaCal modules longitudinally into two sections at the shower maximum. The front section, where showers are less developed, reduces pile-up effects, while the back section provides precise timing information to aid clusterisation across adjacent cells. Additionally, the possibility of adding a dedicated timing layer between the front and back sections, covering both inner and outer regions, is under evaluation. A scheme of the proposed SpaCal is shown in Figure 6.6.

The muon system consists of four stations, divided into regions at increasing distances from the beam line. Under Upgrade II conditions, higher particle fluxes pose major challenges due to high occupancy and dead time. A higher-granularity detectors is needed, the inner region of the detector employ μ -RWELL chambers while improved MWPC are used in the outer region. The readout system is also redesigned, requiring new front-end ASICs.

6.1.3 Upgrade of the readout and trigger systems

Upgrade II studies also focus on improving the LHCb trigger system. A key challenge for an efficient trigger at high luminosity is the enormous data rate generated, over 200 Tb/s, which must be processed in real time and reduced by several orders

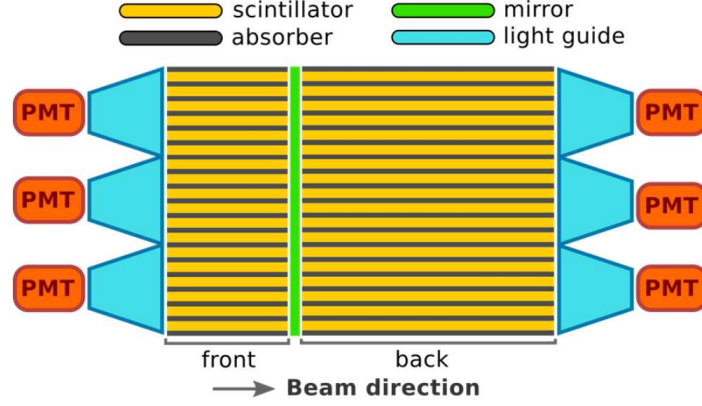


Figure 6.6: Schematic side view of the SpaCal configuration

of magnitude before permanent storage. While the overall computational architecture will largely follow the Run 3 design, several upgrades are required to handle the increased data volume. These include the installation of new electronics links and servers, potential redesigns of readout boards, and the deployment of GPUs or other high-performance processors. The system must remain reliable, scalable, and compatible with heterogeneous computing elements. Cost optimisation is also a priority; unlike Runs 1 and 2, only widely available, mass-produced devices will be considered, as custom computing technologies would be prohibitively expensive. Data processing is the main challenge affecting trigger efficiency, particularly due to the high number of hadrons produced in LHCb acceptance from roughly 40 pp interactions per bunch crossing. A conventional trigger based solely on track topology and vertex displacement is insufficient for Upgrade II, due to the higher production rates of b and c hadrons. Such an approach would fail to efficiently select physics events in the crowded Run 5 environment and would not significantly reduce the data rate at early trigger stages. The Upgrade II trigger strategy instead aims to suppress hits unrelated to pp interactions of interest as early as possible. This is made feasible by the improved timing information provided by the upgraded detectors. Early discrimination of relevant hits accelerates particle reconstruction and allows precise spatial and temporal separation, which is essential for identifying heavy-flavour signatures while reducing combinatorial background. The system will perform initial reconstruction to identify inclusive beauty and charm signatures and high- p_T muons. The trigger lines will largely follow Run 3 designs, including inclusive selections based on displaced tracks, muon identification, p_T , or invariant-mass. Additional lines target charmed hadrons and very rare signals such as diproton or diphoton events. Then it will execute a full, high-quality reconstruction using calibration and alignment information, analo-

gous to Run 3.

6.2 The RICH upgrade

The LHCb RICH pattern recognition algorithm matches observed photodetector hits with expected patterns for each particle type, based on tracking information. While it has proven robust in Runs 1–3, its performance will degrade at Run 5 due to the higher track and photon densities. In high-occupancy regions, multiple photons can hit the same pixel, causing information loss due to binary readout and straining the readout electronics. Performance can be recovered by increasing detector granularity to reduce occupancy and by reconstructing the Cherenkov angle more precisely. Further improvements come from associating tracks with photons in time. Simulation studies for Upgrade II show that photon time-stamps with < 100 ps resolution are needed, compared to the purely 3D reconstruction used at Run 3. New electronics under development can provide 25 ps time bins with a 2 ns gate to suppress out-of-time background and data load. Currently, RICH1 and central RICH2 use MaPMTs with $2.88 \times 2.88 \text{ mm}^2$ pixels, while outer RICH2 pixels are $6 \times 6 \text{ mm}^2$, corresponding to ~ 200000 channels. Upgrade II will require smaller pixels, leading to a higher channel count. The Cherenkov angle resolution depends on several factors: pixel size (detection point error), chromatic dispersion (chromatic error), optical aberrations (emission point error), and tracking. Reducing these is essential for improved performance. Chromatic error can be lowered by shifting the photon spectrum toward the green band, where dispersion is smaller. Emission point error can be reduced with improved optics. Tracking is assumed to remain at least at Run 3 performance, having an impact equal to or smaller than other sources of error

Ongoing work includes a full simulation of the detector and improved reconstruction algorithms, making full use of precision timing and exploring further optimisations in optics, photon yield, and photodetector technologies, especially in the lower-occupancy regions of RICH1 and RICH2.

6.2.1 Scenarios

Three detector scenarios with different photodetector granularity and optics are being studied: Baseline, Middle, and Low. All require a new optical layout for RICH1, since simulations show the current design would limit PID performance even with good timing (35 ps). The new layout includes moving the flat mirrors into acceptance and slightly increasing the curvature radius of the spherical mirrors. Three types of photodetectors are under review: SiPMs, MCP-PMTs (including LAPPDs), and MaPMTs. Each has pros and cons, with ongoing studies to ad-

dress their limits. In all scenarios, finer pixels are foreseen for the high-occupancy inner regions. The Baseline scenario assigns 1/3 of the detector plane to this region, while the Middle and Low scenarios use 1/4. The Low scenario also adds a third, coarser-granularity zone at the outer edges of RICH2 with $5.6 \times 5.6 \text{ mm}^2$ pixels, reducing channel count but lowering performance.

- Baseline: Adds TORCH and new mirror geometry for both RICH1 and RICH2 with carbon-fiber mirrors and gas vessels. Occupancy stays well under control.
- Middle: Adds TORCH and a new RICH2 vessel; occupancy can exceed 30% in parts of RICH1, requiring further optical optimisations.
- Low: Uses MaPMTs in the outer regions to cut cost and risk, but timing is insufficient, reducing performance.

6.2.2 Performance

The PID performance of the three detector options was studied using emulation, since full simulations for the new geometries were not yet available. This approach allows comparison of detector granularity, timing windows, and quantum efficiency, assuming one photon per pixel per bunch crossing and $\sim 90\%$ hardware efficiency. Cherenkov angles are smeared according to the expected resolution, and photon yield is reduced if required by the optical layout. Timing is included, with photons only associated to tracks if they fall inside the chosen time window. To avoid signal loss, the window must be at least four times the photodetector resolution; better timing allows tighter windows and stronger background rejection. Limitations of the emulation include: fixed photon distributions and spectra, only average photon yield reductions, and no simulation of electronic saturation (beyond overlapping hits in one pixel). The Low scenario does not include the reduced granularity at the RICH2 outer edges, but this has negligible impact due to low occupancy in that region. Performance was evaluated with 150 ps and 300 ps time windows, corresponding to single-photon resolutions of 38 ps and 75 ps. Figure 6.7 shows that improved RICH2 optics are crucial: the Baseline and Middle scenarios perform similarly at high momentum. With the wider 300 ps window, Baseline performance drops at low momentum under high luminosity due to occupancy, but this effect is largely reduced with the narrower 150 ps window.

6.2.3 Upgrade II photon detector candidates

The MaPMTs that are being used during Run 3 and will be used in Run 4 are not suitable in any region of the RICH detectors, lacking the required pixel granular-

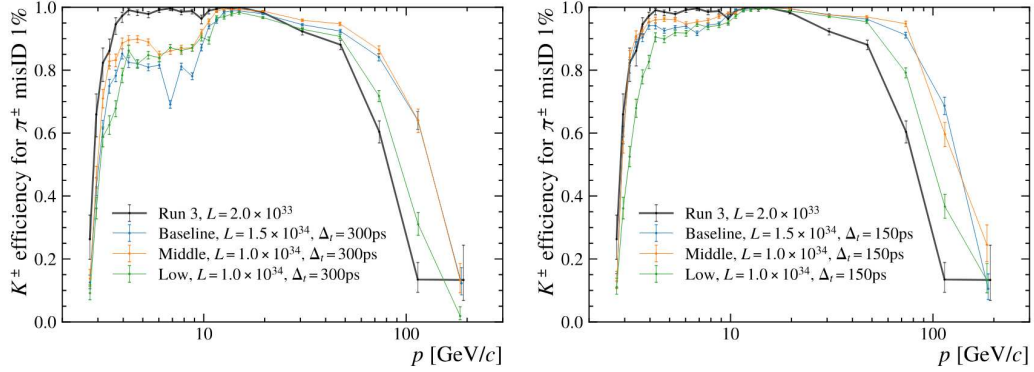


Figure 6.7: Kaon efficiency for 1% pion misidentification versus momentum, for the Run 3 and the three RICH Upgrade II options at different luminosities, for (left) 300 ps and (right) 150 ps time windows, assuming the same average photoelectron yields

ity and timing resolution. Moreover, the current proposal for the RICH1 optics upgrade does not leave enough space for magnetic shielding, requiring detectors which are minimally affected by strong magnetic fields. Thus, new photodetectors are required to meet the requirements on the granularity, the timing resolution and the magnetic resistance. Currently, the most promising candidates to be used are SiPMs and MCPs.

Silicon Photomultiplier overview

A Silicon Photomultiplier (SiPM) is a solid-state photodetector composed of an array of Single-Photon Avalanche Diodes (SPADs). Each SPAD is capable of detecting individual photons with excellent timing resolution [324].

The avalanche photodiode (APD) is a p-n junction operated under reverse bias in the depletion region. Incident photons generate electron-hole pairs via the photoelectric effect, which are separated by the internal electric field. In an APD the field is strong enough to accelerate carriers so that, through impact ionization, they create additional electron-hole pairs. This avalanche process amplifies the primary signal, enabling the detection of weak light intensities. In silicon, electrons dominate the multiplication process, giving a linear relation between the output signal and the number of absorbed photons. The device performance is strongly dependent on the reverse bias voltage. At low voltages, it behaves like a simple photodiode with no gain. In the linear mode, where only electrons contribute significantly to impact ionization, the output current is proportional to the light intensity, resembling the behavior of a photomultiplier tube. At higher voltages, holes also contribute to the avalanche, leading to non-linear signal amplification.

Since multiplication is a statistical process, the gain fluctuates and introduces additional noise.

In an APD, the avalanche process stops once all generated electrons and holes are extracted without further ionization. If the electric field is strong enough that carriers undergo impact ionization faster than they are removed, the avalanche becomes self-sustaining, leading to an exponential increase in charge carriers. This condition is known as avalanche breakdown, and an APD operated above this point is called a Geiger-mode APD (GAPD), due to its similarity to a Geiger counter. The very high electric field results in improved quantum efficiency and excellent time resolution due to the rapid avalanche buildup and minimal carrier transit time variations, making GAPDs capable of detecting single photons. However, they cannot distinguish between one or multiple simultaneous photons, so they behave as digital detectors. For this reason, GAPDs specifically designed for single-photon detection are referred to as Single-Photon Avalanche Diodes (SPADs).

A SiPM consists of multiple identical SPADs connected in parallel to form a single output. This design preserves the high sensitivity and gain of individual SPADs while providing a wider dynamic range, allowing the detection of higher light intensities. The main parameters describing a SiPM are:

- breakdown voltage;
- photo-detection efficiency (PDE);
- Primary and Correlated noise, measured by the Dark Count Rate (DCR) and the Excess Noise Factor (ENF), respectively;
- Single-Photon Timing Resolution (SPTR).

The breakdown voltage of a SiPM is the minimum bias at which the device operates in Geiger mode, increasing with both the depletion region thickness and temperature. The photon detection efficiency (PDE) depends strongly on the silicon absorption coefficient, which varies widely with wavelength, so the SiPM structure must be optimised for the application. PDE is determined by the quantum efficiency (QE), the probability that a photon generates a primary electron-hole pair in the active volume, and the avalanche breakdown triggering probability, as well as the fill factor, which is the ratio of the SPAD active area to its total area. SiPMs exhibit several noise sources, categorised as primary and correlated noise. Primary noise arises from thermally or tunnel-generated carriers that trigger discharges, defining the Dark Count Rate (DCR), which follows a Poisson distribution. Thermal generation dominates, typically following an exponential temperature dependence, but high electric fields and defects increase DCR via tunneling.

Reducing material impurities and cooling the device lowers DCR, which is critical after irradiation. Correlated noise includes optical cross-talk, where an avalanche in one SPAD triggers a neighboring SPAD. Afterpulses, caused by trapped carriers emitted after the avalanche, produce secondary discharge. SiPMs also offer excellent timing resolution, quantified by the Single Photon Timing Resolution, which measures the precision of photon arrival time estimation. The timing distribution typically exhibits a Gaussian peak, arising from avalanche build-up jitter, electric field non-uniformity, charge collection time variations, transit time spread, and front-end electronics, along with an exponential tail caused by carriers generated in the undepleted region.

SiPMs are considered highly promising candidates for the LHCb RICH Upgrade II for several reasons. They offer excellent photon-detection efficiency, with peak sensitivity at longer wavelengths compared to MaPMTs, an important advantage, as shifting the PDE toward the green region helps improve single-photon resolution. In addition, SiPMs are inherently insensitive to magnetic fields, eliminating the need for magnetic shielding, and they operate at relatively low voltages. The main limitation of SiPMs lies in their high DCR, particularly after significant irradiation, expected to reach $6 \cdot 10^{13}$ 1MeV neq/cm² in the RICH1 region at the end of Run 5. To mitigate this, techniques such as cryogenic cooling, neutron shielding, and annealing are necessary [325]. Ongoing studies are exploring additional strategies to further suppress the DCR, including the use of micro-lenses to reduce the effective pixel area, as well as time gating at the front-end electronics stage [316].

MicroChannel Plate overview

A microchannel plate (MCP) is an array of microscopic channels that function as electron multipliers, utilizing the channel walls as a continuous series of dynodes. Depending on the bias voltage applied across the MCP, each channel can amplify an electron signal with a gain ranging from 10^3 to 10^7 . The small transit time spread enables excellent timing resolution of 20-30 ps, while the spatial resolution is primarily limited by the channel diameter and pitch. Figure 6.8 illustrates the basic operating principle of an MCP. Two key geometric parameters define MCP performance. The length-to-diameter ratio largely determines the electrical behavior [326], while the open-area ratio, the fraction of the total MCP area occupied by pores, affects efficiency. When combined with a photocathode for photon-to-electron conversion, MCPs enable single-photon sensitivity over large areas with high spatial and temporal resolution and low dark counts. Additionally, MCPs are minimally affected by magnetic fields [326].

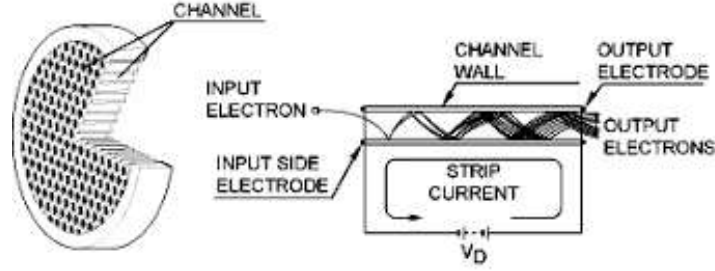


Figure 6.8: Schematic of MCP operating principle.

A primary limitation of MCPs is their limited lifetime, largely due to ion feedback. This occurs when high-charge electron pulses at the MCP output generate residual gas ions, which are accelerated back toward the channel entrance. These ions can degrade the photocathode, reduce QE, and produce secondary electrons, resulting in additional, smaller output pulses. The effect increases channel dead-time and becomes significant at high gains (order of 10^5). The main strategies, illustrated in Figure 6.9 to mitigate this effect include:

- Two MCPs with opposite tilt angles reduce ion feedback and achieve gains of $10^6 - 10^7$. The charge resolution can be further enhanced by minimizing the gap between MCPs [327].
- Z-stack configuration: three MCPs stacked with alternating tilt angles can provide even higher gain.
- Curved channels: prevent ions from reaching the entrance; however, fabrication of small curved channels is challenging.

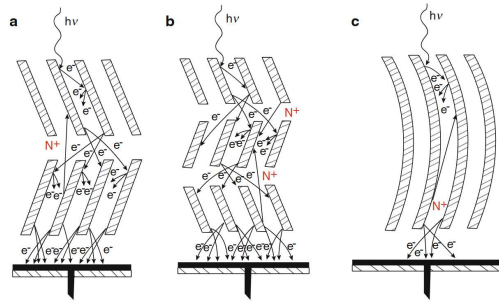


Figure 6.9: MCP configuration: a) Chevron MCP stack. b) Z-plate MCP stack. c) Curved-channel C-plate MCP.

Electron back-scattering at the MCP input or output anode can generate image halos, reducing spatial resolution. Initial mitigation involves minimizing the gaps

between the photocathode and MCP input, and between MCP output and anode. A more advanced solution is end-spoiling, in which electrode material is evaporated onto the MCP input and output surfaces, penetrating each channel to a depth d_{es} relative to the channel diameter d . End-spoiling suppresses secondary emission from channel ends, improving spatial resolution at the cost of some gain. Typical end-spoiling depths are $1d$, $2d$, or $3d$, representing multiples of the channel diameter.

MCPs exhibit a low DCR and, combined with their excellent timing resolution, typically on the order of 30 ps represent a very promising option for the LHCb RICH Upgrade II. The primary limitations of MCPs are their restricted lifetime and gain saturation at high rates, although substantial improvements have been achieved through ALD coating [328]. Ongoing detector development aims to fully exploit MCP advantages while mitigating these drawbacks [316]. Among these efforts is the design of MCP-PMTs with a pixelated anode, incorporating a CMOS ASIC capable of reading charge signals at very high rates. This approach could maintain MCP timing and spatial resolution while operating at lower gain, thereby extending lifetime. Research is also underway on Large Area Picosecond Photo Detectors (LAPPDs) [329], which are single-photon detectors typically measuring 20 cm² and composed of borosilicate glass MCPs, achieving timing resolutions around 50 ps. However, their current spatial granularity, limited by their larger pore sizes, does not yet meet the requirements for the RICH upgrade.

Chapter 7

4DPHOTON project

The development of a new photodetector could enable major advances in fields such as particle physics, life sciences, and optical quantum experiments [330]. Such a device must detect single photons with outstanding timing and spatial resolution, while operating efficiently at high rates. A detector that combines such characteristics would therefore represent a breakthrough for both particle physics and imaging applications. While a four-dimensional charged particle tracker has already been demonstrated [331], an equivalent single-photon detector providing simultaneous precise timing and position information, low noise, high rates, and wide coverage is still lacking.

The 4DPHOTON project aims to fill this gap by developing a hybrid detector that integrates complementary technologies, each optimised for specific functions [332, 333]. Its architecture consists of a vacuum tube with a photocathode, an embedded MCP, and a Timepix4 ASIC serving as a pixelated anode. The ultimate goal is to produce a compact, large-area photodetector capable of measuring single photons with unprecedented spatial and temporal precision, while maintaining low noise at room temperature [332]. Such performance would overcome current limitations in light-based imaging and open new opportunities across disciplines. In high-energy physics in particular, it could enable a new generation of Ring Imaging Cherenkov detectors, ensuring efficient particle identification in challenging high-luminosity environments.

Sec 7.1 provides an overview of the state of the art in pixelated ASICs coupled to MCPs. Sec. 7.2 provides an overview of the 4DPHOTON detector. In Sec.7.3 a brief description of the software to control the Timepix4, read out the data and analyse them is reported, while Sec. 7.4 discusses its possible applications.

7.1 State of the art of pixelated ASIC coupled to MCPs

Pixelated ASICs are typically designed for high-granularity solid-state sensors, but they have also been successfully employed to read out MCPs and Gas Electron Multipliers (GEMs) [334, 335, 336, 337]. Both Medipix2 and Timepix ASICs have been used to read out MCPs for the detection of photons, electrons, and neutrons [338]. Coupling MCPs to pixelated ASICs enables spatial resolution limited only by the MCP pore size and pitch, together with good timing performance. Optical MCP image tube demonstrators have been realised with a single Medipix2 ASIC [335] and with a 2×2 Timepix array [336].

The first optical MCP image tube, consisted of an MCP sealed in a vacuum tube with a photocathode on the input window and a Medipix2 ASIC acting as the pixelated anode. Integrating Medipix2 inside the vacuum imposed strict mechanical and electrical constraints. The MCP–ASIC distance was minimised to preserve spatial resolution. A larger optical MCP-PMT using a 2×2 Timepix array was later developed [336]. Its design mirrored the Medipix2-based tube but was optimised for larger area and better efficiency. It used a planar input window with a semi-transparent bi-alkali photocathode, two Chevron-configured MCPs placed 4.5 mm from the photocathode, and the Timepix array located 0.5 mm from the MCP output.

Spatial resolution, initially limited by the 4.5 mm photocathode–MCP gap, was 165 μm FWHM detecting light passing through a pinhole mask placed in front of the photocathode. However, by diffusing light over the whole MCP, it has been possible to resolve the MCP pore spacing of 32 μm , since electrons entered in each channel, thus the resolution is not limited anymore by the distance between photocathode and MCP, but depends only on the one between MCP and Timepix, and the Timepix spatial resolution itself. To reach this resolution, the Timepix has been operated in Time-over-Threshold (ToT) mode, and using the charge sharing information to estimate the electron clusters centroids, a resolution better than 5 μm has been achieved. Although optical measurements remain limited by the photocathode–MCP separation, optimised geometries could in principle reach sub-pixel resolution as well.

The detector timing resolution is mainly constrained by time-walk, i.e., the dependence of the measured Time-of-Arrival (ToA) on signal amplitude: weaker signals cross the threshold later, introducing delays. Correcting this effect requires simultaneous ToT and ToA information, but Timepix cannot simultaneously record both, limiting achievable timing resolution to 100–200 ns, see Figure 7.1.

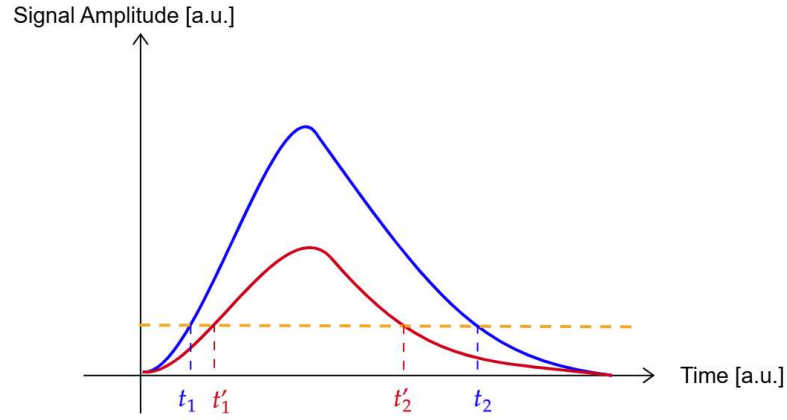


Figure 7.1: Representation of the time-walk effect: a lower signal induced simultaneously with a higher signal will rise above the threshold slightly later than the higher signal.

7.2 4DPHOTON description

The hybrid assembly, shown in Figure 7.2, is based on a vacuum tube that houses the MCP-PMT components under high vacuum conditions ($\sim 10^{-10}$ mbar).

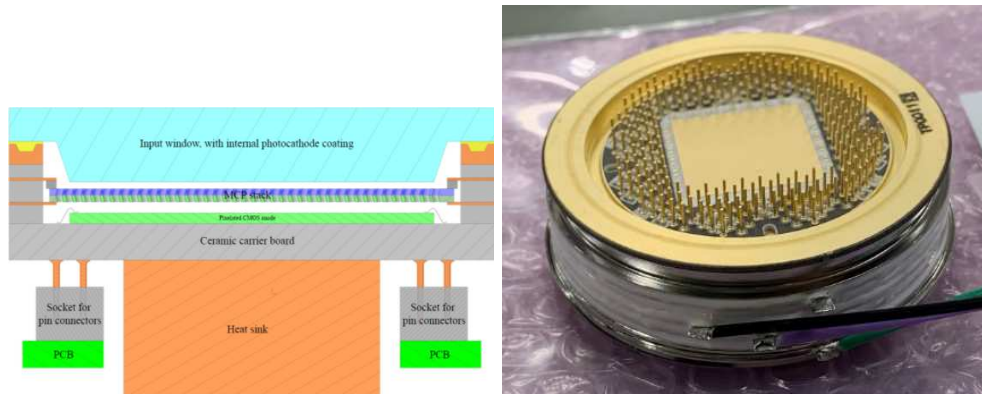


Figure 7.2: (Left) Simplified cross section of a of the detector assembly. (Right) The 4DPHOTON detector prototype.

At the top of the tube, a silicate window, transparent across the IR, visible, and UV ranges, serves as the photon entrance window. A high QE photocathode is deposited on its inner surface. Behind the photocathode, an MCP stack arranged in either Chevron or Z-stack configuration multiplies the single electrons emitted by the photocathode. The amplified electron cloud is collected by a bare Timepix4

CMOS ASIC, which functions as a pixelated anode. The Timepix4 is bonded to a ceramic carrier board, which forms the bottom cap of the tube and provides the interface between the vacuum environment and the external detector components. Power lines, input/output signals, and high-speed data connections are routed through pins, and a custom Pin Grid Array enables the detector to be mounted onto a dedicated socket on a Printed Circuit Board (PCB). The PCB, in turn, links the detector to an FPGA-based data acquisition system responsible for both configuration and data readout. During operation, the Timepix4 can dissipate up to 5 W. To prevent overheating, a heatsink connected to a chiller system provides active cooling, ensuring stable operating conditions and avoiding performance degradation. The operating principle of the detector, schematically illustrated in Figure 7.3, can be summarised as follows:

- A photon entering the input window is converted by the photocathode into a photoelectron.
- The photoelectron is accelerated by an electric drift field onto the MCP stack, where it triggers the emission of secondary electrons, amplifying the signal.
- The resulting electron cloud is transported by a second drift field onto the input bump-bonding pads of the bare Timepix4 ASIC, where it is detected by the readout electronics.
- The Timepix4 then amplifies, discriminates, and digitises the MCP signal.

7.2.1 Optical window and photocathode

The input window is specifically shaped to position the photocathode close to the MCP stack, improving spatial resolution. Electrons emitted from the photocathode possess an initial momentum component parallel to its surface [339], causing them to drift parallel to the photocathode and perpendicular to the drift field. This transverse motion ceases when the electron enters an MCP channel. The effect of this motion is a random shift in the detected electron position, proportional to the initial momentum and the flight time from the photocathode to the MCP. The flight time can be minimised by increasing the electron drift speed, achieved by raising the voltage difference between the photocathode and MCP, and by reducing the flight distance. The baseline photocathode material will be either a bi-alkali variant or a multi-alkali variant. Both materials offer excellent timing capabilities, good quantum efficiency, and negligible dark count rates. The multi-alkali variant provides a broader spectral response than the bi-alkali, covering wavelengths from the ultraviolet to the near-infrared. The photocathode is

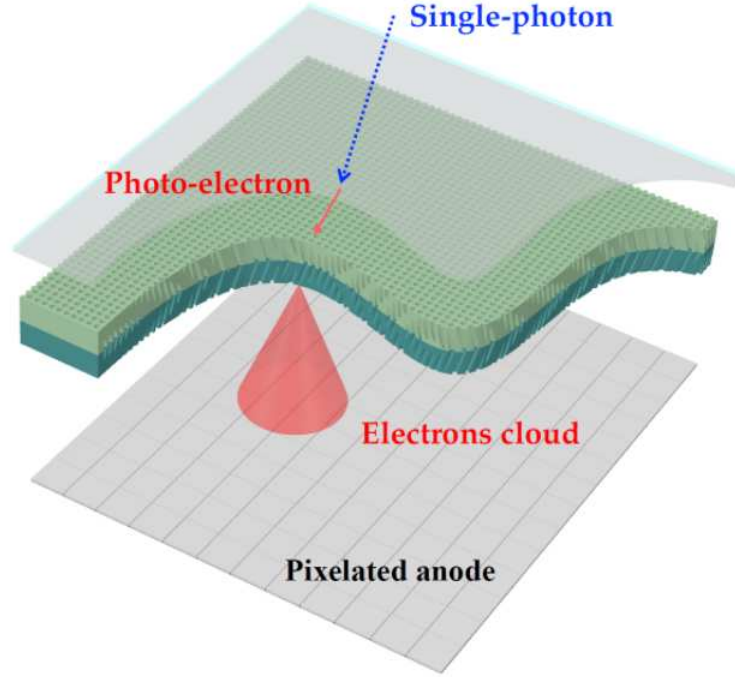


Figure 7.3: Visual representation of the hybrid detector operating principle

selected to have high QE ($\sim 30 - 40\%$) in the Cherenkov photon region and a low dark count rate ($\sim 10^2 - 10^3$ Hz/cm² at 300 K). Using a photocathode to externally convert photons to electrons adds design flexibility, as the spectral sensitivity can be modified simply by replacing the photocathode material.

7.2.2 MicroChannel Plate

To achieve spatial resolution of ~ 10 μm , the microchannels must have a diameter of 6.0 μm and a pitch of 7.5 μm . Considering the Timepix4 diagonal size, the MCP effective diameter must be at least 42 mm, corresponding to an outer diameter of approximately 50 mm. The MCP open-area ratio should be at least 60% to maximise detection efficiency. A careful balance is required between the lateral spread of the electron cloud and the MCP gain: sufficient charge must be delivered to each pixel to fully exploit the Timepix4 timing performance, while spreading the cloud over multiple pixels improves resolution through clustering analysis.

The MCP will operate at a gain of $10^4 - 10^5$ to reduce ion feedback and extend the detector lifetime. The Timepix4 low noise level allows operation at these lower gains, which provides several advantages: the MCP lifetime is increased by more than an order of magnitude compared to standard applications, the MCP

channels recharge faster, and the electronics experience lower dead-time due to reduced time-over-threshold values.

Two main MCP configurations have been considered to achieve the required performance at low gain: a double-stack MCP in Chevron configuration and a three-stack MCP in Z configuration. Both configurations enable higher gains at lower voltages while minimising ion feedback. The MCP channels have a bias angle of 5° , providing a good compromise between gain and spatial resolution [340]. End-spoiling is applied to both the input and output channels. On the output face of the MCP three end-spoiling configurations are under test: $1d$, $2d$ and $3d$. On the input face, end-spoiling is kept below $0.5d$ to minimise the probability that electrons strike the electrode instead of the channel walls.

7.2.3 Timepix4

The pixelated anode is implemented using a bare Timepix4, a CMOS ASIC fabricated in 65 nm technology, developed by the Medipix4 collaboration [334]. Timepix4 comprises 512×448 square pixels with a pitch of $55 \mu\text{m}$, corresponding to an active area of approximately 6.94 cm^2 . This configuration yields a binary spatial resolution determined by the $55 \mu\text{m}$ bins, equivalent to $16 \mu\text{m}$ r.m.s. resolution. The bare ASIC is mounted onto a ceramic carrier board via wire-bonding, with MCP signals coupled through bump-bonding pads. Each pixel includes electronics for amplification and discrimination.

Signal timing is recorded by an integrated Time-to-Digital Converter (TDC), providing time stamps with a bin size of 195 ps, equivalent to a ~ 56 ps r.m.s. resolution, exploiting a local Voltage-Controlled Oscillator (VCO) oscillating at the nominal frequency of 640 MHz. Timepix4 supports extremely high rates, achieving a maximum bandwidth of 163.84 Gb/s, with data transmitted through 16 high-speed links operating at up to 10.24 Gb/s each. The output per event consists of a 64-bit data word encoded with the 64b/66b protocol. The ASIC also ensures a low equivalent noise charge of $\sim 80 e^-$ per pixel. For each pixel, ToA and ToT are measured simultaneously. The ToT not only provides an estimate of the deposited charge but is also used to correct for time-walk effects. As a result, individual pixels achieve spatial and timing resolutions of $\sim 16 \mu\text{m}$ and ~ 56 ps, respectively.

A major advantage of Timepix4 lies in the ability to perform three-dimensional clustering (two spatial dimensions plus time), by exploiting pixel coordinates together with time-stamp information. Multi-pixel sampling within a cluster improves precision: by applying a weighted average of ToA values, with ToT as weight, the centroid timing resolution can be enhanced to $O(10 \text{ ps})$. Similarly, the cluster centroid spatial resolution improves to $\sim 5\text{--}10 \mu\text{m}$, approaching the intrinsic

MCP channel spacing [341].

Finally, since the electron cloud emerging from the MCP spreads over several pixels, the MCP–Timepix4 spacing must be carefully optimised. The goal is to ensure sufficient charge deposition within each pixel to preserve good timing resolution, while at the same time spreading the charge over enough pixels to enable centroid algorithms to achieve improved spatial resolution.

The Timepix4 ASICs integrated into the tubes have been thinned from their original thickness of $\sim 700\ \mu\text{m}$ down to $300\ \mu\text{m}$ due to spatial constrain. Since the ASICs are wire-bonded to the ceramic carrier, the stand-off height of the wire bonds above the carrier imposes an additional constraint on the MCP-to-anode distance, which cannot be reduced below $300\ \mu\text{m}$.

7.2.4 Readout electronics

The Timepix4 ASIC exploits a data-driven architecture: each pixel generates an output signal only when it is hit, after which the signal is encoded and transmitted to the off-detector electronics. The data acquisition (DAQ) system receives this stream, processes the data, and stores the results in PC memory. The detector is connected to the DAQ system via a dedicated board, referred to as the tube card, which houses the custom MCP-PMT PGA socket, provides power supply to the ASIC and transmits input and output (I/O) signals for the communication. The tube card interfaces with the main control board through an FPGA Card (FMC) connector. Within the DAQ system, the Timepix4 I/O signals are routed to an FPGA, which handles communication with a remote server for detector control. Additionally, the DAQ includes an optical transceiver to enable high-speed transmission of the output data.

7.2.5 Vacuum tube

The vacuum tube, developed by Hamamatsu Photonics, has been designed to meet the stringent performance requirements of the project. The key specifications include a spatial resolution of $10\ \mu\text{m}$ and timing resolution below $100\ \text{ps}$ for single-photon measurements, quantum efficiency exceeding 20% around $400\ \text{nm}$, an MCP gain in the range $10^4 - 10^5$ and long-term operational reliability, requiring excellent vacuum tightness, suppression of ion feedback, and thermal stability.

7.2.6 Ceramic carrier

The Timepix4 ASIC is mounted on a ceramic carrier using die-attach material and wire-bonding connections, enabling input/output signals to be routed outside

the tube via PGA connectors. The ceramic carrier, developed jointly by INFN, CERN, and Kyocera and manufactured by Kyocera, is a multilayer ceramic board that provides vacuum feedthrough connections for the ASIC I/O signals. On the back side, the I/O signals are arranged around the perimeter of the Timepix4 in a PGA configuration, allowing insertion into a compatible socket on the external PCB. The carrier is designed with a central opening directly beneath the ASIC, enabling the attachment of a heat sink or cooling element. Signal transmission is achieved via spring-loaded contact pins, which connect to a custom socket on the electronics board. From there, the digital output is transferred to the acquisition system through a combination of electrical and optical high-speed links.

7.3 Data Acquisition and data analysis softwares

The 4DPHOTON project also involves the development of the software to control Timepix4 and acquire its data, as well as the creation of a data analysis framework for decoding, processing, and analysing the information [342].

7.3.1 DAQ software architecture

The software must be high-performance to match the Timepix4 output bandwidth, while remaining user-friendly and flexible enough to allow simple modifications of acquisition tasks and setups. C++ was chosen to meet the speed requirement, offering low-level control over hardware and data. Flexibility is achieved through an object-oriented design, with independent classes for each hardware component and DAQ system, enabling adaptation to different configurations and communication protocols. User-friendliness is addressed by simplifying basic operations while retaining the ability for detailed, bit-level control of the Timepix4, balancing ease of use with full configurability. The software architecture is divided into two main sections: slow control configuration (including slow readout) and high-speed readout. Both are accessible via a Command Line Interface. Slow control relies on the DAQ system protocol using a client-server model, while high-speed readout uses a DAQ-independent UDP "push" mechanism for rapid data transfer. A virtual DAQ control class defines the fundamental read/write methods for Timepix4 registers, allowing any DAQ system to integrate by inheriting this class. Finally, a control readout class handles configuration and polling of registers, whereas high-speed acquisition reads and stores data for online and offline analysis, independent of the specific DAQ system.

7.3.2 Data analysis architecture

Data decoding and event building

A custom data analysis framework has been developed to process Timepix4 data and support a range of applications. It provides low-level access to raw or minimally decoded data, essential for debugging, while remaining user-friendly to enable complex analyses through simple commands. Designed modularly, it allows easy extension with additional classes or objects for new applications. Implemented in C++ with an object-oriented architecture, the software first reads the bitstream either online or from storage, performs basic decoding to extract pixel addresses, timing information, and reconstructs hit events while incorporating external information.

Data clustering

Particles interacting with the detector typically deposit energy across several neighboring pixels, forming traces whose shapes depend on the interaction mechanism. Proper interpretation of these events and extraction of physics information therefore requires a clustering algorithm. The choice of algorithm depends on expected event types, rates, and particle-detector interactions. A density-based clustering approach was chosen, specifically the Density-Based Spatial Clustering of Applications with Noise (DBSCAN) [343] algorithm. DBSCAN identifies clusters by grouping neighboring points within a defined radius and a minimum number of neighbors, expanding clusters iteratively from core points until no further neighbors meet the criteria. Non-core points are assigned to the first cluster satisfying the distance requirement. The algorithm is highly adaptable, allowing clustering in arbitrary phase spaces with appropriately defined distances.

7.4 Future applications

The development of the 4DPHOTON detector will have significant impact across multiple application domains, including high-energy physics, life sciences, and materials science. In general, this detector offers major advantages wherever the measurement of single photons over a large area with simultaneous high spatial and timing resolution is required.

Future high-energy physics experiments will take place in environments with extremely high particle intensities, presenting new experimental challenges. Detectors will be required to achieve improved timing resolution, granularity, and rate capability, which current technologies cannot fully deliver. The 4DPHOTON detector addresses these limitations, providing a substantial benefit for next-

generation experiments. RICH detectors could greatly benefit from the precise timing capability of 4DPHOTON, which enables accurate time association of hits and effective discrimination of overlapping rings.

In the life sciences, the gold-standard technique for fluorescence lifetime measurements is time-correlated single-photon counting, which records individual fluorescence photons emitted by a sample—excited by an ultrafast laser pulse—with picosecond precision [344]. This technique relies on non-imaging fast detectors, though camera-based implementations are already well established. The 4DPHOTON detector, with its picosecond timing resolution and high-rate capability, can significantly advance this technique by overcoming photon losses inherent in frame-based detectors, thanks to its data-driven architecture.

Furthermore, the unique features of the 4DPHOTON hybrid detector open possibilities for novel imaging techniques. For instance, combining it with super-resolution methods in wide-field imaging could enable breakthroughs in resolving complex biological and material structures with unprecedented precision.

Chapter 8

Characterisation of the 4DPHOTON tube

A characterisation of the 4DPHOTON prototype tube, mainly in terms of timing resolution and DCR, has been performed in the laboratory and also during a testbeam campaign at the CERN SPS North Area.

Sec. 8.1 presents some characterisation performed in the laboratory are presented. In Sec. 8.2 the Geant4 simulation developed to study and optimise the testbeam setup is described. Finally, Sec. 8.3 presents the final testbeam setup and the result obtained.

8.1 Laboratory characterisation

Several tests were carried out to ensure the proper functionality and performance of the system components. These included the verification of the off-detector electronics, as well as the quality control of ceramic carrier boards. In addition, extensive phototube quality tests were performed, which covered the quality test setup and DAQ system and electronics tests. The detailed procedures and results of these tests will not be reported in this thesis but are documented in [345].

The following subsections reports the measurements performed to the 4DPHOTON prototype with 3 MCP stages and 1*d* end-spoiling depth.

8.1.1 Dark count rate measurements

The dark count rate and gain were evaluated through data-driven acquisition under dark conditions. Measurements were carried out at different operating points

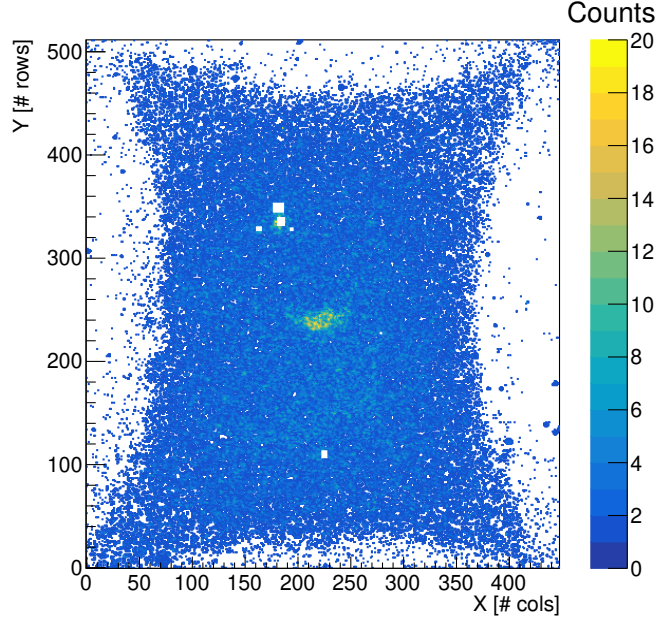


Figure 8.1: Dark count spatial distribution obtained with a 10 minutes long acquisition using $V_{\text{MCP}} = 2100$ V, $V_{\text{MCP-PHC}} = 75$ V and $V_{\text{Tp4-MCP}} = 150$ V.

by varying the voltages applied between the detector components: specifically, between the Timepix4 and the MCP, between the MCP stacks top and bottom faces, and between the MCP and the photocathode.

When a photon generates an electron cloud, it typically spreads across multiple pixels, forming a cluster. The cluster size depends both on the MCP gain and on the voltage applied between the MCP bottom surface and the Timepix4. The dark count rate was estimated as the number of reconstructed clusters. An example of the resulting photon-counting distribution is shown in Figure 8.1, obtained with the MCP biased at $V_{\text{MCP}} = 2100$ V, $V_{\text{MCP-PHC}} = 75$ V and $V_{\text{Tp4-MCP}} = 150$ V.

The origin of the observed dark count rate structure is still under investigation. Several possible effects are being considered. Field distortions have been excluded by scanning the detector with a micro-focused laser across specific pixel regions. The MCPs have been biased at low voltage during this study, since it has been observed that doing so the dark count rate structure was more evident. In this configuration, the effect is more pronounced. The distances between illuminated cluster centroids matched the programmed motion stage steps, with an error smaller than the Timepix4 pixel size, demonstrating that field distortions are negligible. At higher MCP gain, the effect becomes less visible but does not vanish. Notably, clusters appear smaller in regions where they were missing at low gain, pointing

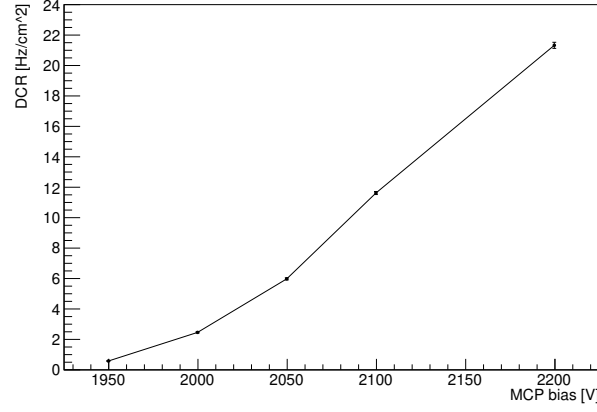


Figure 8.2: Dark count rate dependence on the MCP bias

to local gain inhomogeneities. A combined effect is currently under study. On one hand, the variations in cluster size and charge indicate non-uniform gain, possibly due to imperfect co-planarity of the MCP stacks. On the other hand, the cushion-shaped structure observed suggests an additional effect related to the rectangular geometry of the Timepix4, potentially arising from a non-perfect planarity of the ASIC bonded to the carrier. Several values of $V_{\text{Tp4-MCP}}$ were tested, but no noticeable changes in the dark count structure were observed, suggesting that this parameter has no measurable effect. The structure observed in the central region was not observed in later measurements, both in the laboratory and during the testbeam (see Figure 8.20), indicating that it is not a generic properties of the prototype.

Figure 8.2 shows the dependence of dark count rate to the bias applied. As expected, increasing the gain results in a higher dark count rate. The dark count rate also depends strongly on the threshold setting: increasing the threshold cuts off a larger fraction of low-charge events. To minimise threshold-induced suppression, all characterisations were performed at a threshold of $1000 e^-$.

8.1.2 Gain measurements

Gain estimation was based on the reconstructed cluster charge. At low gain, the spectra follow an exponential distribution. The gain was estimated as the mean cluster charge, corresponding to $E[Q] = \frac{1}{\lambda}$ where λ is the slope parameter of the exponential distribution. At higher MCP gain, of order of $\mathcal{O}(10^5 - 10^6)$, a distinct peak emerges at higher cluster charge. In this case the gain was estimated as the mean of the peak rather than from an exponential fit. Two cluster charge spectra,

at low and high gain, are presented in Figure 8.3.

As with the dark count rate, the gain shows a strong dependence on threshold, particularly at high MCP gain. This occurs because a significant fraction of pixels in the cluster halos receive only a small charge, often below threshold, reducing the reconstructed cluster charge.

The gain extracted from these distributions is affected by spatial non-uniformities. To mitigate this, the analysis was repeated using only the central matrix region. At high bias the discrepancy is pronounced, as the inclusion of low-gain regions reduced the average. These results are summarised in Figure 8.4.

8.1.3 Timing resolution

Preliminary measurements on Timepix4 assembly

A preliminary characterisation of a Timepix4 assembly bonded to a 100 μm thick planar n-on-p Si detector has been performed [346]. The measurements have been performed by sending simultaneous signals to different pixels and measuring the time-difference between them to estimate the timing resolution.

An infrared laser pulse with a FWHM of ~ 36 ps and a trigger jitter below 20 ps r.m.s. has been used. The beam is attenuated, collimated, focused, and aligned to the sensor back side using three remotely controlled micrometric translation stages in a 3D setup.

To isolate the timing resolution contribution of the laser setup, the laser jitter was measured with an oscilloscope. The result, 10.9 ± 0.1 ps r.m.s., is negligible.

The achievable timing resolution is influenced by several factors. Some of these factors, such as the TDC resolution, are intrinsic and cannot be eliminated, while others, can be corrected. These contribution arise from: VCO frequency variations, reference signal resolution and time walk effect.

As described in Sec.7.2.3, VCOs are a circuit whose output clock frequency depends on a control voltage, oscillating at the nominal frequency of 640 MHz, and it defines the timing resolution of each TDC. By analysing the number of bins between a signal edge and the next 40 MHz clock, it was found that the expected range of 128 bins varied across pixels. These deviations indicate that VCO frequencies differ from their nominal value, which introduces errors in ToA measurements. To correct this, each VCO frequency must be measured and calibrated using internal digital test pulses. The VCO frequency of each TDC is calibrated by sending synchronised internal digital test pulses to selected pixels and varying their phase relative to the 40 MHz reference clock, allowing to extract the correct frequency.

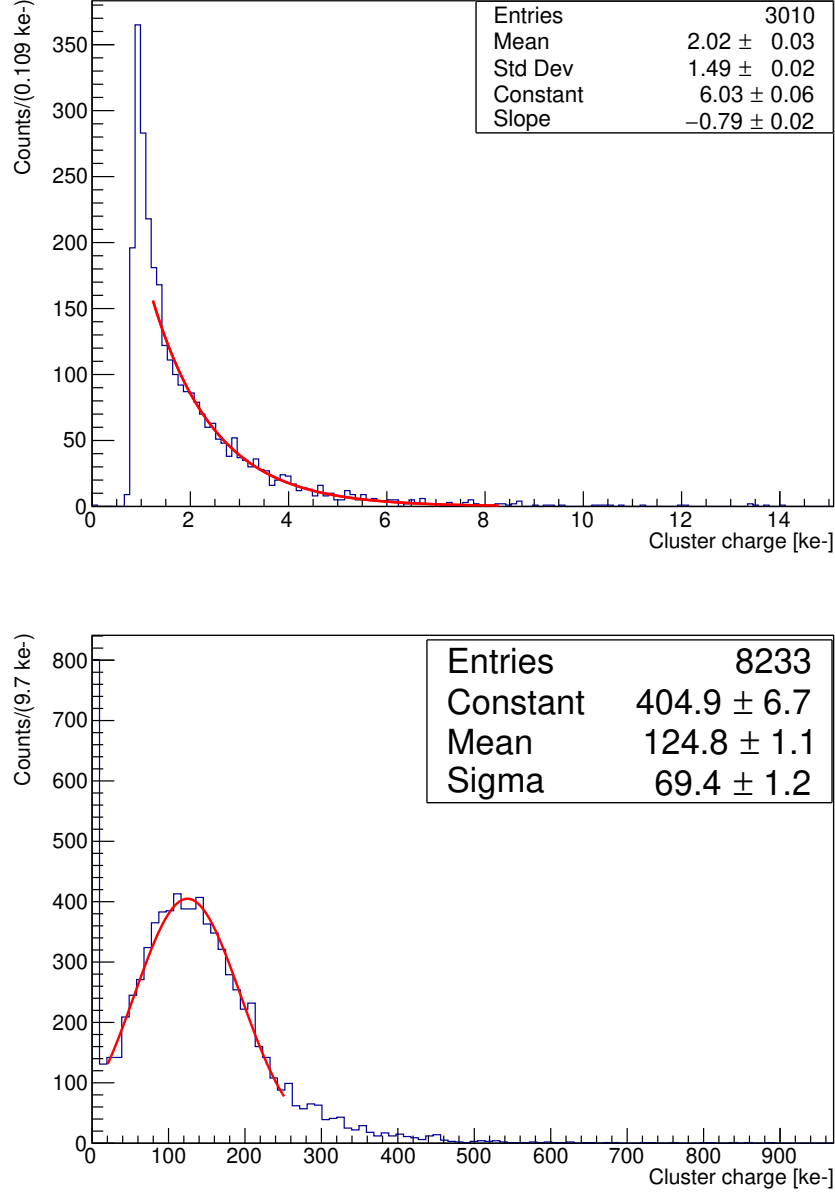


Figure 8.3: Cluster charge spectra measured applying a voltage difference of $V_{\text{MCP-PHC}} = 75$ V and $V_{\text{TPx4-MCP}} = 150$ V and a MCP bias of (left) $V_{\text{MCP}} = 2000$ V and (right) $V_{\text{MCP}} = 2300$ V.

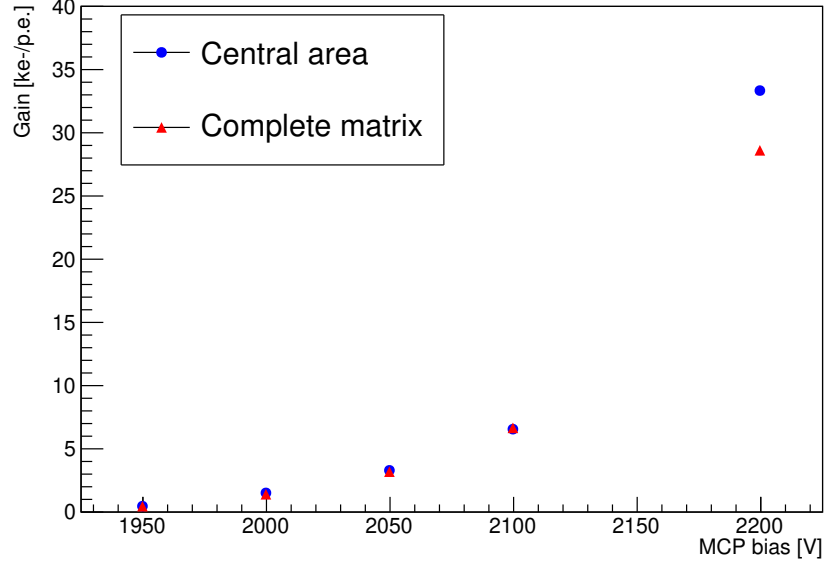


Figure 8.4: Comparison between the gain estimations obtained considering the complete matrix (red triangles) and the central region alone (blue circles).

Without accounting for VCO frequency variation, the measured timing resolution is ~ 200 ps r.m.s.. After using the proper VCO oscillation frequency the standard deviation of the ToA difference is:

$$\sigma_{\text{VCO corrected}} = 126 \pm 1 \text{ ps r.m.s.} \quad (8.1)$$

The resolution of the reference signal has been measured using test pulses generated externally. The standard deviation of the $(\text{ToA}_{n+1} - \text{ToA}_n)$ distribution is measured to be $\sigma(\text{ToA}_{n+1} - \text{ToA}_n) = 102 \pm 1$ ps r.m.s.. Because this result reflects the difference between two signals measured by the same TDC, the resolution of the reference signal can be estimated as follows:

$$\sigma_{\text{Ref}} = \sigma(\text{ToA}_{n+1} - \text{ToA}_n) / \sqrt{2} = 72 \pm 1 \text{ ps r.m.s.} \quad (8.2)$$

The timing resolution of a single Timepix4 pixel can be determined by subtracting the reference signal contribution σ_{Ref} in quadrature:

$$\sigma_{\text{ToA}} = \sqrt{\sigma_{\text{VCO corrected}}^2 - \sigma_{\text{Ref}}^2} = 103 \pm 1 \text{ ps r.m.s.} \quad (8.3)$$

To achieve precise timing, the time-walk effect in individual pixels must be cor-

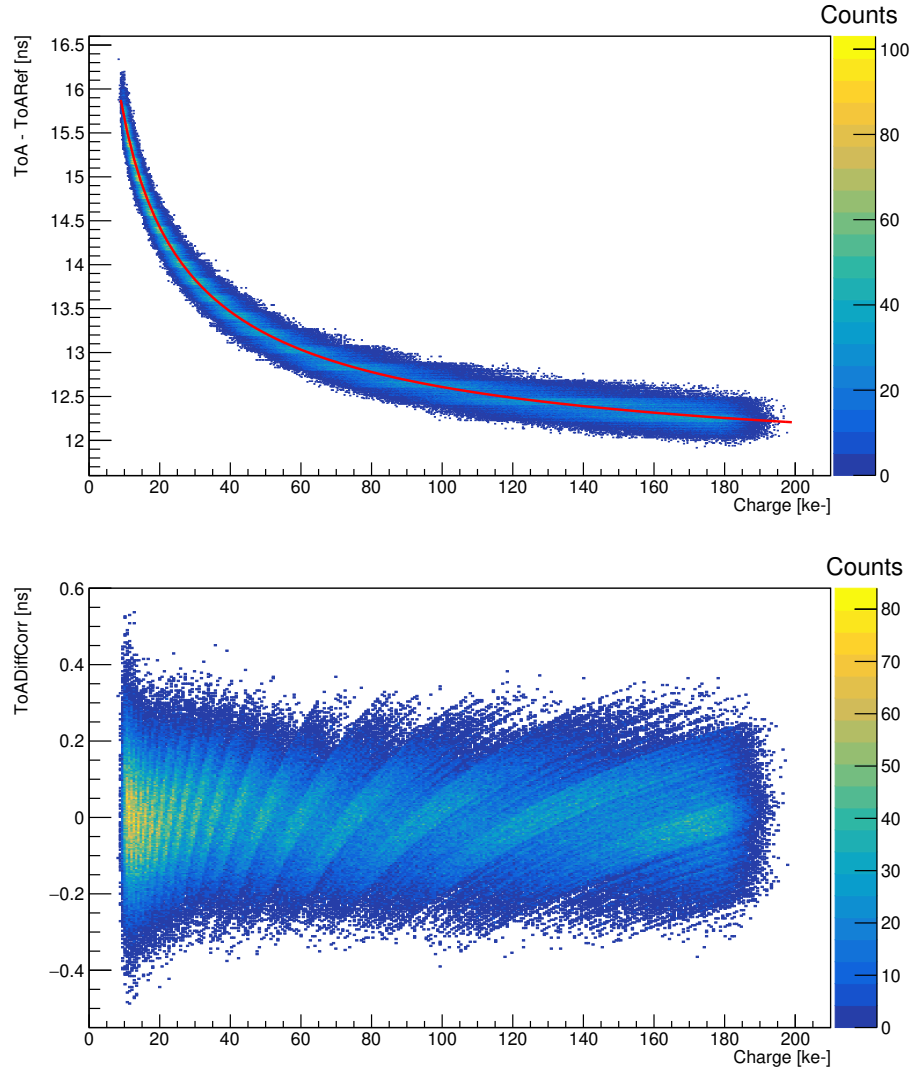


Figure 8.5: (Top) Single pixel time walk distribution superimposed with the fitting function. (Bottom) Single pixel Δ ToA distribution as a function of the charge after time walk correction.

rected [347, 348, 349, 350]. This was studied by varying the laser attenuation to span charges from $\sim 10 \text{ ke}^-$ to $\sim 200 \text{ ke}^-$, producing the spectrum in Figure 8.5 for one pixel, fitted using

$$f(Q) = \Delta\text{ToA}(Q) = \frac{p_0}{Q^{p_1} + p_2} + p_3 \quad (8.4)$$

Each ToA value is then corrected by subtracting the charge-dependent contribution, yielding the distribution in Figure 8.5.

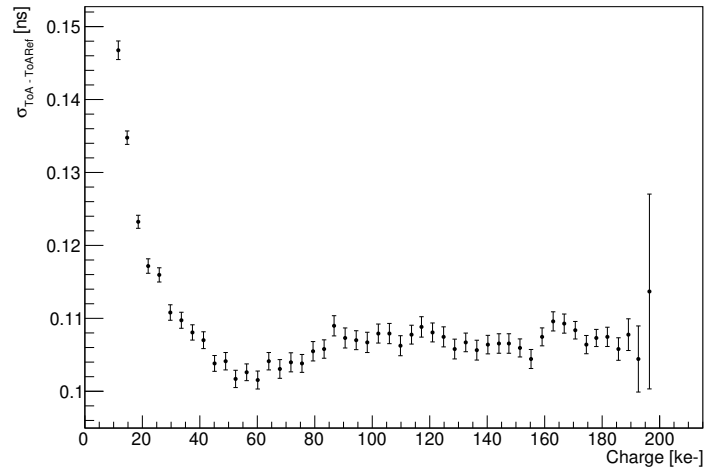


Figure 8.6: Timing resolution dependence with respect to the calibrated charge for a single pixel, after all correction applied.

The corrected data were divided into $\sim 4 \text{ ke}^-$ charge slices, and the timing resolution for each slice was extracted from the standard deviation of the ΔToA projection, see Figure 8.6. After subtracting the reference signal contribution in quadrature, the intrinsic single-pixel resolution is $\sigma_{\text{ToA}} = 103 \pm 3 \text{ ps}$ r.m.s. at $\sim 60 \text{ ke}^-$, with a plateau of $109 \pm 3 \text{ ps}$ r.m.s. at higher charges. The degradation of the resolution at high charges can be explained considering that, at fixed laser intensity, charge spectra follow a Gaussian distribution due to electronic noise and charge collection fluctuations, while timing measurements show quantisation effects from the 195 ps binning, enhanced by small VCO frequency mismatches. At high charges ($Q \geq 70 \text{ ke}^-$), overlapping charge distributions and quantisation broaden the timing spectra, explaining the observed resolution degradation.

For multi-pixel clusters, combining data from individual pixels improves timing precision. As described in Sec. 7.3, the DBSCAN density-based algorithm [351] is used, as it groups hits by spatial and timing proximity. For Timepix4, parameters

are tuned to handle pixel masking and particle timing ranges, ensuring clusters are well separated. Each pixel ToA is corrected as previously described, and the cluster ToA ($\text{ToA}_{\text{cluster}}$) is calculated as the charge-weighted average of the pixels ToAs, while the cluster charge is the sum of individual pixel charges. Figure 8.7 shows $\text{ToA}_{\text{cluster}}$ as a function of cluster charge after time walk corection.

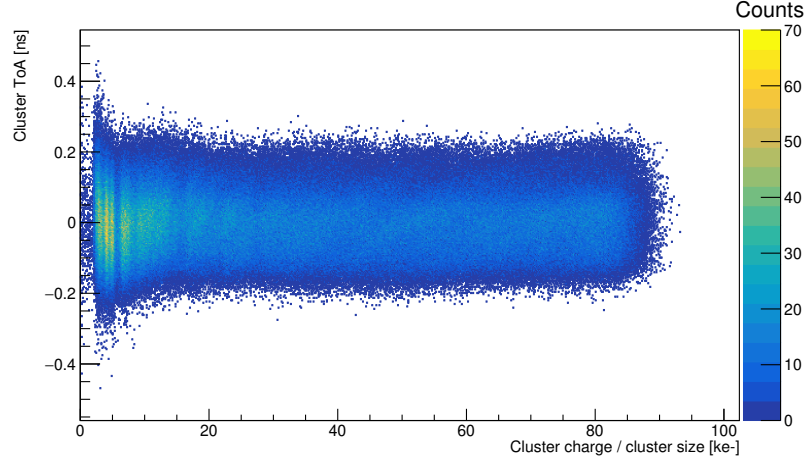


Figure 8.7: Cluster ToA distribution after time walk corection as a function of the cluster charge.

A slice analysis over the full charge range yields the cluster timing resolution $\sigma_{\text{ClusterToA}}$, computed as the standard deviation of the difference $\text{ToA}_{\text{cluster}} - \text{ToA}_{\text{Ref}}$. The resolution reaches a minimum at $\sim 900 - 1300 \text{ ke}^-$ corresponding to $79 \pm 1 \text{ ps}$. Subtracting the reference contribution gives the true minimum cluster resolution:

$$\sigma_{\text{ToA}}^{\text{Cluster}} = 33 \pm 3 \text{ ps} \quad (8.5)$$

Figure 8.8 shows $\sigma_{\text{ClusterToA}}$ plotted as a function of the cluster charge. Minor fluctuations are due to overlap between ToA distributions in different timing bins. These results are reported in Ref [346].

4DPHOTON measurements

The first photodetector prototypes feature a photocathode with high quantum efficiency in the blue/UV range. A PicoQuant LDH-P-C-405 laser ($\lambda = 405 \text{ nm}$) is used. The laser has a FWHM of 56 ps and jitter $< 20 \text{ ps}$ r.m.s., and its intensity is adjusted with a variable fiber optic attenuator. The focused spot covers a few pixels, with a minimum-intensity spot size of 1.2 pixels r.m.s.. The

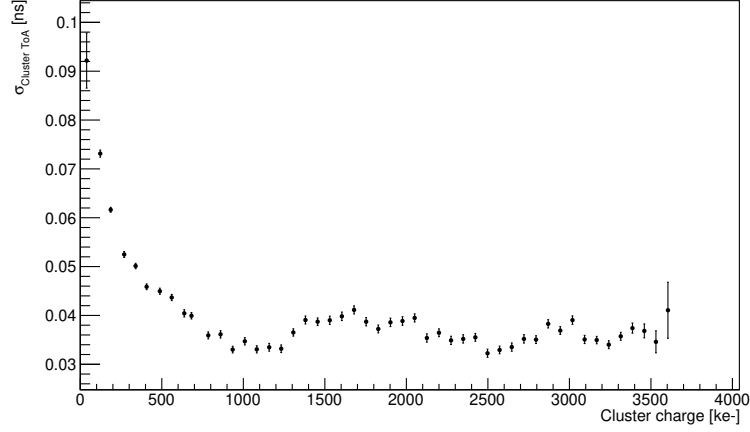


Figure 8.8: Cluster timing resolution $\sigma_{\text{ClusterToA}}$ plotted as a function of the cluster charge.

photocathode is illuminated at a fixed position with 10 Hz pulses. Voltages are set to $V_{\text{MCP-PHC}} = 75$ V and $V_{\text{TPx4-MCP}} = 150$ V and $V_{\text{MCP}} = 2400$ V.

Timing analysis is analogous to the one described previously. Each pixel ToA is compared to the reference ToA, and time-walk corrections are applied. The resulting per-pixel timing resolution, after subtracting the reference contribution, reaches at best 107 ± 6 ps at $Q \sim 40$ ke⁻, consistent with previous Si-bonded measurements. Contributions from the MCP and photocathode are < 50 ps, the digital front-end ~ 55 ps, and the laser/trigger ~ 10 ps. The cluster ToA distribution after corrections is shown in Figure 8.9.

Cluster analysis is performed by computing the charge-weighted average ToA and total cluster charge. The timing resolution distribution shows slight improvement, reaching a plateau at $\sigma_{\text{ToA}}^{\text{4DPHOTON}} = 88 \pm 9$ ps. The distribution of $\sigma_{\text{ToA}}^{\text{4DPHOTON}}$ as a function of the total cluster charge is shown in Figure 8.10. Most of the charge in the clusters generated by the MCP's electron clouds is collected by a single pixel, a wider halo of pixels gathers only a small fraction of the charge. As a result, the enhancement obtained with the cluster analysis is not as evident as in situations where many pixels register a large amount of charge, for this reason the cluster timing resolution is close to the single pixel one for the 4DPHOTON detector.

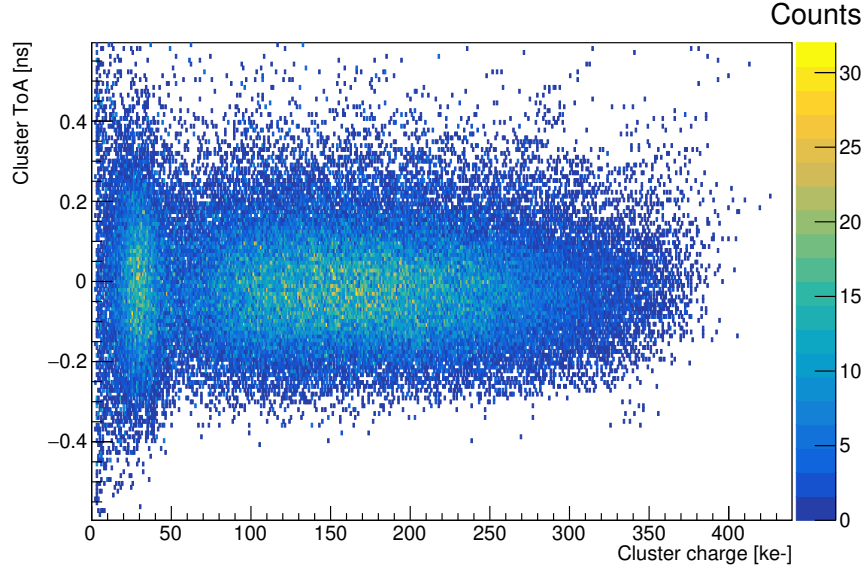


Figure 8.9: Cluster ToA distribution as a function of the cluster charge.

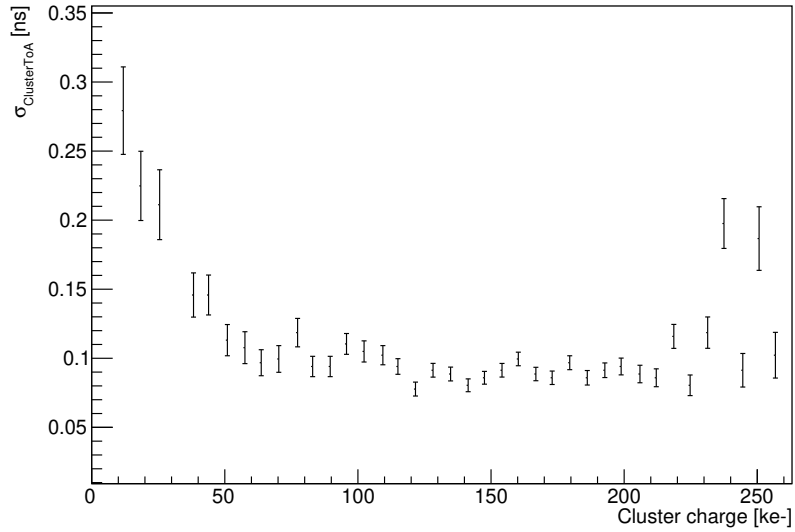


Figure 8.10: Cluster timing resolution as a function of the cluster charge. The reference signal contribution has already been subtracted.

8.2 Geant4 simulation of a Cherenkov setup

Geant4 [352, 353, 354] is a software toolkit designed to simulate the interactions of particles with matter and their propagation through it. The simulation process can be finely controlled, including aspects such as detector geometry, material properties, the types of particles and their interactions, primary event generation, particle tracking, detector responses, as well as event storage and visualisation. At its core, Geant4 relies on a suite of physics models that accurately describe particle interactions across a wide range of energies.

The Geant4 application comprises the complete experimental setup, as will be described later. It models the beam configuration, its interaction with the setup components, the photon production and transportation. The application is highly flexible, enabling easy adjustments such as beam composition and setup configuration. Its geometry definition also allows adding optical elements, modifying the number of 4DPHOTON detectors, and optimising the spatial arrangement of components.

The goal of this simulation is to test and optimise several setup to characterise the performance of the first detector prototypes using a RICH configuration.

8.2.1 Physics List Configuration

The PhysicsList is built by combining several Geant4 physics lists to cover electromagnetic, hadronic, optical, and decay processes. For the electromagnetic sector, the default choice is `G4EmStandardPhysics_option4`. Hadronic interactions are handled by a set of physics lists including `G4HadronPhysicsQGSP_BIC_HP`, `G4HadronElasticPhysicsHP`, `G4IonPhysics`, and `G4StoppingPhysics`. Processes like gamma nuclear interactions are managed by `G4EmExtraPhysics`. Optical processes are explicitly constructed, with Cherenkov, Scintillation, Absorption, Rayleigh, MieHG scattering, and Boundary interactions included. The decay sector is covered by `G4DecayPhysics` for generic particle decays. Additionally, atomic deexcitation, fluorescence and PIXE are activated through `G4UAtomicDeexcitation`, ensuring a detailed treatment of secondary photon and electron production. Combined, these physics lists form a comprehensive model capable of simulating the optical, electromagnetic, hadronic, and nuclear decay processes that may occur following the interaction of the beam with the radiator lens.

8.2.2 Geometry

The testbeam setup has been fully simulated and consists of two trackers, a RICH system, and two timing detectors. For each material used, the refractive index,

transmittance, absorbance, and reflectance have been taken into account over the relevant energy range, corresponding to the Cherenkov wavelength spectrum (200–800 nm). The following sections provide a detailed description of the simulation approach for each components.

Trackers

The two Timepix4 assemblies that make up the tracking station have not been simulated at the electronic level; however, their material has been included to account for the correct on-beam material budget. Each assembly is modeled with three parallelepiped layers, measuring $24.7 \text{ mm} \times 29.96 \text{ mm} \times 300 \text{ }\mu\text{m}$, attached to a second parallelepiped of $24.7 \text{ mm} \times 29.96 \text{ mm} \times 700 \text{ }\mu\text{m}$ both made of silicon. The back side includes a PCB made of FR4 with a thickness of 1.6 mm. The two Timepix4 assemblies are 1 m apart, to grant a high resolution on the reconstruction of particle direction.

Several tests were carried out to assess the impact of the trackers on the beam properties, with particular attention to the beam position and transverse momentum. Examples of these results are presented in Appendix P.

RICH system

The RICH system is located within a dark box downstream with respect to the tracking system. It includes a Cherenkov radiator, the optics system used to drive the Cherenkov photons to the detector, and a single prototype of hybrid MCP-PMT used as photodetector

Two main configurations have been simulated, with the primary goal of achieving a well focused ring and minimising the time dispersion of the photons. The reference system used consist of $(\hat{x}, \hat{z})$, where $\hat{z}$ is directed along the beam axis with the same verse, and $\hat{x}$ is horizontal and perpendicular to $\hat{z}$, and it points in such a verse as to form a right-handed system with a vertical $\hat{y}$ axis directed upwards.

The first configuration has been developed using Zemax [355]. This configuration comprises an aspheric plano-convex lens of N-BK7 [356], serving both as a radiator and as the first refractive optical element. This lens has a curvature radius of 31.81 mm and its vertex is placed in position (0,0) of the reference system. The conic constant is -4.46 and its front lens diameter is 40 mm. Cherenkov radiation undergoes total internal reflection on the flat surface and is then reflected by the concave surface, which is properly metallised. Due to the small angle of incidence, total internal reflection is not possible on this surface, so metallisation is required. The chosen coating is aluminum with a protective SiO_2 layer. A folding mirror

of aluminium, placed in (0,111.00) mm and tilted at an angle of 45° , allows the optical axis to be bent by 90° . Then two spherical fused lenses corrects the lateral chromatic aberration introduced by the first lens, achromatising the system. The first lens, made of N-LAK33 [357], is shaped to accommodate the second lens. It features a curvature radius of 51.26 mm and a front diameter of 80.00 mm. The second lens, composed of SF59 [358], is carved to form a concave surface with a curvature radius of 112.80 mm and has an external curvature radius of 35.69 mm. Its front lens diameter measures 60 mm. The vertex of this doublet is placed in (59.64,111.00) mm. The MCP-PMT prototype is described as an absorber plane with the dimension of the Timepix4 coupled to an input window. The input window is composed of two separated parts. The first window is a simple cylinder with a diameter of 63 mm and a thickness of 3.6 mm. The second window is shaped as a truncated cone. The larger face has a diameter of 45.2 mm, while the smaller face has a diameter of 43.8 mm. Its thickness is 2.1 mm, and it is positioned directly on top of the first window to form a continuous optical assembly. The coordinates of the center of the MCP-PMT prototype is placed in (221.84,110.00) mm. The distribution of the quantum efficiency have been provided by Hamamatsu Photonics as a function of the wavelength for the optical window of the detector and have been implemented in the simulation.

This configuration of the RICH system was implemented in Geant4 using two complementary approaches. In the first approach, the geometry was represented with triangular mesh models imported from .obj files via the CADMesh library [359]. In the second approach, the geometry was constructed using native Geant4 classes. In particular, the aspherical lens was modeled by calculating its thickness as a function of the radial coordinate according to:

$$z = \frac{c \cdot r^2}{1 + \sqrt{1 - (1 + k) \cdot c^2 \cdot r^2}}, \quad (8.6)$$

where c is the curvature (the inverse of the curvature radius) and k is the conic constant. The resulting profile points were then used with the `G4Polycone` class to create the lens in Geant4.

The results obtained from the Geant4 simulation have been compared with the one of Zemax, which expected a Cherenkov ring of radius ~ 10 mm and width of $\sim 100 \mu\text{m}$. The Geant4 results however showed great discrepancies, actually no Cherenkov rings were observed because the optical system did not focus the ring correctly. This is probably due to the fact that the geometry imported from Zemax is approximated and Geant4 does not correctly describe the interaction of the photons with the surfaces of the lenses. Several studies were performed, such

as lowering the step dimension used in Eq. 8.6 or augmenting the number of mesh of the models, but the results were always in disagreement with Zemax.

For this reason, a second configuration was developed using exclusively spherical lenses. Due to time constraints for the upcoming testbeam, only commercially available lenses were employed in the new setup.

All optical elements of the second configuration are made of fused silica, with a refractive index ranging from 1.45 to 1.50 in the wavelength range of Cherenkov photons. The Cherenkov radiator is a LA4545 plano-convex lens [360], with a curvature radius of 46.00 mm, a maximum thickness of 10.70 mm and a front lens diameter of 50.80 mm. As in the previous configuration, photons are reflected in the plane surface onto the concave surface. An aluminum coating on the spherical surface reflects photons back towards the subsequent optical elements. The radiator vertex is positioned at (0.0, 0.0) mm. A LA4078 plano-convex lens [361] is placed next, with its vertex at (0.0, 25.0) mm and the planar surface upstream. It has a curvature radius of 34.50 mm, a maximum thickness of 14.2 mm and the same front lens diameter of 50.8 mm. Following the first doublet, an Edmund Optics cylindrical mirror 84-425 [362] with diameter 75.0 mm and thickness 7.5 mm is placed. The mirror center is at (2.0, 52.0) mm and tilted 45° , redirecting the optical axis from the $\hat{z}$ to the $\hat{x}$ direction. The first lens encountered along the redirected optical axis is a LA4795 plano-convex lens [363], with curvature radius of 92.00 mm, maximum thickness 11.00 mm and front lens diameter 70.50 mm. Its vertex is at (70.00 mm, 50.00 mm). Two additional LA4795 lenses, with vertices at (98.00, 50.00) mm and (120.20, 50.00) mm, form a doublet. Finally, a LA4384 plano-convex lens [364] has its vertex positioned at (121.0, 50.0) mm, with curvature radius of 41.40 mm, maximum thickness 26.9 mm and front lens diameter 75.00 mm. The detector, described as in the previous configuration, is centered at (180.00 mm, 50.00 mm). Figure 8.11 shows the simulated lens system described.

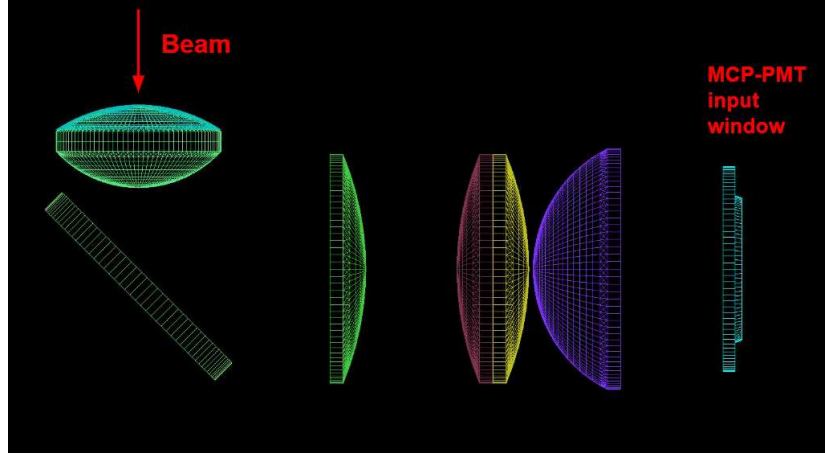


Figure 8.11: Geant4 simulation of the optics system designed for the RICH system of the testbeam. The beam direction and the MCP-PMT prototype are also labelled.

Timing system

Downstream of the RICH system, along the beam direction, a pair of light guide coupled with photodetectors is installed to provide precise timing measurements. Each lightguide consists of a single solid element shaped as a parallelepiped with dimensions $12.5, \text{mm} \times 11.0, \text{mm} \times 100.0, \text{mm}$. One end of the parallelepiped continues into a truncated pyramid, which continues into a cylindrical section designed to couple with the photomultiplier, all made of Polyallyl diglycol carbonate (CR39), enclosed in a dark box with optimised geometry. These detectors are included in the simulation through `.obj` files via the CADMesh library [359]. Due to their simple geometry the issues due to the approximation of the shape is not a problem. Cherenkov photons are generated and totally internally reflected on the surface until they reach the end of the light guide where they are detected by the photodetector.

Several studies have been performed in order to optimise resolution of the system in terms of the angle between the light guide and the beam axis and also in terms of the distance between the center of the detectors and the beam. Results of these studies are reported in Appendix Q. Given these results the detectors are tilted at 60° with respect to the beam axis and the center is shifted of 1.5 cm from the axis. The geometrical contribution to the time resolution of the timing system is approximately 20 ps as shown in Figure 8.12.

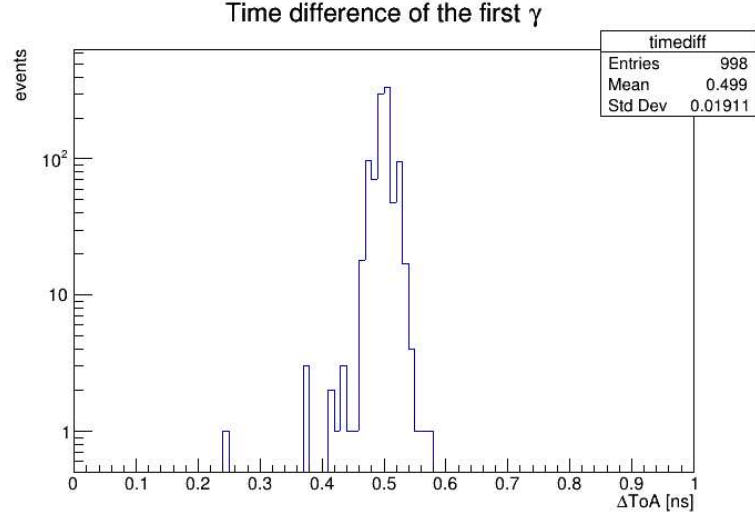


Figure 8.12: Distribution of the difference in ToA for the first photon between the two detectors that make up the timing system.

8.2.3 Beam configuration

Several beam configurations were tested to evaluate their impact on the setup before finalising the testbeam location. Once the test beam location and corresponding properties were defined, two types of beams were simulated. The first is a point-like beam, with no divergence, where all particles are generated at the fixed position $(0, 0)$ in the reference system described in Sec. 8.2.2. The second is a realistic beam model, characterised by an angular divergence of $\sigma = 1$ mrad, and a generation point distributed according to a Gaussian with mean 0 and $\sigma = 5$ mm for both $\hat{x}$ and $\hat{y}$ axis. In both cases, the beams consist of protons with an energy of 180 GeV. Given the radiator lens refractive index in the range 1.45–1.50, the Cherenkov threshold corresponds to $\beta = 0.69$. Since protons with energies above 180 GeV have $\beta > 0.99$, they effectively saturate the Cherenkov angle, and thus all are emitted at the same angle.

8.2.4 Simulation results

The distribution of Cherenkov photons on the detector surface has been studied in the two-beam configuration, as shown in Figures 8.13. The expected ring radius is $R = 8.60 \pm 0.01$ mm and $R = 8.69 \pm 0.01$ mm for a point-like and realistic beam, respectively.

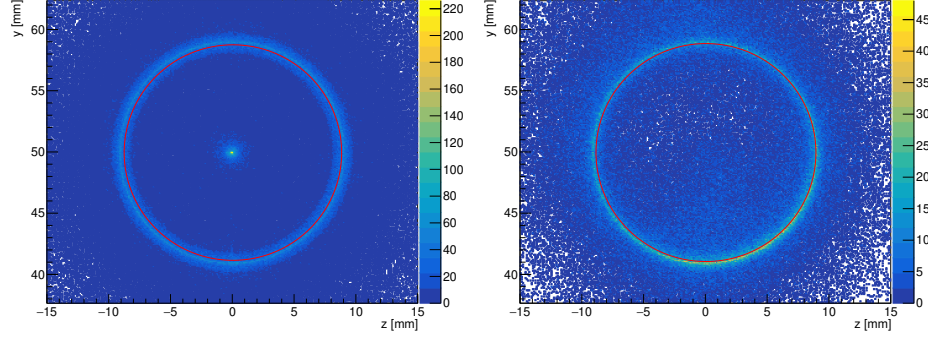


Figure 8.13: Simulated Cherenkov photons distribution on the detector prototype using (left) point-like beam model and (right) realistic beam model. The fit used to extract the value of the expected radius is superimposed.

The wavelength spectrum of the detected photons follows the expected Cherenkov emission shape, modulated by the transmission of the optical system, validating the photon generation and transport in the simulation, as shown in Figure 8.14.

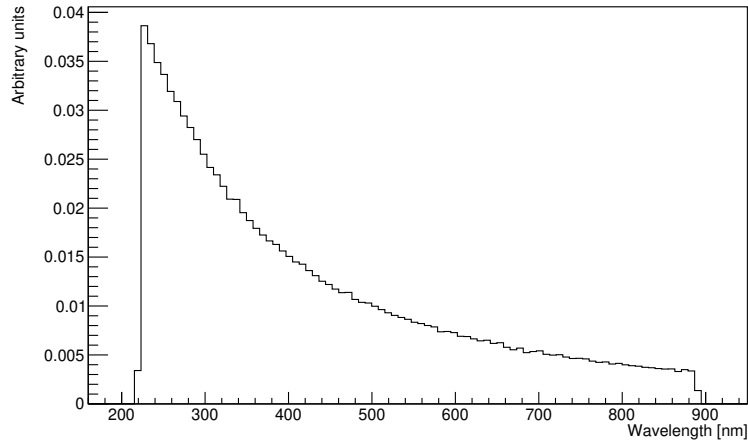


Figure 8.14: Wavelength distribution of the simulated Cherenkov photons.

The arrival time distribution of photons on the detector exhibits a well-defined peak, shown in Figure 8.15, the expected contribution to the measured time distribution is $\sigma = 19.5 \pm 0.1$ ps and $\sigma = 23.5 \pm 0.1$ ps for the point-like and realistic beam models, respectively.

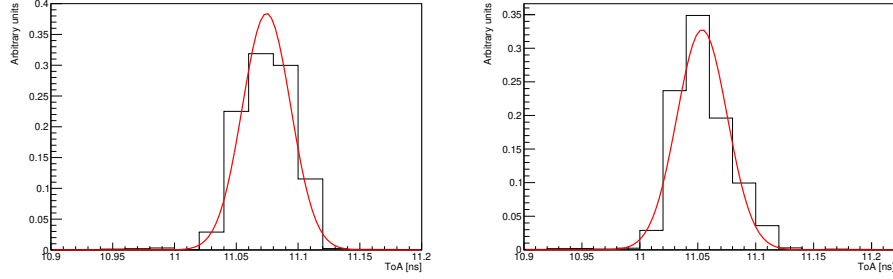


Figure 8.15: Time of arrival distribution of the Cherenkov photons on the detector with a gaussian fit superimposed using (left) point-like beam model and (right) realistic beam model.

A background component arising from nuclear interactions within the lens has been identified, resulting in a broader Cherenkov ring and an increased photon yield at the detector, of the order of $\mathcal{O}(10)$ photons. These events amount to about 5% of the total statistics. Finally, the total photon yield on the detector has been quantified, taking into account the quantum efficiency of the sensor and the open-area ratio of the MCP. The distribution is shown in Sec. 8.3 and the expected number of photons per interacting particle is 15 ± 1 .

8.3 Testbeam with Cherenkov setup

The test beam was performed at the CERN SPS, in the H8 beamline. This line provided a hadron secondary beam composed of approximately 80% protons, 20% pions, 1% muons, and a negligible fraction of electrons. The particle energy was measured to be $E = (140 \pm 1.8)$ GeV. The maximum beam intensity is about 10^5 particles per spill, with each spill lasting 4.8 s. The beam profile is Gaussian in both directions perpendicular to the beamline.

As already described in the previous section, the experimental setup consists of a tracking system, a RICH detector, and a timing system. A schematic representation of the setup is shown in Figure 8.16.

8.3.1 Detector Setup

The tracking system consists of two Timepix4 v2 assemblies, each bonded to 300 μm thick p-on-n planar silicon sensors, and is positioned upstream of the full setup. During the test beam campaign, the silicon sensors were operated at the bias voltage of 100 V. The two Timepix4 assemblies are separated by 1 m to ensure high precision in reconstructing the particle direction. The notations

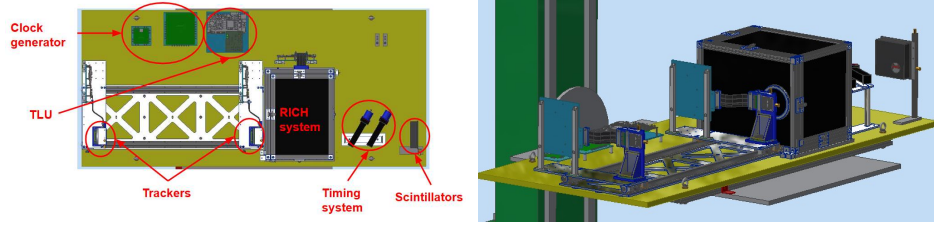


Figure 8.16: (left) Schematic of the test-beam setup seen from above: it comprises a tracking station composed of two Timepix4 assemblies bonded to Si sensors, a RICH system enclosed within a dark-box, a timing system consisting of two Cherenkov detector, and two scintillators to align the setup to the beam. An external clock generator and a Trigger Logic Unit (TLU) are used to provide an external reference clock and external T0 sync and shutter signal to the three Timepix4. (right) Lateral view of the testbeam setup

tracker 1 and tracker 2 are used to refer to the Timepix4 assemblies positioned farther from and closer to the RICH system, respectively. This system provides crucial information on the incident position and direction of particle hitting on the Cherenkov radiator.

The RICH system is housed in a dark box positioned downstream of the tracking system. It consists of a Cherenkov radiator, an optical system that directs the Cherenkov photons to the detector, and a single prototype hybrid MCP-PMT serving as the photodetector. The Cherenkov radiator, along with the optical system are the one described in Sec. 8.2.2. The Detector Under Test (DUT) is the 4DPHOTON prototype with 3 MCP stages and $1d$ end-spoiling depth.

The timing system is positioned downstream of the RICH system and has the geometry described in Sec. 8.2.2. During data taking, several issues were identified in the output of the timing reference system, as shown in Figure 8.17, these issues prevented the use of those data in the following analysis.

8.3.2 DAQ and data storage setup

The DAQ and storage network is organised into four subnets, as illustrated in Figure 8.18. These subnets are referred to as the *Slow control*, *Fast links*, *Storage*, and *Remote* subnets.

The *Slow control* subnet handles the configuration of all instruments. It includes the SPIDR4 DAQ systems [365] controlling the three Timepix4 assemblies: the two bonded to the silicon detectors of the tracking station, and the one mounted within the MCP-PMT prototype under test. Two low-voltage power supplies pro-

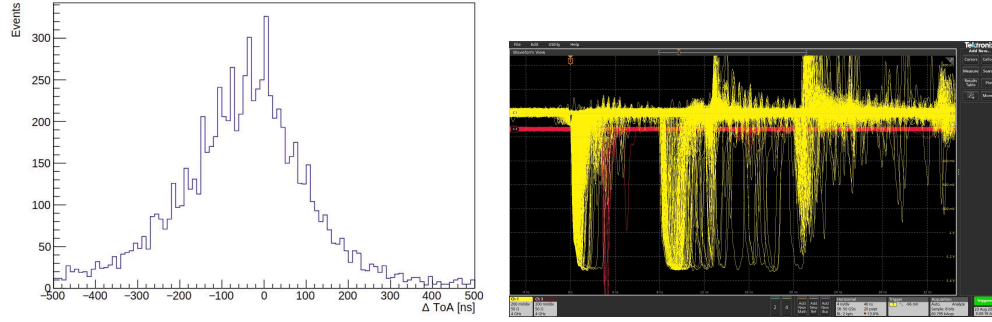


Figure 8.17: (Left) Difference of ToA distribution between the two detectors of the timing system, measured from data collected in a single spill and recorded with the online monitoring tool during the test beam. The timing resolution is ~ 150 ps. (Right) Measurements performed using an oscilloscope on the timing system detectors. As can be seen, the signals exhibit multiple reflections of not yet identified cause.

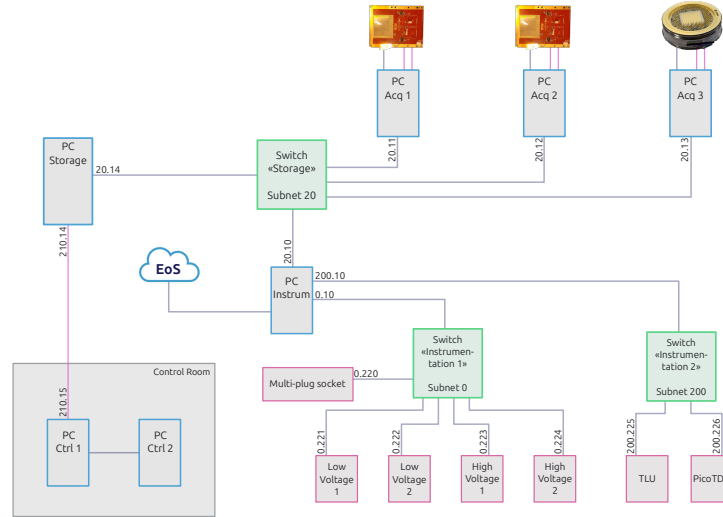


Figure 8.18: Schematic of the test-beam DAQ and data storage setup.

vide power to the Timepix4 boards, while a high-voltage HiVolta DT1415ET unit powers the silicon detectors, MCP-PMT elements, and the standard MCP-PMTs used for timing.

A custom Trigger Logic Unit (TLU) receives SPS Extraction Warning signals and generates T0_sync and shutter signals, which are sent to all three SPIDR4 boards. An external clock generator synchronises the three SPIDR4 boards and the Timepix4 assemblies. A PicoTDC precisely measures timing reference signals and TLU outputs to correlate Timepix4 measurements with those of the standard

MCP-PMTs. All instrumentation is powered via programmable Power Distribution Units.

Each Timepix4 is connected to a dedicated PC to separate configuration and acquisition, facilitating management of the high expected rates. All other devices are connected via 1 GbE to a switch, which is in turn connected to the control PC.

Data readout occurs through high-speed links to PCIe boards mounted on the PCs connected to the SPIDR4 DAQ systems, forming the *Fast links* subnet. The three acquisition PCs are connected via 10 GbE to a 10 GbE switch, which is linked to the storage PC, forming the *Storage* subnet. Finally, the control and storage PCs connect to remote PCs located in the control room (*Remote* subnet).

8.3.3 DAQ software

A dedicated application was developed to adapt to the test-beam setup, enabling synchronisation of acquisition runs with the beam spill injections. The configuration executable has been separated from the acquisition executable, allowing the Timepix4 to be configured once for an arbitrary number of spills, while performing separate acquisitions for each injected spill. The Timepix4 is configured to use external T0 synchronisation and external shutter signals. Extraction Warning signals are fed into the TLU, where they are used to generate both the external T0 sync signal and a 5-second external shutter. With this setup, acquisitions start immediately before the spill injection and stop at the end of the spill. Data are saved immediately after each spill acquisition.

8.3.4 Methodology and testbeam results

The detector was operated at a temperature of -10°C . The trackers were powered at 100 V, while for the DUT, the MCP was biased at 2100 V, with voltage differences of $V_{\text{MCP-PHC}} = 75 \text{ V}$ and $V_{\text{Tp4-MCP}} = 150 \text{ V}$. Laboratory tests were conducted with MCP voltages up to 2400 V; however, 2100 V was chosen as the working point during the test beam to ensure greater stability during data taking. No hardware triggers were used during data acquisition, except for the shutter signal at the start of each spill.

The first criterion for event selection is based on the tracker system. Since a particle is expected in the beam approximately every $\sim 1 \mu\text{s}$, and the time of flight between the two trackers is around $\sim 3 \text{ ns}$, a coincidence window of 10 ns between the two trackers is required. The time difference between the two tracker is displayed on Figure 8.19. Since the trackers were operated at a relatively low bias

voltage, the time resolution of the tracking system is suboptimal, corresponding to approximately ~ 1 ns. Figure 8.19 also shows that the mean of the distribution is 1.5 ns, whereas the expected value is approximately ~ 3 ns. This discrepancy is given by the Timepix4 ASIC. When receiving the 40 MHz clock, a 640 MHz clock is generated by the Timepix and then compared with the input one. The discrepancy arises by the fact that the 40 MHz clock locks on a random 640 MHz clock cycle, thus the resulting 40 MHz clock may not be in phase with the input one. The differences in these phases in the two assemblies result in a difference in the measured events ToA.

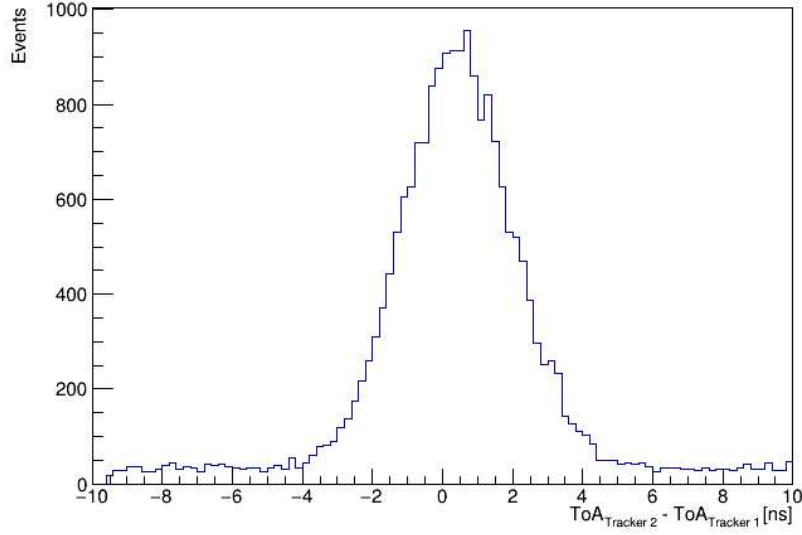


Figure 8.19: Distribution of the ToA difference between the two trackers from 50 spills.

Furthermore, all hits in the DUT had to occur within 50 ns of the hit in the tracker closest to the RICH system. Figure 8.20 shows the cumulative distribution on the DUT from data collected over 50 spills. The Cherenkov ring appears with the expected size and position, while a background pattern is also visible, consistent with that observed during laboratory characterisation. The vetoed region correspond to noisy areas of the MCP that were masked.

Figures 8.21 show the cumulative distribution on the trackers from data collected over 50 spills. The x and y projections of each tracker have been fitted simultaneously using a gaussian distribution. The projections, alongside the superimposed fit, are displayed in Figure 8.22. The fit results are reported in Table 8.1; the Geant4 simulation beam description has been updated using this result and it is

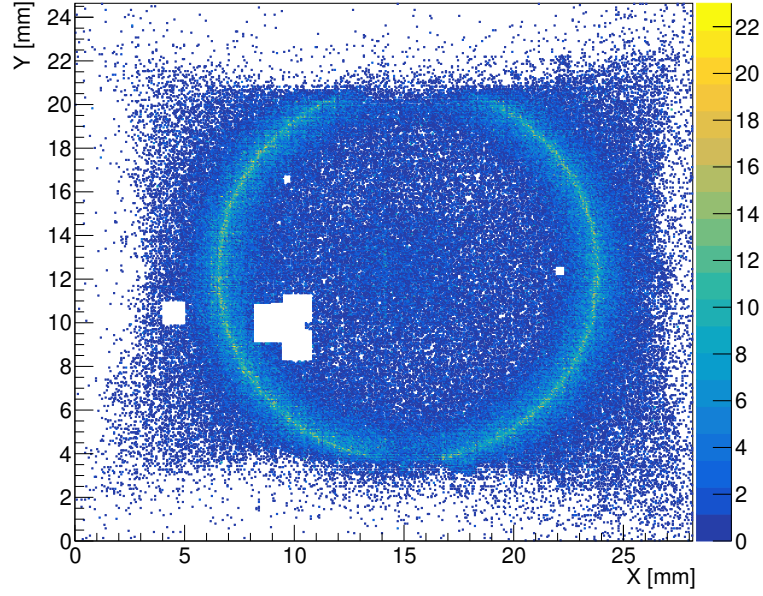


Figure 8.20: Cumulative distribution on the detector prototype from data collected over 50 spills.. The vetoed region correspond to noisy areas of the MCP that were masked.

used for comparison hereafter.

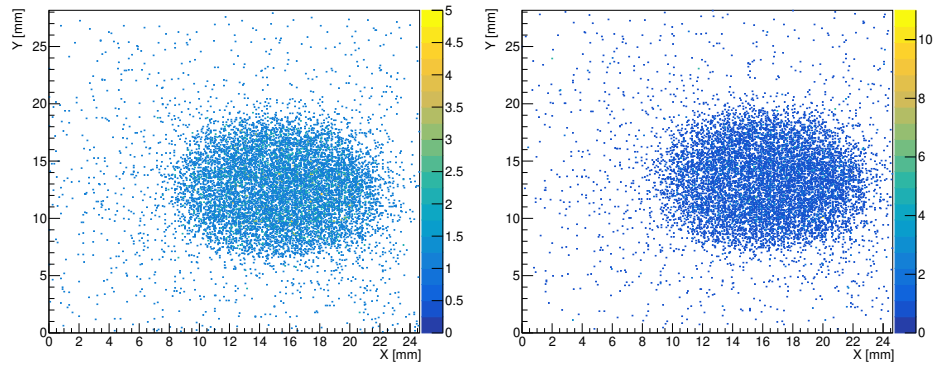


Figure 8.21: Cumulative hit distribution on (left) tracker 1 and (right) tracker 2 from data collected over 50 spills.

A dedicated ring-fitting procedure was developed to extract the Cherenkov ring radius and compare it with the value predicted by simulation. Each ring candidate is fitted independently, with the ring center coordinates (x, y) and the radius as

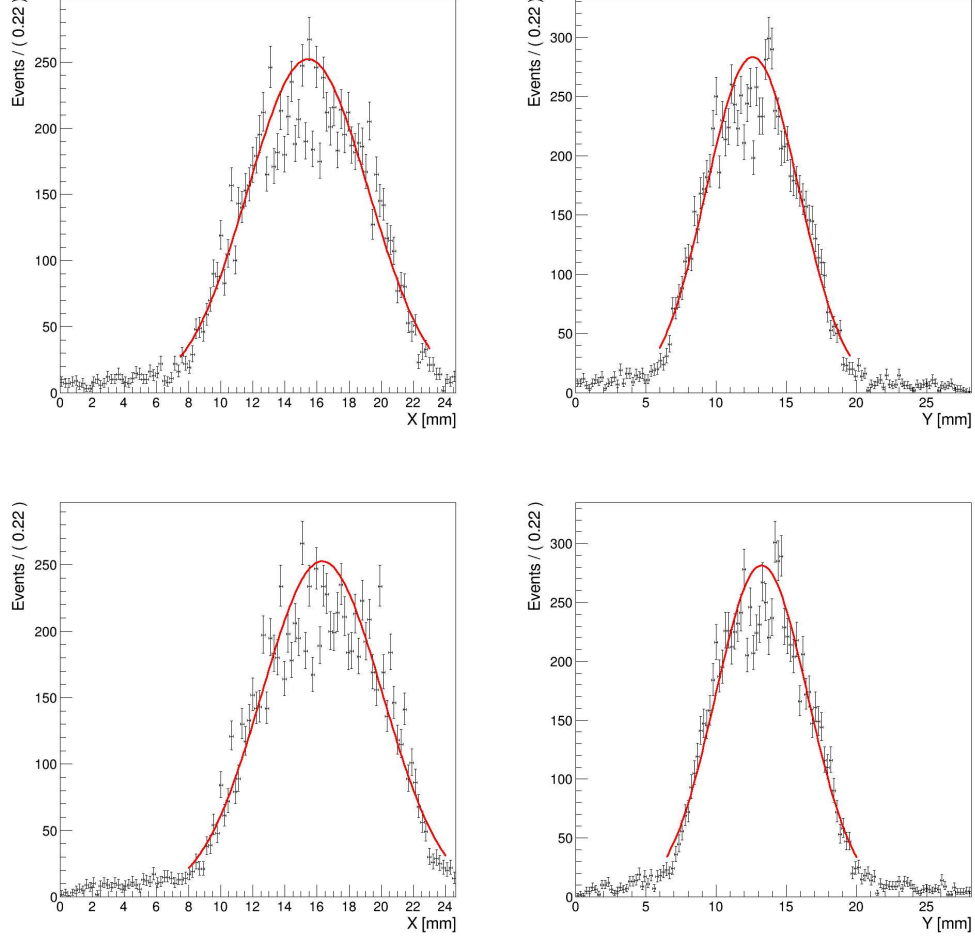


Figure 8.22: (Top) Tracker 1 and (bottom) tracker 2 (left) x and (right) y hit distribution from data collected over 50 spills. The gaussian fit are superimposed.

Parameter	Tracker 1 value [mm]	Tracker 2 value [mm]
μ_x	15.44 ± 0.04	16.32 ± 0.04
μ_y	12.60 ± 0.04	13.24 ± 0.04
σ_x	3.77 ± 0.03	3.75 ± 0.04
σ_y	3.29 ± 0.04	3.28 ± 0.03

Table 8.1: Parameter values from the simultaneous $x - y$ fit for tracker 1 and tracker 2.

free parameters. For each candidate, the fit minimises the distance between the hits and the ring, and only successful fits are retained. An example of a single-event fit is shown in Figure 8.23. The distribution of radii obtained from data,

compared with that from simulation, is presented in Figure 8.24. The mean radius measured from data is $R = 8.62 \pm 0.01$ mm, in good agreement with the simulated value, while a standard deviation of $\sigma = 0.54 \pm 0.01$ mm is measured. The two distributions are in very good agreement, indicating that the main contribution to the resolution comes from the setup geometry. Based on this, the spatial resolution of the DUT, expected to be on the order of ~ 10 μm , cannot be measured.

The distribution of recorded photon counts has also been analysed to estimate the number of detected Cherenkov photons. This estimate is based on the single-event fits described before, considering only hits within $5\sigma_{\text{fit}}$ of the fitted Cherenkov ring, where σ_{fit} is the error on the radius parameter extracted from the fit. Figure 8.24 shows the resulting distribution of photon counts per event, alongside the corresponding distribution from the Geant4 simulation. The data distribution appears less peaked and exhibits a more pronounced tail compared to the simulation. This difference is likely due to detector effects and lens imperfections that are not accounted for in the simulation. The mean number of photons per event is measured to be 15 ± 1 , compared with 13 ± 1 obtained from the simulation with the corrected beam configuration.

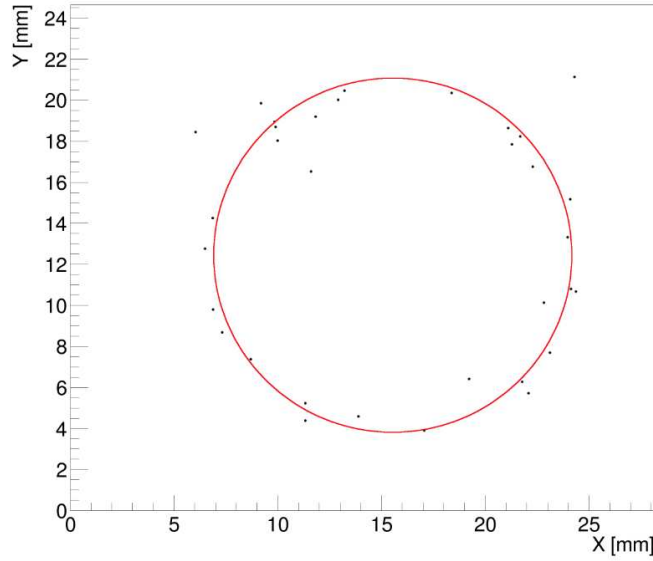


Figure 8.23: Plot of the hits of a single ring candidate with the fitted ring superimposed.

The initial strategy to measure the DUT time resolution was to use the pair of Cherenkov radiators placed downstream of the lens system as a timing reference. However, as already described, issues observed in the output of the timing reference

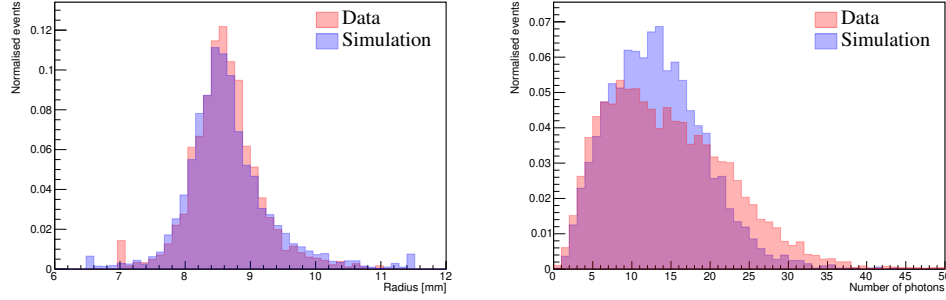


Figure 8.24: (Left) Radius distributions of reconstructed Cherenkov rings with the simulated ring superimposed. (Right) Distribution of the number of photons reconstructed from data with the expected distribution obtained from the simulation superimposed.

system during data taking made these data unusable. The trackers could not serve as an alternative reference either, as shown in Figure 8.19, since their low operating voltage limited their timing resolution to about 1 ns.

Therefore, the DUT time resolution was estimated using only the timing information from candidate Cherenkov rings. For each ring candidate, hits were randomly split into two subsets. The time of each hit in one subset was randomly subtracted from that of a hit in the other subset. This also helps mitigate systematic effects, such as variations in the optical path and chromatic dispersion. The method has been tested on the simulation data, the result is in good agreement with the results reported in Sec.8.2.4. The resulting distribution was fitted with an unbinned maximum likelihood method using a Crystal Ball function [293] to account for possible asymmetric tails. Figure 8.25 shows the measured time difference distribution together with the fit result. The fit yields a timing resolution of $\sigma_{\text{ToA}} = 400 \pm 10$ ps. Since this value comes from the difference of two statistically independent subsets, the intrinsic DUT resolution is obtained by dividing by $\sqrt{2}$, giving $\sigma_{\text{ToA}}^{\text{Testbeam}} = 283 \pm 7$ ps. This value differs from the timing resolution of $\sigma_{\text{ToA}}^{\text{4DPHOTON}} = 88 \pm 9$ ps measured in laboratory. This discrepancy is primarily due to the low gain of the MCP at an operating voltage of 2100 V, compared to the 2400 V bias used in laboratory measurements. Moreover, this result should be considered preliminary, as no external time reference was used in the analysis as described previously.

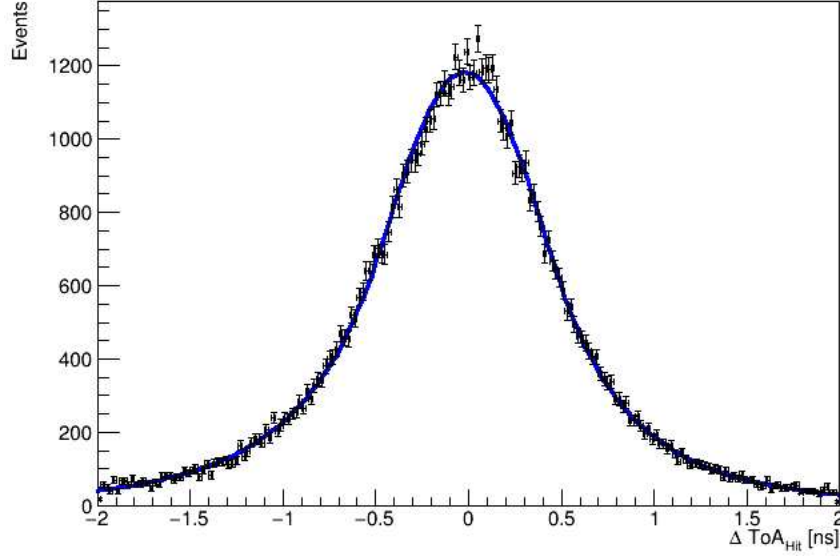


Figure 8.25: Distribution of the time of arrival difference of the photons with the fit results distribution superimposed.

8.4 4DPHOTON characterisation overview

A set of characterisation measurements has been carried out to verify the correct operation of the first MCP-PMT prototype. These measurements include assessments of the dark count rate and gain under dark conditions, as well as timing resolution measurements using a 405 nm picosecond laser setup. The average gain and dark count rate values across the matrix are consistent with the reference values provided by Hamamatsu Photonics Japan (HPK) and meet the project target specifications. However, further investigations are needed to better understand the sources of gain variations observed across the matrix.

Timing resolution results are currently higher than expected. Ongoing efforts aim to reduce the dominant contributions to these results, including improvements in VCO and timewalk calibrations, testing at higher MCP bias to increase the charge collected by each pixel and reduce signal jitter, and optimising electron cloud focusing to distribute the charge across more pixels, thereby enhancing the multi-sampling effect in cluster timing resolution estimation. A precise evaluation of spatial resolution is still in progress; preliminary laser measurements already indicate sub-pixel resolution.

A new testbeam campaign has been performed during November 2025 at the CERN SPS North Area, using the same setup as previously presented but with improvements, including in particular enhancements to the timing system in order to obtain a more precise and reliable time resolution measurement. Tests have been performed over a wider range of bias voltages for both the trackers and the DUT, in order to study the dependence of the measured quantities. Moreover, several prototypes with different properties of the DUT have been tested. A detailed analysis is ongoing.

Chapter 9

Conclusions

During my PhD I have been involved in various activities related to the LHCb experiment. I worked on a new analysis of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decays, with $\eta_c \rightarrow p\bar{p}$, to search for an exotic candidate in the $\eta_c \pi^-$ system, of which I am the main contributor and corresponding author for the relative publication. With respect to the previous analysis [173] the sample is doubled in size, allowing a more precise measurement of the branching ratio of the decay and improving the amplitude analysis description. The measurement of the $B^0 \rightarrow \eta_c K^+ \pi^-$ branching ratio has been performed leading to:

$$\mathcal{B}(B^0 \rightarrow \eta_c K^+ \pi^-) = (5.82 \pm 0.20_{stat} \pm 0.23_{syst} \pm 0.55_{ext}) \cdot 10^{-4}$$

where the first uncertainty is statistical, the second systematic, and the third is due to the limited knowledge of external branching fractions. The data are better described by including a resonance decaying to $\eta_c \pi^-$. However, this resonance is found to be not significant when systematic uncertainties are taken into account. New measurements of the branching fractions of the $B^0 \rightarrow \eta_c K^*$ decays have also been performed. The results of this analysis supersede those obtained in Ref. [173] and, at the time of submission of this thesis, are submitted to EPJC and can be found in Ref. [300].

In parallel to the analysis activity, I also contributed to the LHCb RICH operations and future upgrades. I developed a tool to compute online charged-hadron identification performance. Using data processed by the second level trigger, the yield of the $D^{*+} \rightarrow D^0(\rightarrow K^- \pi^+) \pi^+$ and $\Lambda \rightarrow p \pi^-$ decays are calculated, using a fitting procedure optimised to be robust and consistent, and are used to calculate the identification and misidentification efficiencies of the p , K and π hadrons. This tool have been deployed during the 2025 data taking period of LHCb and used

since then. It is the first tool computing quasi-online (within a day after the data taking) high-level variables for the RICH detectors.

In addition, I contributed to the development of a novel photon detector, based on the Timepix4 ASIC and an MCP, within the 4DPHOTON project. I contributed to the software development of the Timepix4 analysis framework and to the measurements performed to characterise the ASIC, reported in Ref. [346]. I contributed to the measurements performed in the laboratory on the first prototypes of the photon detector and I also developed the Geant4 simulation for the setup used during the testbeam campaign. The testbeam has been carried out at the end of 2024: I participated to the data taking and I performed the offline analysis of the data. The testbeam results are in agreement with the ones obtained from the simulation. The timing resolution of the detector is measured to be $\sigma_{\text{ToA}}^{\text{Testbeam}} = 283 \pm 7$ ps differing from the laboratory measurement of $\sigma_{\text{ToA}}^{\text{4DPHOTON}} = 88 \pm 9$ ps mainly because of the lower bias applied to the MCP during the testbeam.

Addition related to the physics analysis

A Analytical expressions for PDFs

A.1 Double CrystallBall

$$f(x, \mu, \sigma_{left}, \sigma_{right}, a_{left}, n_{left}, a_{right}, n_{right}) = \begin{cases} A_{left} \left(B_{left} - \frac{x - \mu}{\sigma_{left}} \right)^{-n_{left}}, & \text{if } \frac{x - \mu}{\sigma_{left}} < -a_{left} \\ \exp \left(-\frac{1}{2} \left[\frac{x - \mu}{\sigma_{left}} \right]^2 \right), & \text{if } \frac{x - \mu}{\sigma_{left}} \leq 0 \\ \exp \left(-\frac{1}{2} \left[\frac{x - \mu}{\sigma_{right}} \right]^2 \right), & \text{if } \frac{x - \mu}{\sigma_{right}} \leq a_{right} \\ A_{right} \left(B_{right} + \frac{x - \mu}{\sigma_{right}} \right)^{-n_{right}}, & \text{otherwise} \end{cases} \quad (1)$$

A.2 Hypatia

$$f(x, \mu, \sigma, l, \zeta, \beta, a_{left}, n_{left}, a_{right}, n_{right}) = \begin{cases} \frac{G(\mu - a_{left}\sigma, \mu, \sigma, l, \zeta, \beta)}{\left(1 - \frac{x}{n_{left}G(\dots)/G'(\dots) - a_{left}\sigma}\right)^{n_{left}}}, & \text{if } \frac{x - \mu}{\sigma} < a_{left} \\ \frac{G(\mu + a_{right}\sigma, \mu, \sigma, l, \zeta, \beta)}{\left(1 - \frac{x}{-n_{right}G(\dots)/G'(\dots) - a_{right}\sigma}\right)^{n_{right}}}, & \text{if } \frac{x - \mu}{\sigma} > a_{right} \end{cases} \quad (2)$$

where

$$G(x, \mu, \dots) = ((x - \mu)^2 + A_l^2(\zeta)\sigma^2)^{\frac{1}{2}l - \frac{1}{4}} e^{\beta(x - \mu)} K_{l - \frac{1}{2}} \left(\zeta \sqrt{1 + \left(\frac{x - \mu}{A_l(\zeta)\sigma} \right)^2} \right) \quad (3)$$

and

$$A_l^2(\zeta) = \frac{\zeta K_l(\zeta)}{K_{l+1}(\zeta)} \quad (4)$$

$K_l(\zeta)$ are the modified Bessel functions of the second kind.

B DecayTreeFitter variables

The DecayTreeFitter (DTF) is employed to recalculate the four-momenta of the final state particles by applying specific mass and/or vertex constraints. This process enhances the mass resolution. A DTF fit is applied in which the B^0 mass is constrained to its PDG value and the B^0 momentum is required to point to the primary vertex. No constraints were imposed on the DTF χ^2 .

The following variables have been obtained through DTF:

- $m_{p\bar{p}}$, used as control variable in the 1D sPlot procedure and as discriminant variable in the 2D sPlot procedure;
- The DP variables $m_{K^+\pi^-}^2$, $m_{\eta_c\pi^-}^2$ and $m_{\eta_c K^+}^2$ and the SDP variables m' and θ' ;

C MVA input variables and BDT output cross-checks

In order to check if the variables used in the MVA analysis are compatible between data and MC a comparison after the multivariate selection is performed. The comparison is not possible before the multivariate selection because the data are dominated by the combinatorial background. The data distribution are also reweighted using the SWeights from the 1D fit described in sec. 4.5.2 and showed in figures 4.22. The comparison are shown in figure C.1 and C.2.

C. MVA input variables and BDT output cross-checks

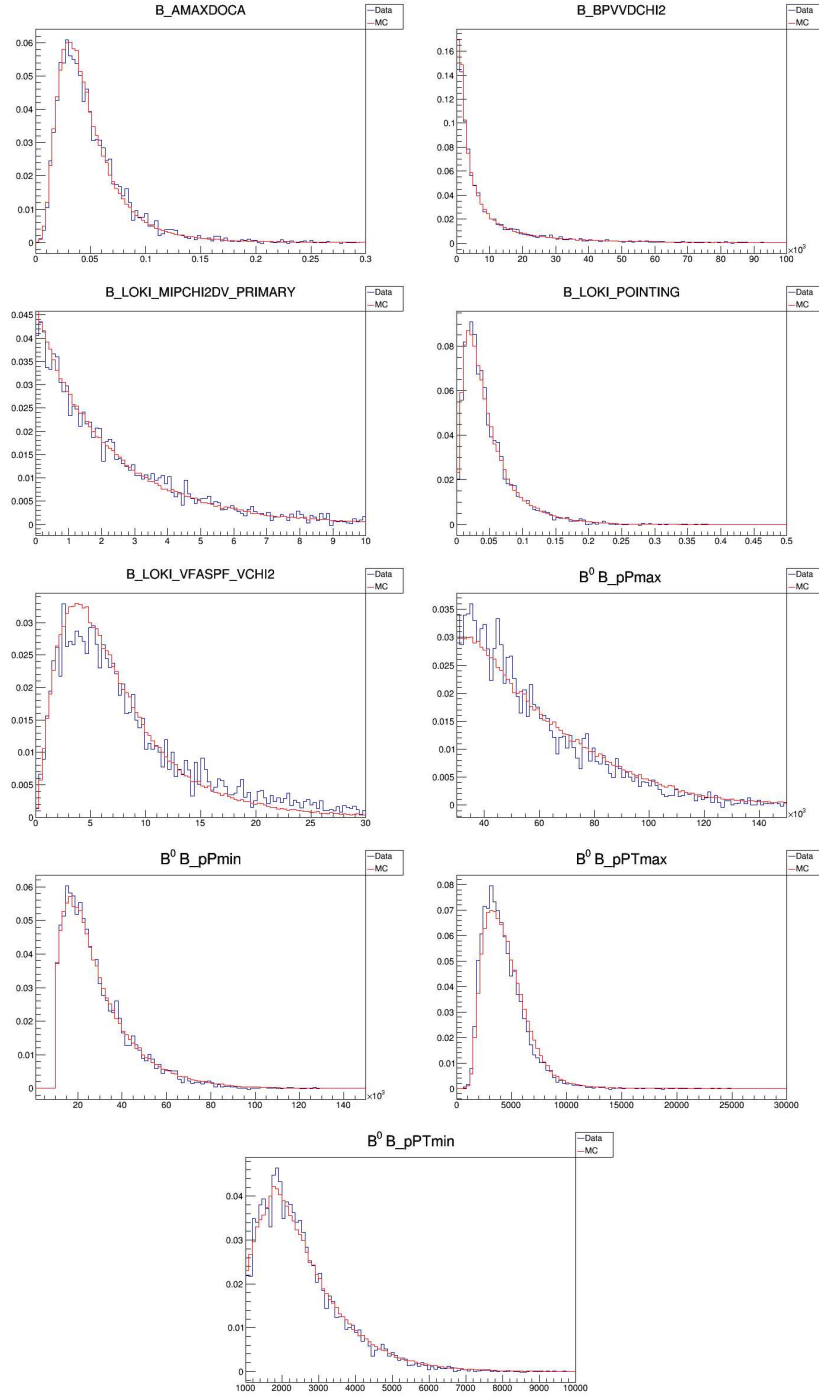


Figure C.1: Comparison for Run1 MVA discriminating variables for data (blue) and MC (red).

C. MVA input variables and BDT output cross-checks

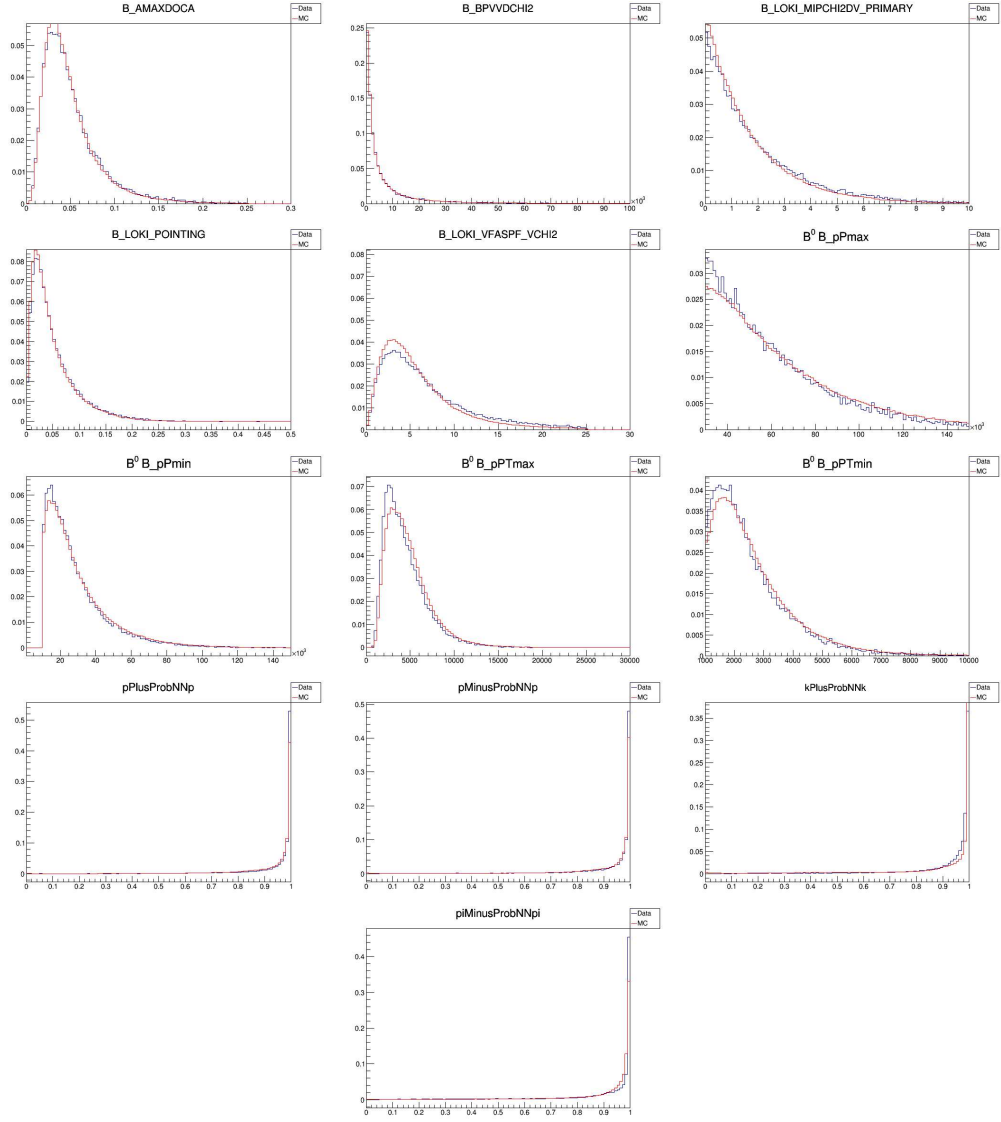


Figure C.2: Comparison for Run2 MVA discriminating variables for data (blue) and MC (red).

D Fit models and MC fit

D.1 $B^0 \rightarrow p\bar{p}K^+\pi^-$

The $B^0 \rightarrow p\bar{p}K^+\pi^-$ component has been parametrised using the Hypatia PDF. The fits to the Run1 and Run2 sample are shown in Figure D.1 and the resulting parameters are shown in Tab. 1.

Parameter	Value
Run1	
μ	5280.90 ± 0.19
σ	13.57 ± 0.08
β	$(-1.75 \pm 1.18) \cdot 10^{-3}$
z	$(7.14 \pm 9.46) \cdot 10^{-4}$
l	-4.27 ± 0.23
a_{left}	2.34 ± 0.05
n_{left}	2.15 ± 0.14
a_{right}	2.52 ± 0.09
n_{right}	2.43 ± 0.25
Run2	
μ	5281.60 ± 0.03
σ	13.68 ± 0.05
β	$(-4.18 \pm 0.48) \cdot 10^{-3}$
z	$(1.05 \pm 7.52) \cdot 10^{-4}$
l	-3.98 ± 0.11
a_{left}	2.39 ± 0.03
n_{left}	2.05 ± 0.07
a_{right}	2.43 ± 0.05
n_{right}	2.11 ± 0.15

Table 1: Parameters extracted from the fit to the Run1 and Run2 simulated $m_{p\bar{p}K^+\pi^-}$ invariant mass distribution.

D.2 $B^0 \rightarrow p\bar{p}\pi^+\pi^-$

The $B^0 \rightarrow p\bar{p}\pi^+\pi^-$ component has been parametrised using the CrystallBall function. The fits to the Run1 and Run2 sample are shown in Figure D.2 and the resulting parameters are shown in Tab. 2.

D.3 $B_s^0 \rightarrow p\bar{p}K^+\pi^-$

The $B_s^0 \rightarrow p\bar{p}K^+\pi^-$ component has been parametrised using the Hypatia PDF. The fits to the Run1 and Run2 sample are shown in Figure D.3 and the resulting

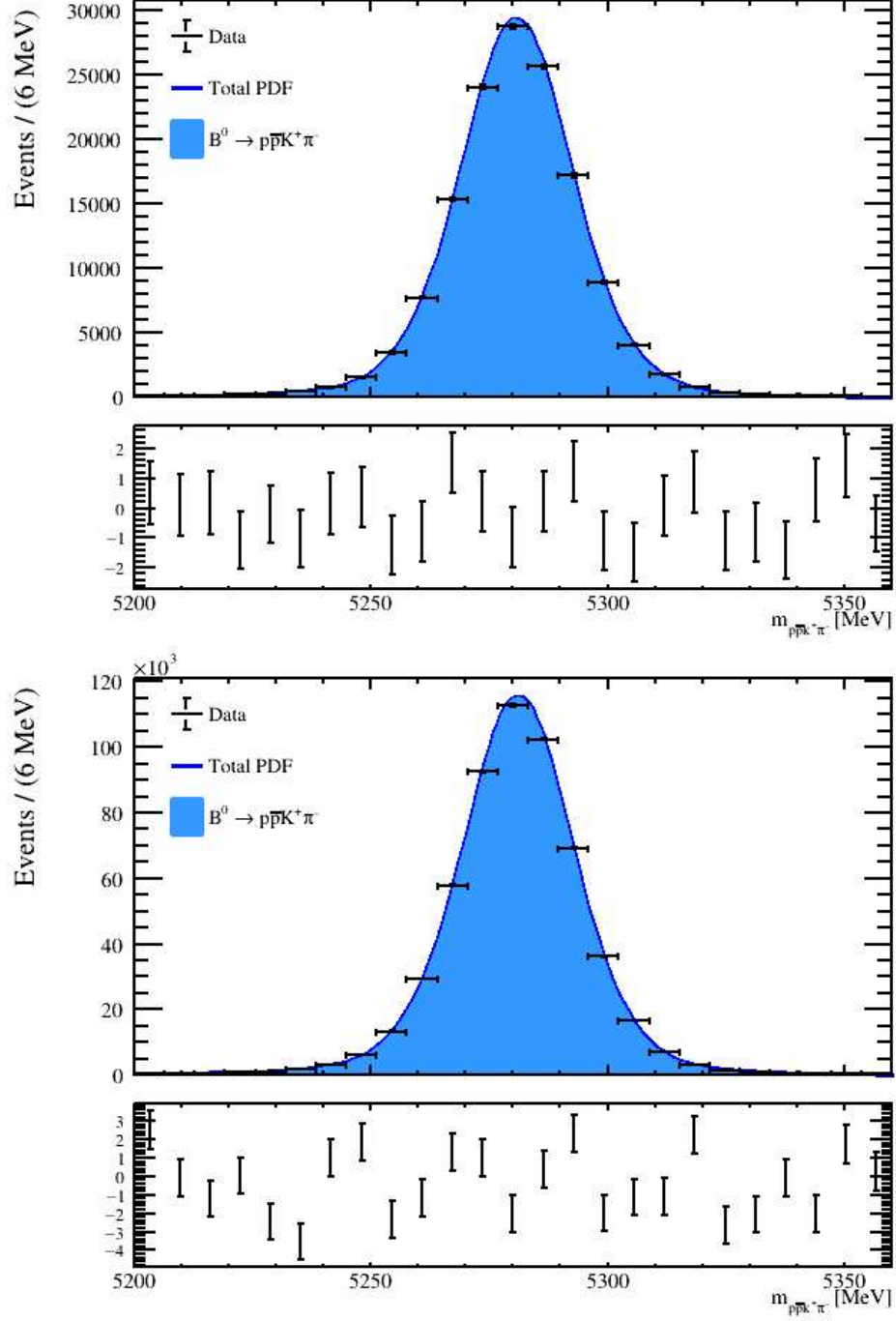


Figure D.1: Fit to Run1 (top) and Run2 (bottom) $m_{p\bar{p}K^+\pi}$ -MC events. The pull distribution are also shown.

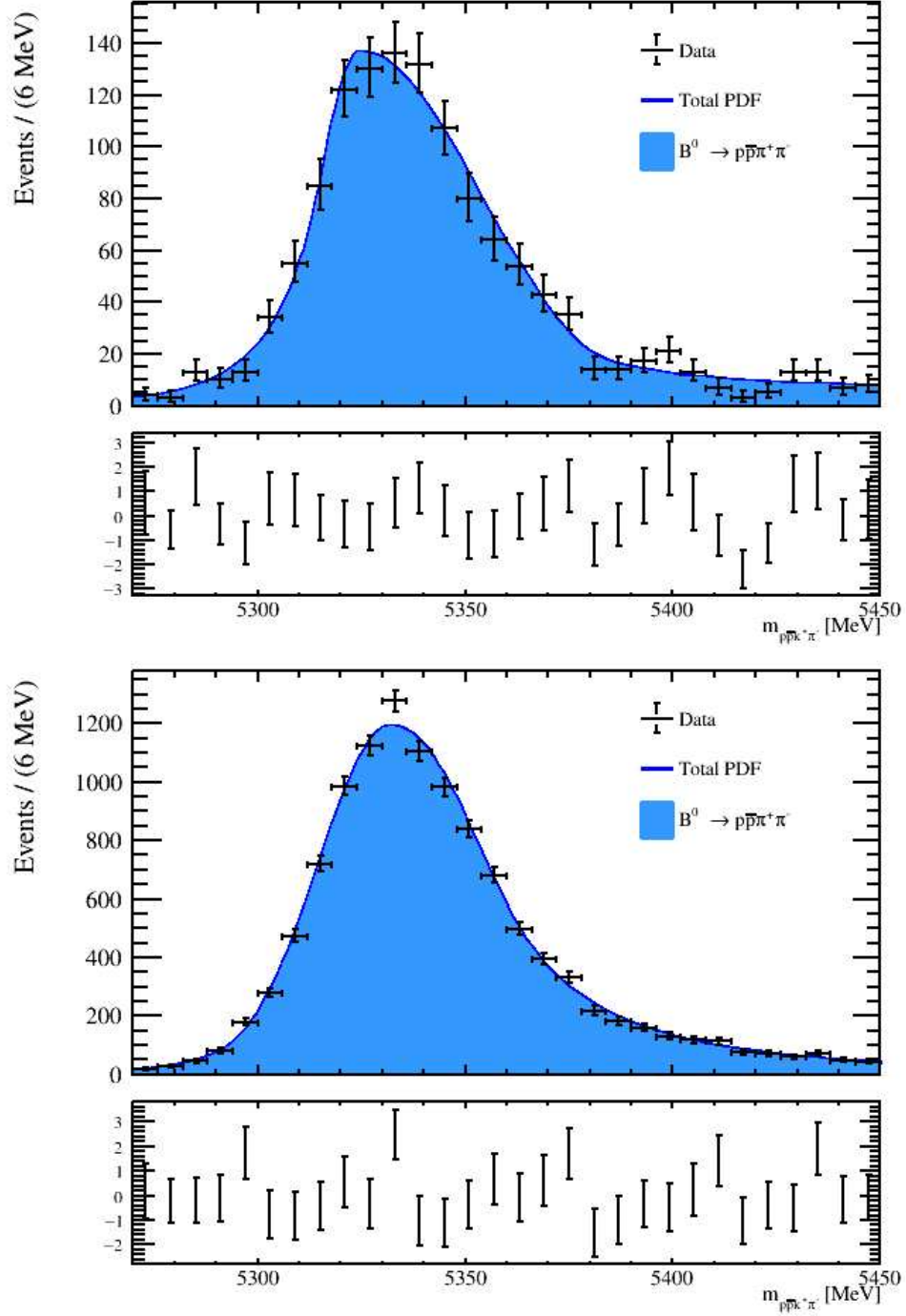


Figure D.2: Fit to Run1 (top) and Run2 (bottom) $B^0 \rightarrow p\bar{p}\pi^+\pi^-$ invariant mass distribution. The pull distribution are also shown.

Parameter	Value
Run1	
μ	5325 ± 3
σ_{left}	10 ± 4
σ_{right}	28 ± 2
α_{left}	0.9 ± 0.3
n_{left}	8 ± 7
α_{right}	1.9 ± 0.1
n_{right}	0.5 ± 0.3
Run2	
μ	5332.1 ± 0.8
σ_{left}	17.3 ± 0.6
σ_{right}	23 ± 1
α_{left}	1.9 ± 0.2
n_{left}	8 ± 8
α_{right}	1.14 ± 0.08
n_{right}	2.8 ± 0.5

Table 2: Parameters extracted from the fit to the Run1 and Run2 simulated $B^0 \rightarrow p\bar{p}\pi^+\pi^-$ invariant mass distribution.

parameters are shown in Tab. 3.

D.4 $B_s^0 \rightarrow p\bar{p}K^+K^-$

The $B_s^0 \rightarrow p\bar{p}K^+K^-$ component has been parametrised using the Crystal Ball function. The fits to the Run1 and Run2 sample are shown in Figure D.4 and the resulting parameters are shown in Tab. 4.

D.5 $J/\psi \rightarrow p\bar{p}$

The $J/\psi \rightarrow p\bar{p}$ component has been parametrised using the a Hypatia PDF. The fits to the Run1 and Run2 sample are shown in Figure D.5 and the resulting parameters are shown in Tab. 17.

D.6 $\eta_c \rightarrow p\bar{p}$

The $\eta_c \rightarrow p\bar{p}$ component has been parametrised using the a Crystal Ball function convoluted with a relativistic Breit-Wigner. The fits to the Run1 and Run2 sample are shown in Figure D.6 and the resulting parameters are shown in Tab. 6.

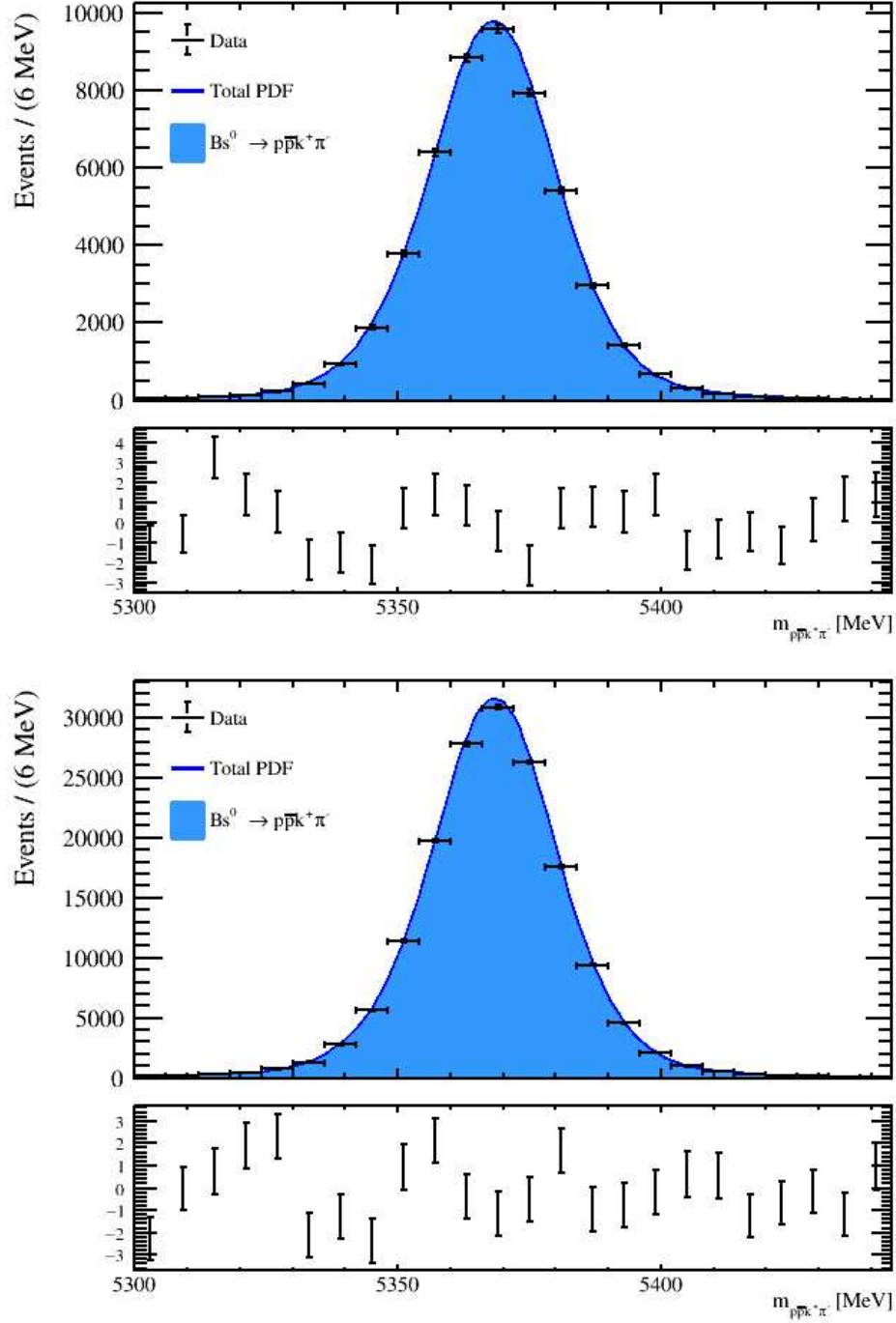


Figure D.3: Fit to Run1 (top) and Run2 (bottom) $B_s^0 \rightarrow p\bar{p}K^+\pi^-$ invariant mass distribution. The pull distribution are also shown.

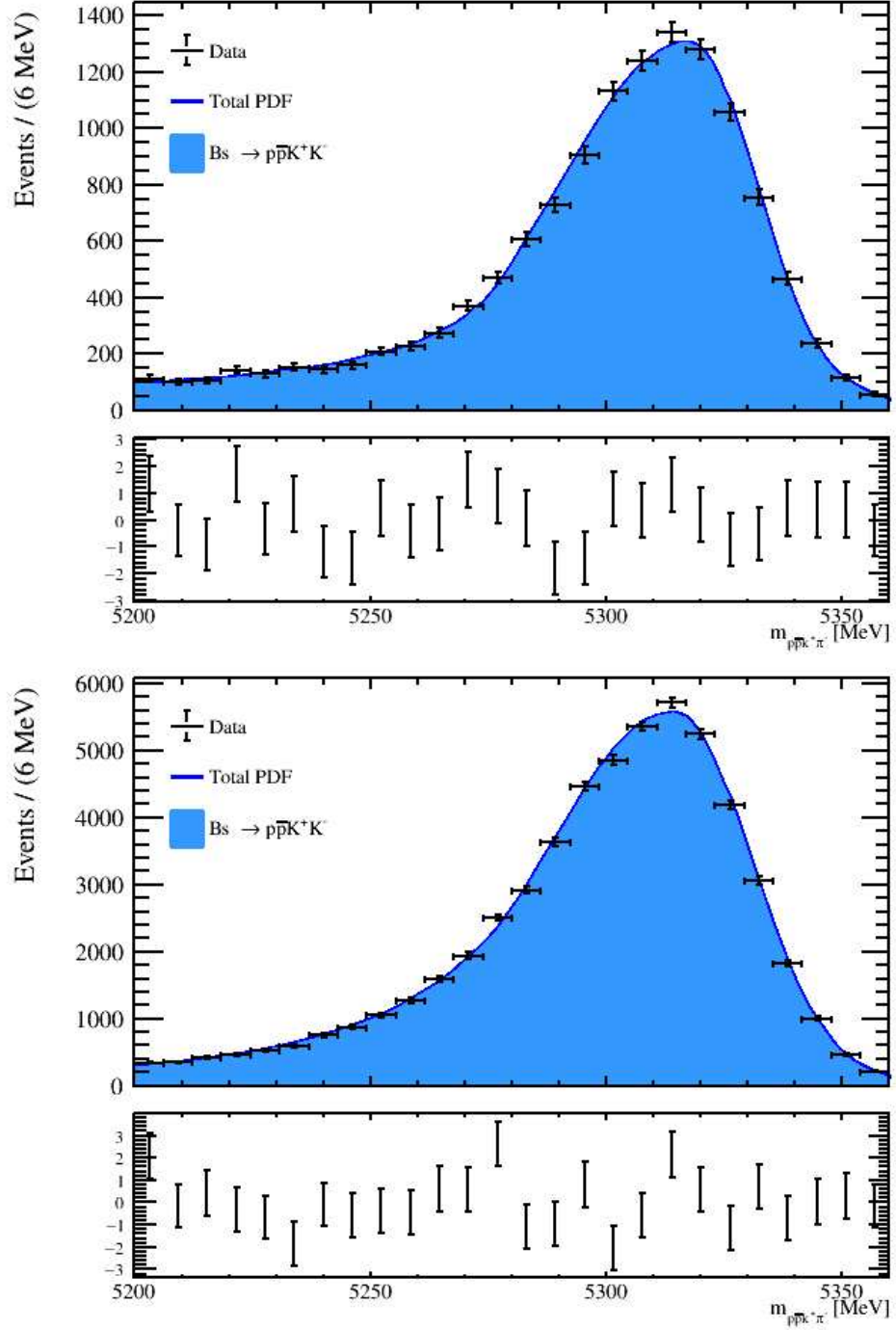


Figure D.4: Fit to Run1 (top) and Run2 (bottom) $B_s^0 \rightarrow p\bar{p}K^+K^-$ invariant mass distribution. The pull distribution are also shown.

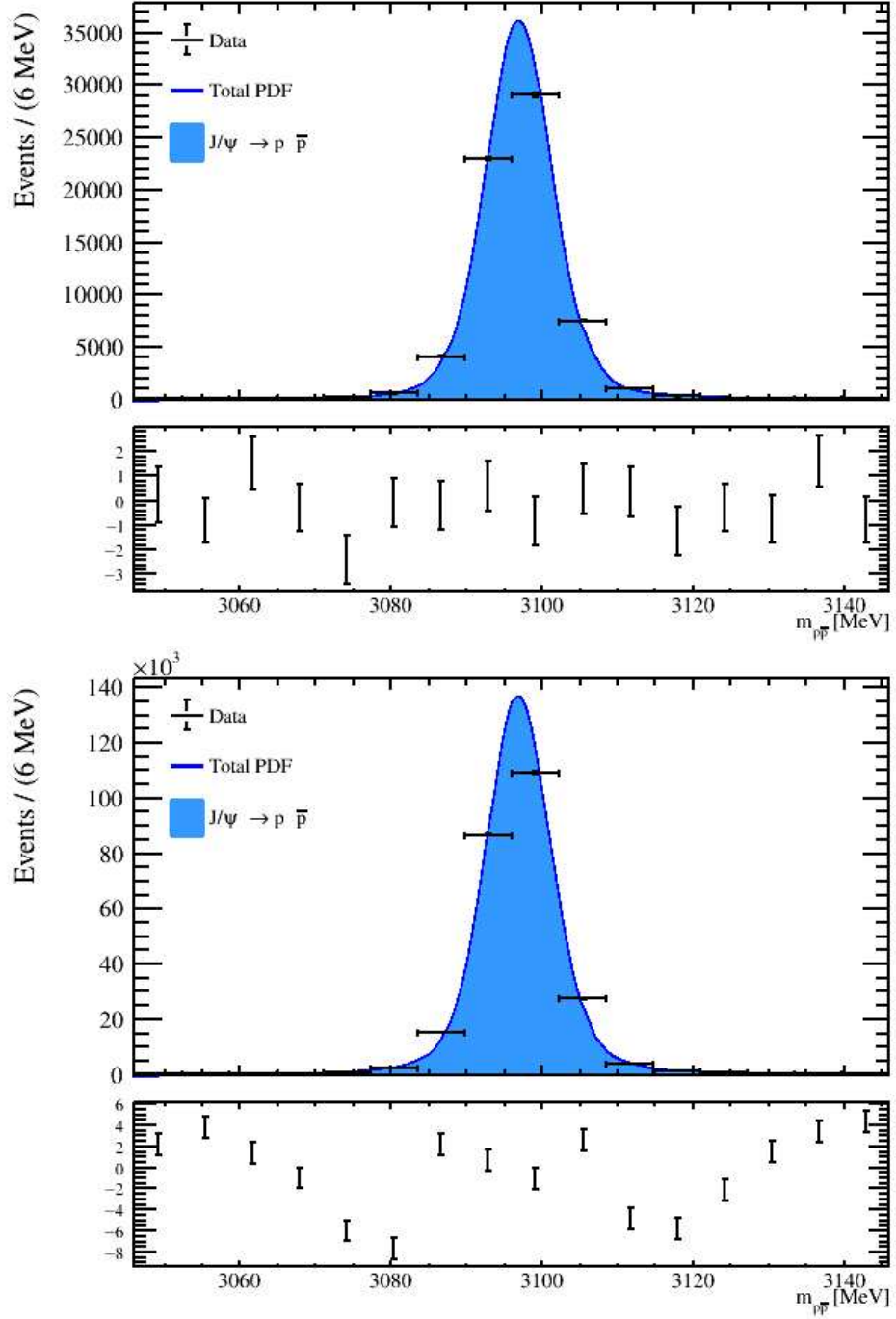


Figure D.5: Fit to the Run1 and Run2 simulated $J/\psi \rightarrow p\bar{p}$ invariant mass distribution from $B^0 \rightarrow J/\psi K^+ \pi^-$. The pull distribution are also shown

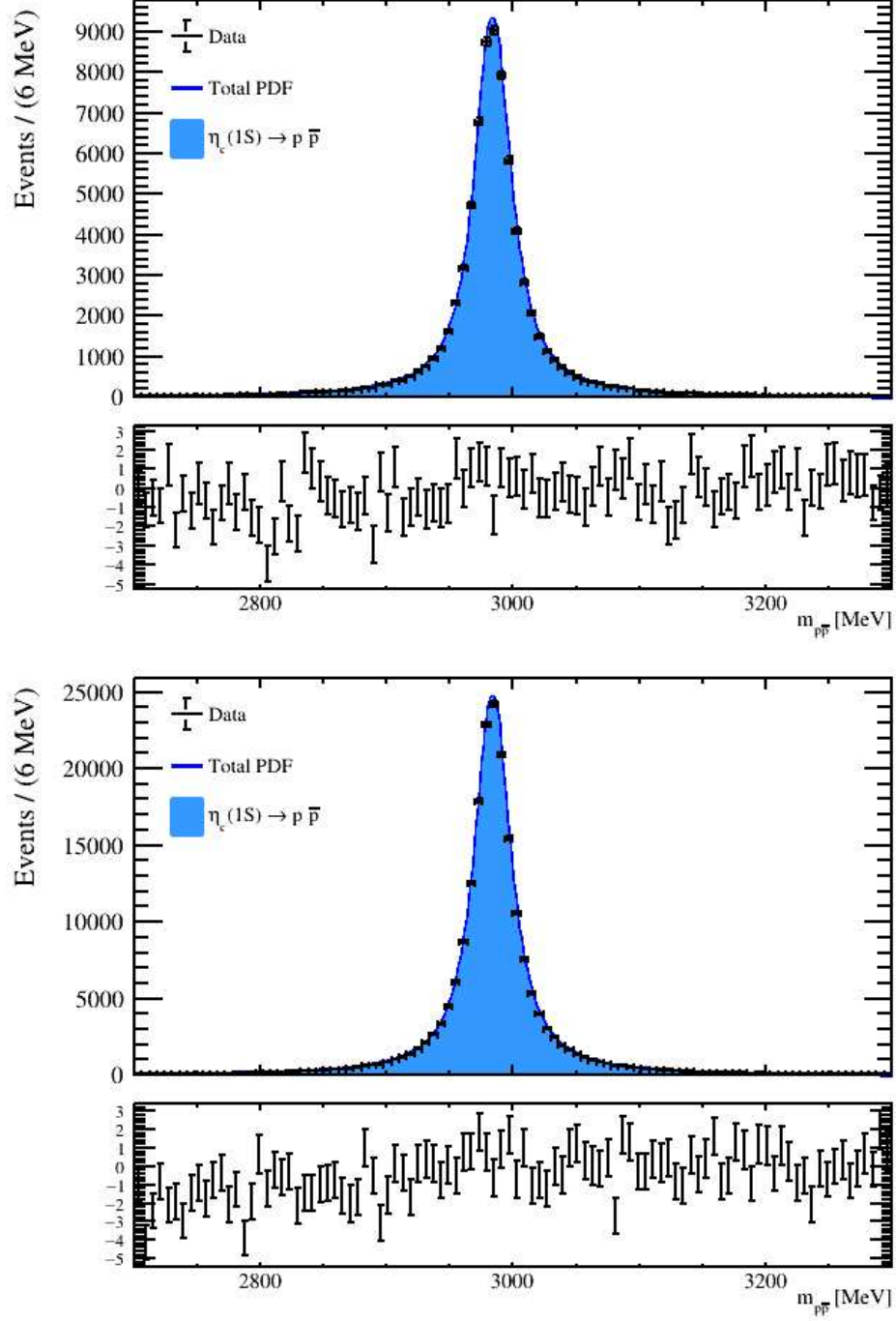


Figure D.6: Fit to the Run1 and Run2 simulated $\eta_c \rightarrow \bar{p}p$ invariant mass distribution from $B^0 \rightarrow \eta_c K^+ \pi^-$. The pull distribution are also shown

Parameter	Value [MeV]
Run1	
μ	5369.45 ± 0.01
σ	14.01 ± 0.01
β	-0.009 ± 0.001
z	0.099 ± 0.001
l	-4.017 ± 0.001
a_{left}	3.835 ± 0.001
n_{left}	0.0 ± 0.9
a_{right}	1.819 ± 0.001
n_{right}	4.659 ± 0.001
Run2	
μ	5369.09 ± 0.04
σ	14.05 ± 0.09
β	-0.005 ± 0.001
z	0.30 ± 0.01
l	-3.57 ± 0.04
a_{left}	3.01 ± 0.01
n_{left}	0.9 ± 0.2
a_{right}	2.4 ± 0.1
n_{right}	2.33 ± 0.09

Table 3: Parameters extracted from the fit to the Run1 and Run2 simulated $B_s^0 \rightarrow p\bar{p}K^+\pi^-$ invariant mass distribution.

Parameter	Value [MeV]
Run1	
μ	5317.3 ± 0.8
σ_{left}	28 ± 1
σ_{right}	14.5 ± 0.6
α_{left}	1.45 ± 0.09
n_{left}	0.9 ± 0.2
α_{right}	1.8 ± 0.2
n_{right}	10 ± 8
Run2	
μ	5314.3 ± 0.4
σ_{left}	27.8 ± 0.7
σ_{right}	16.2 ± 0.3
α_{left}	1.06 ± 0.05
n_{left}	4.3 ± 0.7
α_{right}	2.2 ± 0.2
n_{right}	9 ± 6

Table 4: Parameters extracted from the fit to the Run1 and Run2 simulated $B_s^0 \rightarrow p\bar{p}K^+K^-$ invariant mass distribution.

Parameter	Value [MeV]
Run1	
μ	3096.72 ± 0.07
σ	5.1 ± 0.1
l	-3.1 ± 1
z	1 ± 2
$beta$	0.008 ± 0.003
a_{left}	2.43 ± 0.09
n_{left}	2.44 ± 0.06
a_{right}	2.2 ± 0.1
n_{right}	1.9 ± 0.1
Run2	
μ	3096.60 ± 0.02
σ	4.83 ± 0.03
l	3 ± 3
z	3 ± 2
$beta$	0.013 ± 0.001
a_{left}	1.96 ± 0.02
n_{left}	2.69 ± 0.05
a_{right}	2.05 ± 0.02
n_{right}	2.69 ± 0.05

Table 5: Parameters extracted from the fit to the Run1 and Run2 simulated $B^0 \rightarrow J/\Psi K^+ \pi^-$ with $J/\Psi \rightarrow p\bar{p}$ invariant mass distribution.

Parameter	Value [MeV]
Run1	
μ_{DCB}	0 (fixed)
σ_L	2.1 ± 0.9
σ_R	5.2 ± 0.6
α_{left}	4.4 ± 0.4
α_{right}	2.2 ± 0.1
n_{left}	15 ± 12
n_{right}	2.2 ± 0.2
μ_{RBW}	2981 ± 1
Γ	30.46 ± 0.05
Run2	
μ_{DCB}	0 (fixed)
σ_L	3.0 ± 0.4
σ_R	0.8 ± 0.2
α_{left}	1.0 ± 0.2
α_{right}	0.7 ± 0.1
n_{left}	15 ± 2
n_{right}	2.3 ± 0.2
μ_{RBW}	2985.4 ± 0.3
Γ	30.43 ± 0.05

Table 6: Parameters extracted from the fit to the Run1 and Run2 simulated $B^0 \rightarrow \eta_c K^+ \pi^-$ with $\eta_c \rightarrow p\bar{p}$ invariant mass distribution.

E Check on PID resample

The resampling is applied to the PID variables using the available calibration samples. In figure E.1 and E.2 are shown the distribution of the ProbNN variables before and after the resampling. The resampled variables show a clear bias. Another check for a possible bias in the resampling due to the cut applied in the stripping line for Run2 is performed. Tighter cut are applied with respect to the one reported in table 4.3, the cut are increased of 0.05 before applying the resampling. The comparison between the MC resampled variables used in the analysis and the MC resampled variables obtained by increasing the cuts are shown in figure E.3, E.4, E.5 and E.6. No difference are present between the two distribution so the cuts applied in the stripping line do not bias the resampling.

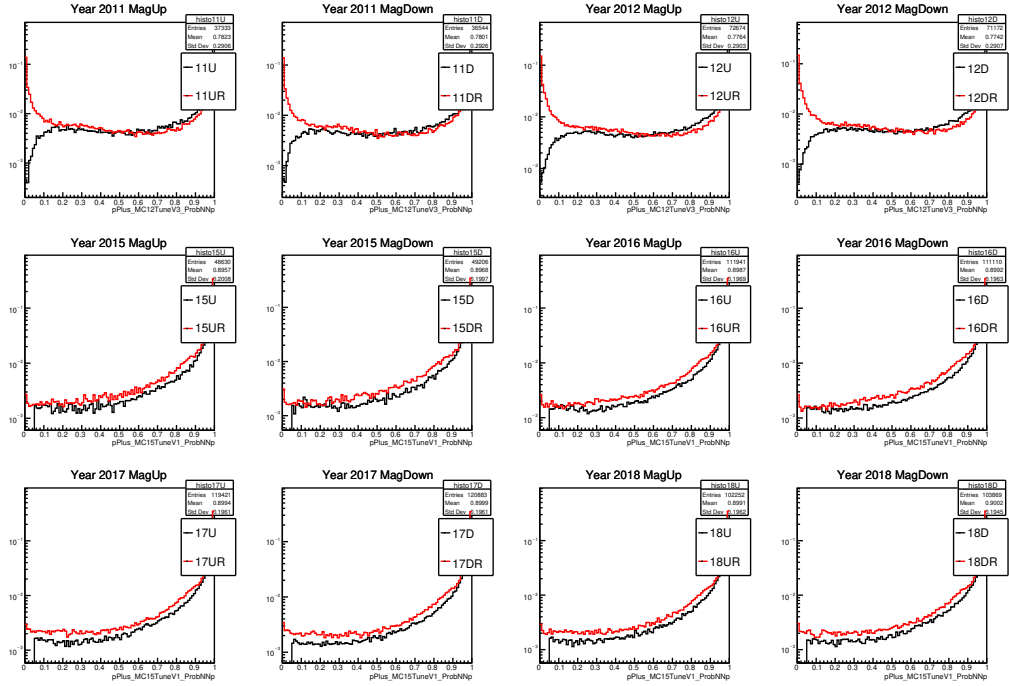


Figure E.1: Comparison for pPlusProbNNp variables before (black) and after (red) the resampling, the Run1 variables show a clear bias.

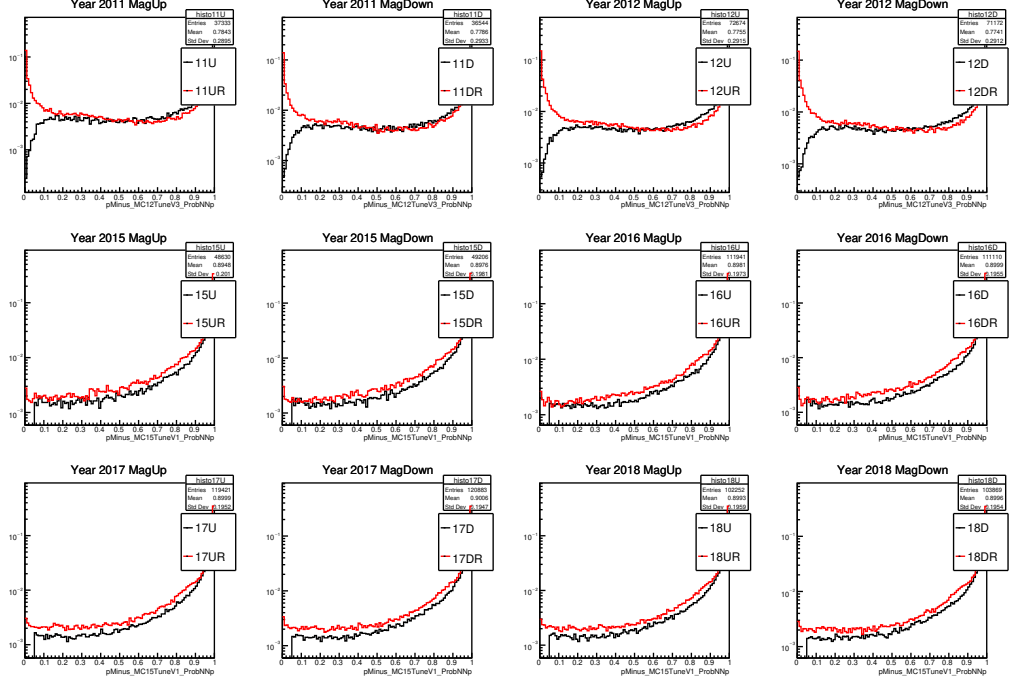


Figure E.2: Comparison for pMinusProbNNp variables before (black) and after (red) the resampling, the Run1 variables show a clear bias.

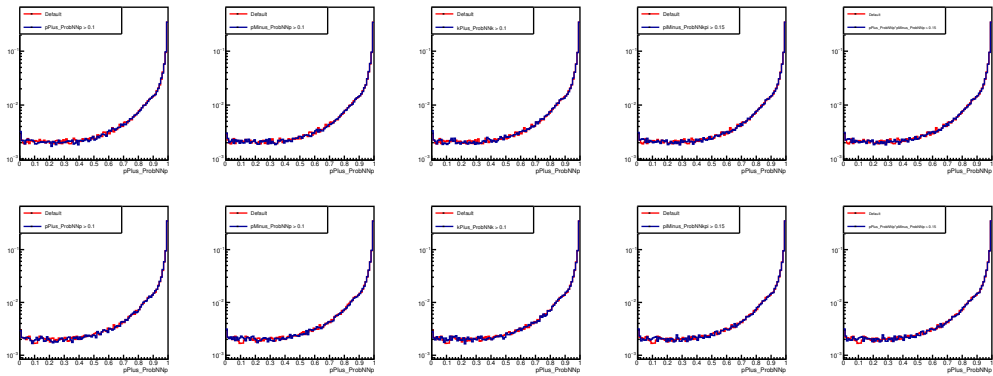


Figure E.3: Comparison for pPlusProbNNp resampled variables with the nominal stripping cut (blue) and with the stripping cut increased by 0.05 (red).

E. Check on PID resample

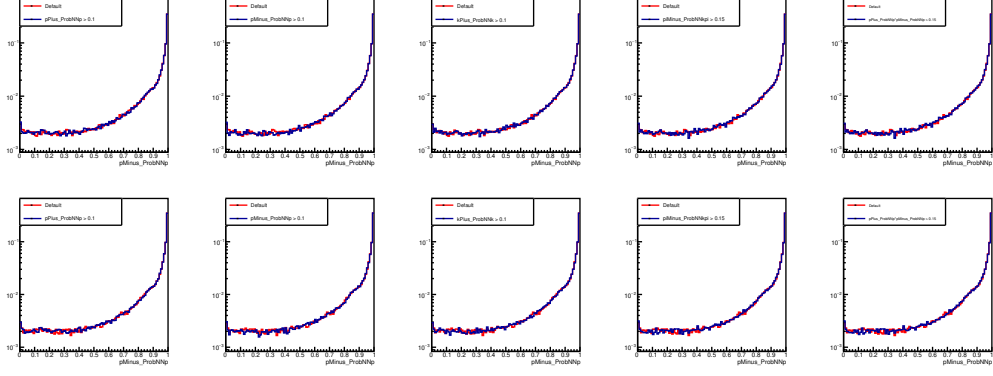


Figure E.4: Comparison for pMinusProbNNp resampled variables with the nominal stripping cut (blue) and with the stripping cut increased by 0.05 (red).

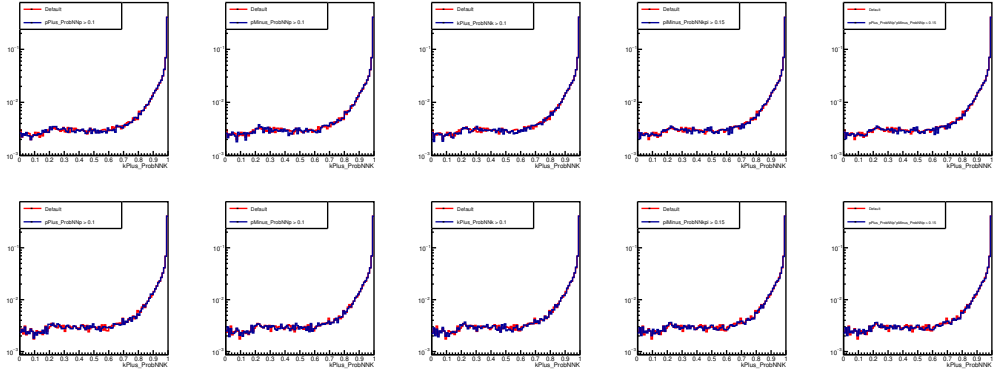


Figure E.5: Comparison for kPlusProbNNK resampled variables with the nominal stripping cut (blue) and with the stripping cut increased by 0.05 (red).

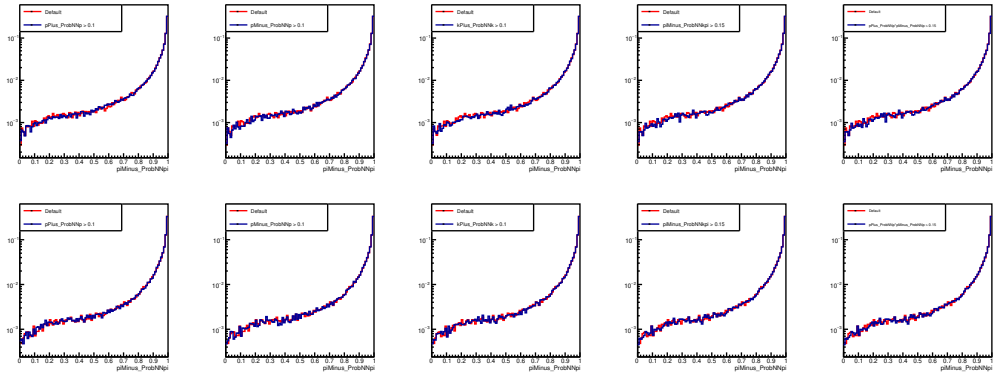


Figure E.6: Comparison for piMinusProbNNpi resampled variables with the nominal stripping cut (blue) and with the stripping cut increased by 0.05 (red).

F Multiple candidates distributions

The 2D invariant mass distribution $m_{p\bar{p}}VS m_{p\bar{p}K^+\pi^-}$ of the rejected multiple candidates is shown in figure F.1. The majority of events are from the $B^0 \rightarrow J/\psi K^+\pi^-$ decay which would not be selected after the cut on the η_c mass. The number of events rejected as multiple candidates that are in the signal region ($m_{p\bar{p}K^+\pi^-}$ in [5220,5340] MeV and $m_{p\bar{p}}$ in [2908,3058] MeV) are 45 for Run 1 and 187 for Run 2.

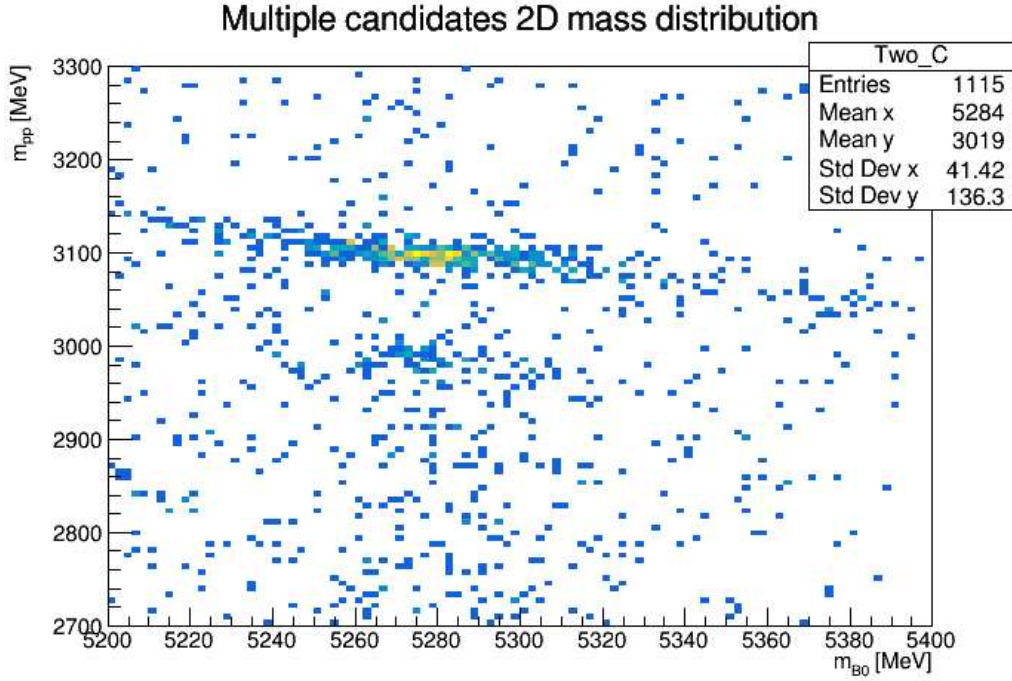


Figure F.1: $m_{p\bar{p}}VS m_{p\bar{p}K^+\pi^-}$ distribution for the multiple candidates eliminated from the sample.

G Alternative 2D fit models

The 2D fit results to the $m_{p\bar{p}}$ and $m_{p\bar{p}K^+\pi^-}$ with the addition of the component describing η_c combined with spurious K^+ and π^- are shown in figure G.1 and G.2. In table 7 are shown the yields extracted. As can be seen the added component has yield always compatible with 0.

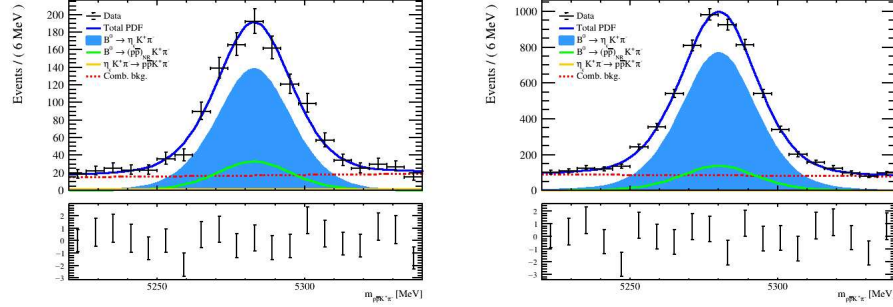


Figure G.1: Run1 (left) and Run2 (right) $m_{p\bar{p}K^+\pi^-}$ invariant mass distribution fit projection with the component describing η_c combined with spurious K^+ and π^- . The pull plot are also shown.

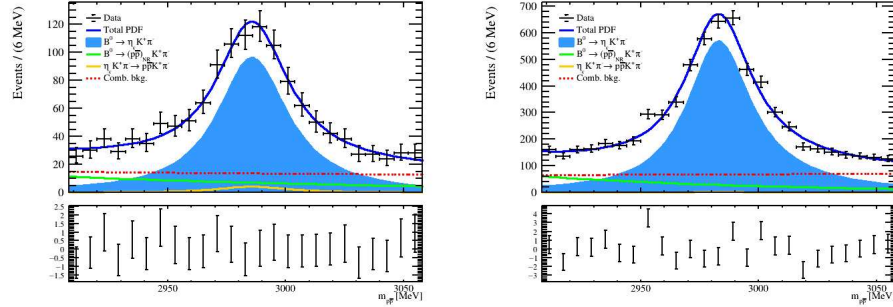


Figure G.2: Run1 (left) and Run2 (right) $m_{p\bar{p}}$ invariant mass distribution fit projection with the component describing η_c combined with spurious K^+ and π^- . The pull plot are also shown.

Yield	Value
Run1	
N_{sig}	780 ± 72
N_{bkg}^{NR}	186 ± 68
N_{bkg}^{comb}	344 ± 44
$N_{bkg}^{\eta_c NR}$	34 ± 39
Run2	
N_{sig}	4384 ± 124
N_{bkg}^{NR}	782 ± 104
N_{bkg}^{comb}	1708 ± 87
$N_{bkg}^{\eta_c NR}$	18 ± 67

Table 7: Yield extracted from the 2D mass fit with the component describing η_c combined with spurious K^+ and π^- .

H Alternative signal extraction strategy

The new strategy consists of: first perform a fit to the $m_{p\bar{p}}$ distribution and then, from the SWeights obtained, the fit on the B^0 mass distribution using the same ranges defined in the note. In figure H.1 are shown the plot to the m_{pp} invariant mass distribution. In figure H.2 are shown the fit performed on the B^0 distribution reweighted using the SWeights from the previous fit, in table 8 are reported the yields extracted. Table 8 shows that all the components are negligible except for the $B^0 \rightarrow \eta_c K^+ \pi^-$. Also in this case the non-resonant contribution of the η_c is negligible.

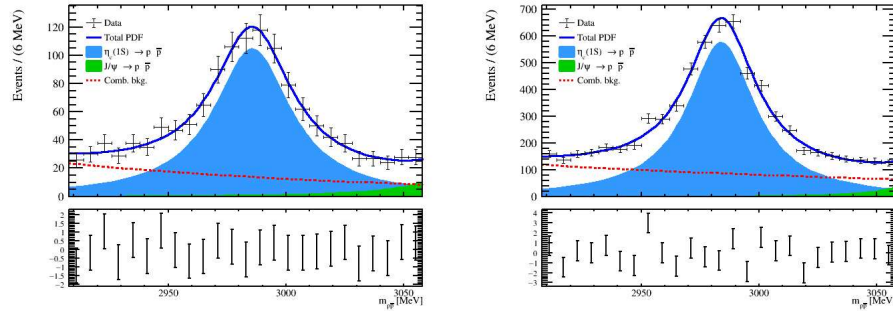


Figure H.1: Fit to the Run1 (left) and Run2 (right) M_{pp} invariant mass distribution. The pull distribution are also shown.

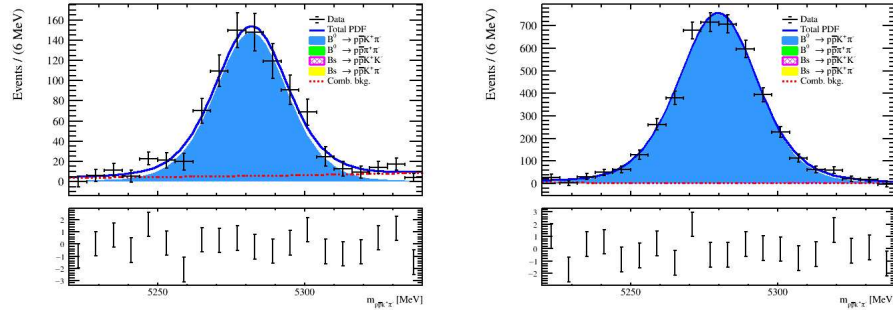


Figure H.2: Fit to the weighted Run1 (left) and Run2 (right) M_{B^0} invariant mass distribution. The pull distribution are also shown.

Yield	Value
Run1	
$N_{B2ppK\pi}$	802 ± 38
$N_{B2pppipi}$	0 ± 34
$N_{Bs2ppK\pi}$	0 ± 6
$N_{Bs2ppkk}$	3 ± 229
N_{bkg}	120 ± 49
Run2	
$N_{B2ppK\pi}$	4397 ± 81
$N_{B2pppipi}$	0 ± 13
$N_{Bs2ppK\pi}$	0 ± 1
$N_{Bs2ppkk}$	78 ± 43
N_{bkg}	60 ± 32

Table 8: Yield extracted from the SPlot fit to the B^0 distribution.

I Data-MC discrepancies studies for efficiencies calculation

The impact of the different tracking response between data and MC in the efficiency maps have been studied. In particular, checks for different multiplicity of the tracks or number of SPD hits between data and MC, affecting the trigger selection, have been performed. The efficiency maps, described in sec. 4.7, have been computed with only the trigger selection applied. Two new sets of maps have been calculated reweighting each event with the ratio:

$$\frac{n_{Tracks}^{data}}{n_{Tracks}^{MC}} \quad \text{or} \quad \frac{n_{SPDhits}^{data}}{n_{SPDhits}^{MC}}$$

The normalised histograms are shown in figures I.1 and I.2. The maps do not show a clear pattern, the distribution is uniform so the effects of this data and MC discrepancies are negligible.

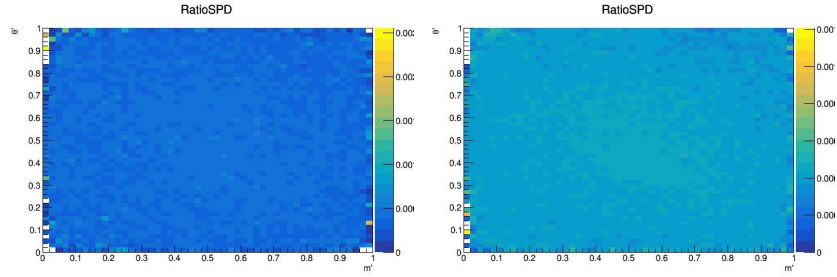


Figure I.1: Ratio of the reweighted and non-reweighted trigger efficiency maps based on the SPD hits number, Run1 (left) and Run2 (right). Z-axis is shown in arbitrary units.

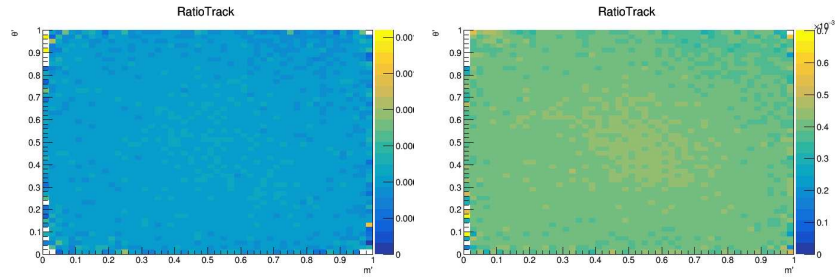


Figure I.2: Ratio of the reweighted and non-reweighted trigger efficiency maps based on the number of tracks, Run1 (left) and Run2 (right). Z-axis is shown in arbitrary units.

J Control and discriminating variable correlation

A check for possible correlation between the control and discriminating variables used in the SWeights methods is performed. In Figure J.1 are shown the 2D distribution of the variables. In table 9 are shown the correlation factor of the control and discriminating variables. The qualitative inspection and the correlation coefficient indicates that the variables are not correlated.

Set of variables	Correlation
$m_{pp} V S m_{K^+\pi^-}$	-0.057
$m_{pp} V S m_{\eta_c \pi^-}$	0.088
$m_{pp} V S m_{\eta_c K^+}$	-0.018
$m_{B^0} V S m_{K^+\pi^-}$	0.017
$m_{B^0} V S m_{\eta_c \pi^-}$	-0.031
$m_{B^0} V S m_{\eta_c K^+}$	0.009

Table 9: Correlation factor between discriminating and control variables used for the SWeight fit.

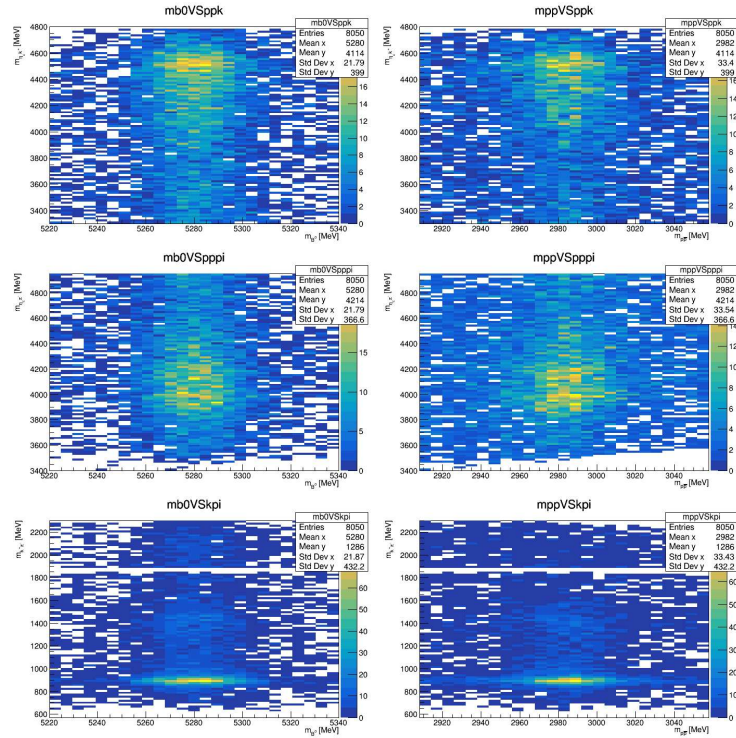


Figure J.1: 2D distribution of the discriminating and control variables used in the SWeights method.

K Comparison with the results of the previous analysis

As already described in 4.1, an analysis has been performed studying the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay and found the evidence for a state decaying to $\eta_c \pi^-$ [173]. The mass and width found are:

$$m_{T_{c\bar{c}}(4100)^-} = 4096 \pm 20_{-22}^{+18} MeV$$

$$\Gamma_{T_{c\bar{c}}(4100)^-} = 152 \pm 58_{-35}^{+60} MeV$$

with a fit fractions of $3.3 \pm 1.1_{-1.1}^{+1.2}$, the J^P favourite assignement could not be discriminated once systematic uncertainties are taken into account.

In order to cross check the results obtained in this analysis, the same fit strategy has been applied to the Run1 and 2016 data only and the amplitude fit has been performed in this data sample. The results for the new resonance are reported in table 10. No systematics uncertainties are taken into account.

As can be seen the results are consistent with each other, the only discrepancy is on the width of the $J^P = 1^-$ hypothesis, even though they are compatible within $\sim 1\sigma$. The reason for this discrepancy is probably due to some differences in the signal selection and the different fitter and minuit version, from ROOT 6.30 Minuit2 is the default minimizer.

Variable	Value from current analysis	Value from the previous analysis
$J^P = 0^+$ hypothesis		
$m_{T_{c\bar{c}}(4100)^-}$	$4090 \pm 14 MeV$	$4059 \pm 30 MeV$
$\Gamma_{T_{c\bar{c}}(4100)^-}$	$178 \pm 60 MeV$	$163 \pm 67 MeV$
FF	$3.1 \pm 1.0\%$	$3.5 \pm 1.6\%$
$J^P = 1^-$ hypothesis		
$m_{T_{c\bar{c}}(4100)^-}$	$4096 \pm 21 MeV$	$4096 \pm 20 MeV$
$\Gamma_{T_{c\bar{c}}(4100)^-}$	394 ± 173	$152 \pm 58 MeV$
FF	$3.6 \pm 1.2\%$	$3.3 \pm 1.1\%$

Table 10: Mass, width and fit fraction of the $T_{c\bar{c}}(4100)^-$ candidate from the Run1 and 2016 data sample measured from the current and previous analysis. Only statistical uncertainties are taken into account.

L Trigger selection

The number of events selected by each trigger condition have been calculated and reported in tables 1112. The efficiencies of signal selection, background rejection and the ratio $\frac{S}{\sqrt{S+B}}$ have been calculated and are shown in figure L.1L.2L.3 respectively.

Trigger Condition	MC	Data sidebands	Data signal
Stripping + preselection	217723	321257	24328
L0			
B_L0MuonDecision_TIS	41094	79514	5694
B_L0ElectronDecision_TIS	22558	47041	3676
B_L0PhotonDecision_TIS	12695	18798	1491
B_L0DiMuonDecision_TIS	8811	34539	2254
B_L0HadronDecision_TIS	80242	126904	9445
L0_TIS	128034	232530	16982
B_L0HadronDecision_TOS	140378	151802	12663
HLT1			
B_Hlt1TrackAllL0Decision_TOS	128034	321257	24328
HLT2			
B_Hlt2Topo2BodyBBDTDecision_TOS	180119	224960	16447
B_Hlt2Topo3BodyBBDTDecision_TOS	198977	209626	17012
B_Hlt2Topo4BodyBBDTDecision_TOS	150207	104031	10064

Table 11: Number of events selected by the single trigger conditions for Run1. Events in data sidebands are in the range $5450 < m_{B^0} < 5550$ MeV. Events in data signal are in the range $5220 < m_{B^0} < 5340$ MeV $2908 < m_{\eta_c} < 3058$ MeV

Trigger Condition	MC	Data sidebands	Data signal
Stripping + preselection	767312	5271240	549977
L0			
B_L0MuonDecision_TIS	123895	871354	90395
B_L0ElectronDecision_TIS	141207	1095902	120366
B_L0PhotonDecision_TIS	57701	248364	26237
B_L0DiMuonDecision_TIS	30252	1017806	94647
B_L0HadronDecision_TIS	268847	1608139	172275
L0_TIS	465791	3709530	380696
B_L0HadronDecision_TOS	484673	2906596	324503
HLT1			
B_Hlt1TwoTrackMVADecision_TOS	762279	4945950	518550
B_Hlt1TrackMVADecision_TOS	745293	4768570	503602
HLT2			
B_Hlt2Topo2BodyDecision_TOS	698533	4086620	395245
B_Hlt2Topo3BodyDecision_TOS	747098	4065040	451860
B_Hlt2Topo4BodyDecision_TOS	639765	2030982	243900

Table 12: Number of events selected by the single trigger conditions for Run2. Events in data sidebands are in the range $5450 < m_{p\bar{p}K^+\pi^-} < 5550$ MeV. Events in data signal are in the range $5220 < m_{B^0} < 5340$ MeV and $2908 < m_{\eta_c} < 3058$ MeV

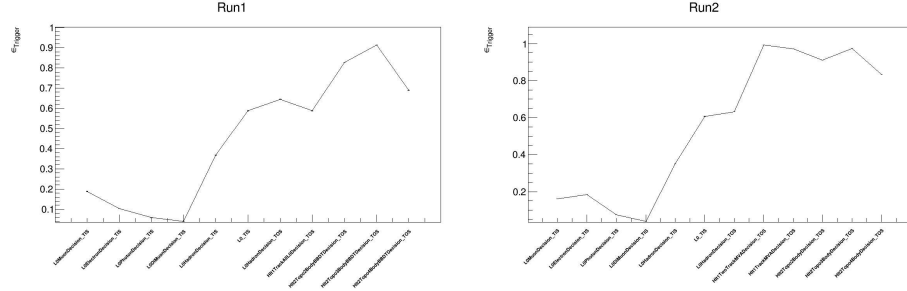


Figure L.1: Single trigger condition efficiency calculated from the $B^0 \rightarrow p\bar{p}K^+\pi^-$ MC sample. (Left) the Run 1 distribution (right) the Run 2 distribution

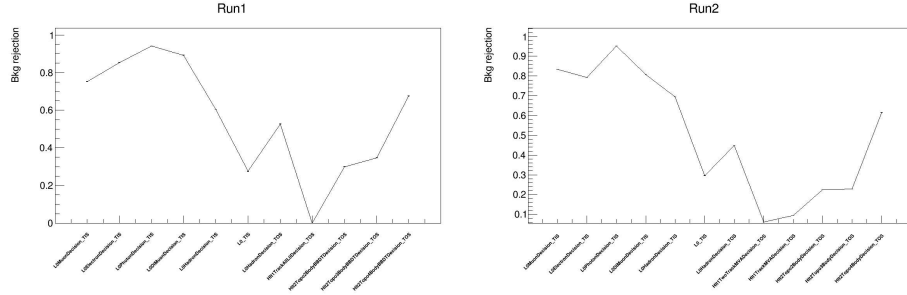


Figure L.2: Single trigger condition background rejection, calculated as $1 - \frac{N_{\text{selected}}}{N_{\text{total}}}$ calculated from the data sideband ($5450 < m_{p\bar{p}K^+\pi^-} < 5550$ MeV). (Left) the Run 1 distribution (right) the Run 2 distribution

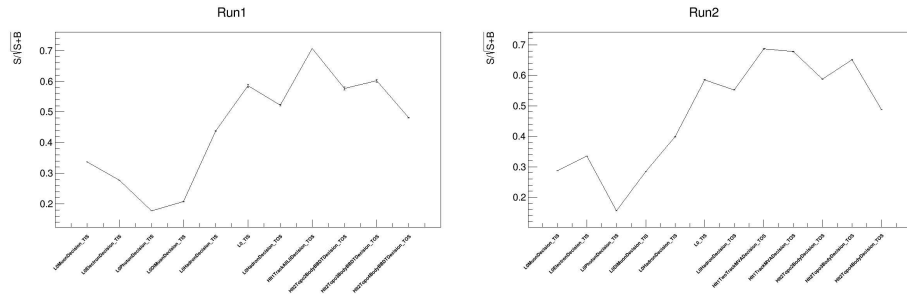


Figure L.3: Single trigger condition ratio $\frac{S}{\sqrt{S+B}}$, B is the number of event in the data sideband ($5450 < m_{p\bar{p}K^+\pi^-} < 5550$) and S is the number of event in the data signal region ($5220 < m_{p\bar{p}K^+\pi^-} < 5360$ MeV & $2908 < m_{p\bar{p}} < 3058$ MeV). (Left) the Run 1 distribution (right) the Run 2 distribution

M Systematics uncertainties studies result from the branching ratio measurement

This section presents the results of studies on the systematic uncertainties affecting the branching fraction calculation.

M.1 Shape parameters

Parameter	Value Run1	Value Run2
σ_L	1.17 ± 0.69	4.67 ± 0.58
σ_R	6.37 ± 0.61	3.39 ± 0.75
α_{left}	3.99 ± 0.41	2.82 ± 0.10
α_{right}	2.56 ± 0.09	1.95 ± 0.22
n_{left}	0.10 ± 0.73	3.45 ± 4.78
n_{right}	1.78 ± 0.16	1.87 ± 0.18

Table 13: Shape parameters mean and sigma extracted from the distribution obtained from the covariance matrix of the fit performed to the Run1 and Run2 MC samples of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay with $\eta_c \rightarrow p\bar{p}$.

Parameter	Value Run1	Value Run2
l	-3.27 ± 1.29	-3.13 ± 0.82
z	1.33 ± 1.39	0.002 ± 0.003
$beta$	0.011 ± 0.002	0.011 ± 0.002
a_{left}	2.39 ± 0.05	2.34 ± 0.02
n_{left}	2.18 ± 0.11	1.88 ± 0.04
a_{right}	2.44 ± 0.05	2.56 ± 0.03
n_{right}	2.04 ± 0.09	1.68 ± 0.03

Table 14: Shape parameters mean and sigma extracted from the distribution obtained from the covariance matrix of the fit performed to the Run1 and Run2 MC samples of the $B^0 \rightarrow J/\Psi K^+ \pi^-$ decay with $J/\Psi \rightarrow p\bar{p}$.

Parameter	Value Run1	Value Run2
l	-4.09 ± 0.31	-4.00 ± 0.25
z	0.0005 ± 0.0005	0.0001 ± 0.0001
β	0.0004 ± 0.0016	-0.005 ± 0.001
a_{left}	2.33 ± 0.07	2.42 ± 0.05
n_{left}	1.98 ± 0.22	2.03 ± 0.12
a_{right}	2.98 ± 0.09	2.45 ± 0.14
n_{right}	1.31 ± 0.14	1.72 ± 0.31

Table 15: Shape parameters mean and sigma extracted from the distribution obtained from the covariance matrix of the fit performed to the Run1 and Run2 MC samples of the $B^0 \rightarrow p\bar{p}K^+\pi^-$ decay.

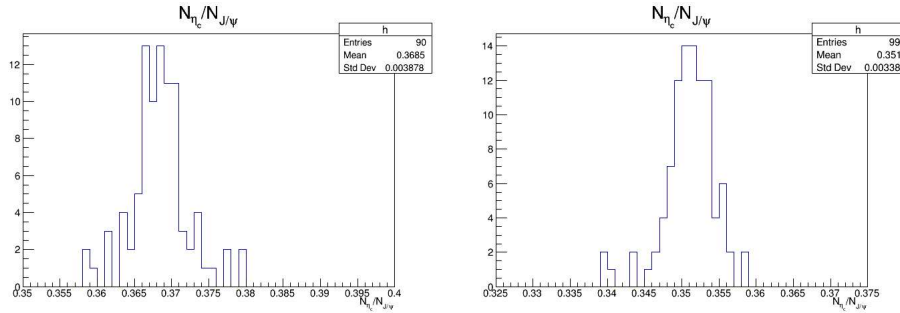


Figure M.1: $\frac{N_{\eta_c}}{N_{J/\psi}}$ ratio distributions obtained varying the shape parameters of the PDFs used to describe the B^0 , J/ψ and η_c candidates. Run1 on the left and Run2 on the right.

M.2 Signal and background parametrization

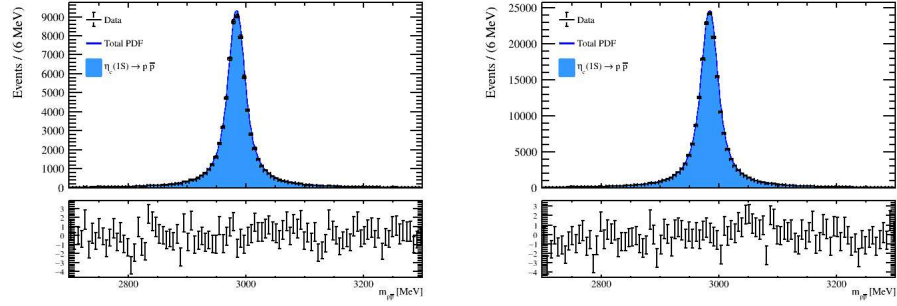


Figure M.2: Fit to the Run1 and Run2 simulated $\eta_c \rightarrow \bar{p}p$ invariant mass distribution from $B^0 \rightarrow \eta_c K^+ \pi^-$ using as pdf a RooVoigtian. The pull distribution are also shown

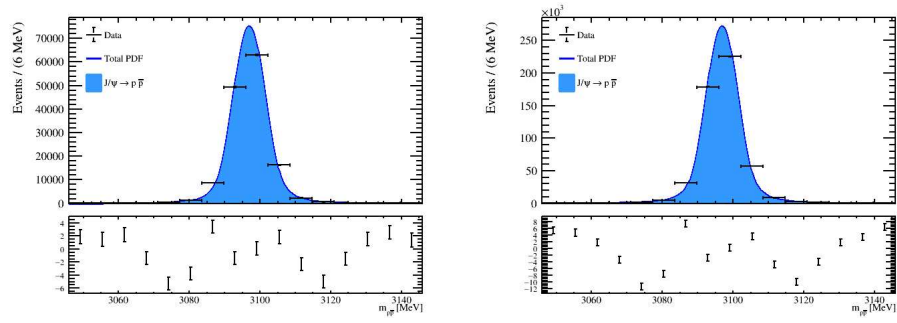


Figure M.3: Fit to the Run1 and Run2 simulated $J/\psi \rightarrow \bar{p}p$ invariant mass distribution from $B^0 \rightarrow J/\psi K^+ \pi^-$ using as pdf a CrystallBall PDF. The pull distribution are also shown

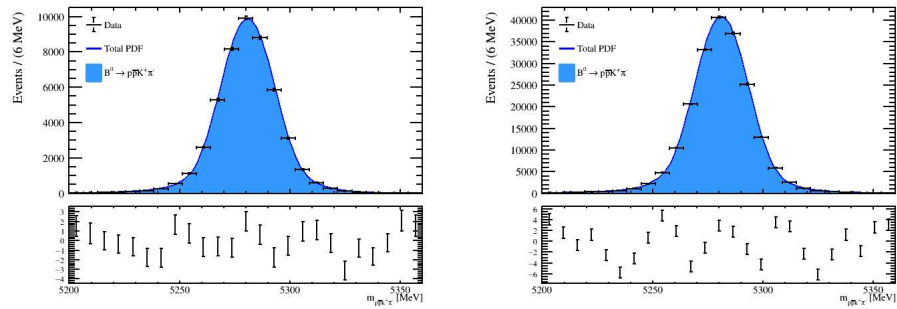


Figure M.4: Fit to the Run1 and Run2 simulated $B^0 \rightarrow \bar{p}p K^+ \pi^-$ invariant mass distribution using as pdf a CrystallBall PDF. The pull distribution are also shown

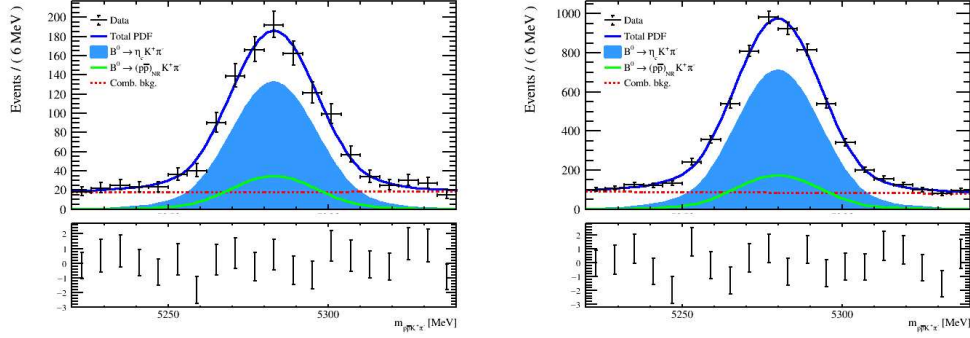


Figure M.5: Run1 (left) and Run2 (right) 2D fit to extract the $B^0 \rightarrow \eta_c K^+ \pi^-$ signal. $m_{p\bar{p}K^+\pi^-}$ invariant mass distribution fit projection. The pdf parametrising the B^0 and η_c candidate have been changed to a CrystallBall PDF and RooVoigtian, respectively. The pull plot are also shown.

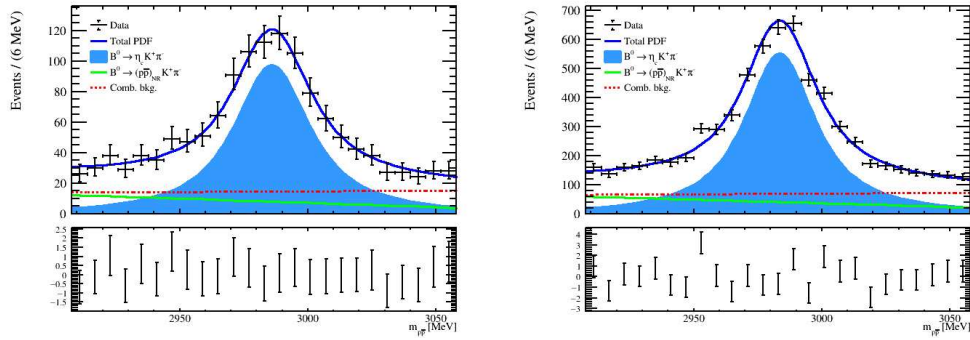


Figure M.6: Run1 (left) and Run2 (right) 2D fit to extract the $B^0 \rightarrow \eta_c K^+ \pi^-$ signal. $m_{p\bar{p}}$ invariant mass distribution fit projection. The pdf parametrising the B^0 and η_c candidate have been changed to a CrystallBall PDF and RooVoigtian, respectively. The pull plot are also shown.

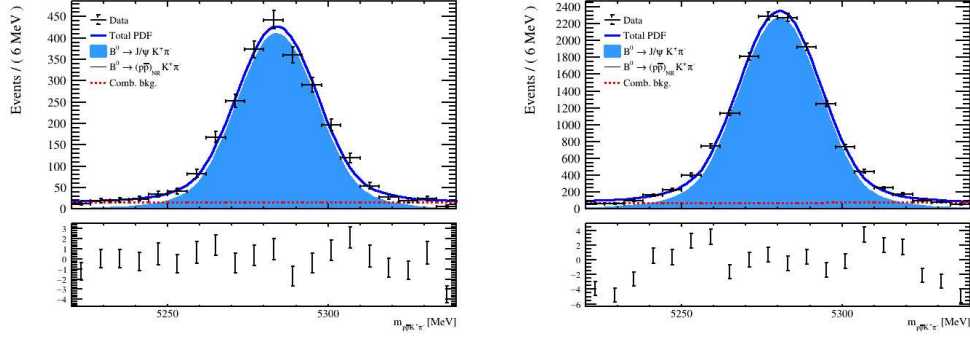


Figure M.7: Run1 (left) and Run2 (right) 2D fit to extract the $B^0 \rightarrow J/\psi K^+ \pi^-$ signal. $m_{p\bar{p}K^+\pi^-}$ invariant mass distribution fit projection. The pdf parametrising the B^0 and J/ψ candidate have been changed to a CrystallBall PDF. The pull plot are also shown.

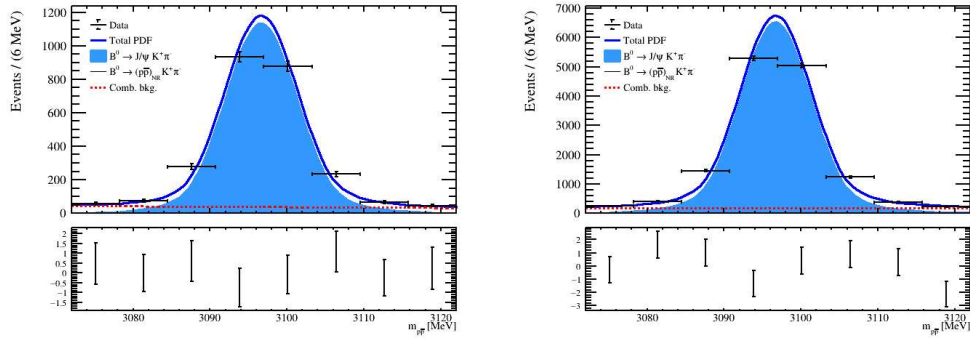


Figure M.8: Run1 (left) and Run2 (right) 2D fit to extract the $B^0 \rightarrow J/\psi K^+ \pi^-$ signal. $m_{p\bar{p}}$ invariant mass distribution fit projection. The pdf parametrising the B^0 and J/ψ candidate have been changed to a CrystallBall PDF. The pull plot are also shown.

Parameter	Value Run1	Value Run2
$N_{J/\psi}$	2256 ± 50	12790 ± 119
N_{η_c}	777 ± 43	4157 ± 97

Table 16: Yields obtained from the 2D fit taking into account the PDF choice as source of systematic uncertainty.

M. Systematics uncertainties studies result from the branching ratio measurement

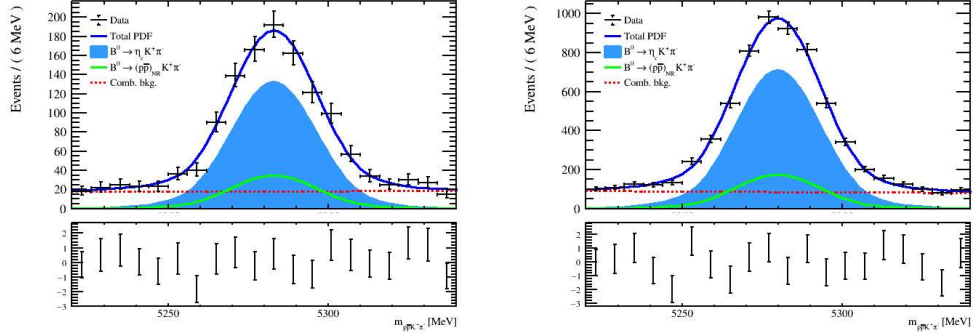


Figure M.9: Run1 (left) and Run2 (right) $m_{p\bar{p}K+\pi^-}$ invariant mass distribution fit projection. Fit applied in the mass range $2924 < m_{p\bar{p}} < 3044$ MeV. The pull plot are also shown.

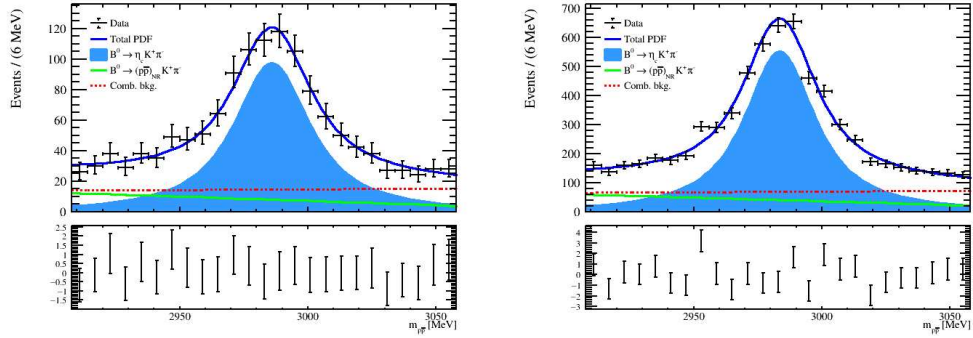


Figure M.10: Run1 (left) and Run2 (right) $m_{p\bar{p}}$ invariant mass distribution fit projection. Fit applied in the mass range $2924 < m_{p\bar{p}} < 3044$ MeV. The pull plot are also shown.

M.3 Mass windows

Parameter	Value Run1	Value Run2
$N_{J/\psi}$	2181 ± 51	12413 ± 122
N_{η_c}	808 ± 44	4111 ± 100

Table 17: Yields obtained from the 2D fit taking into account the mass windows choice as source of systematic uncertainty.

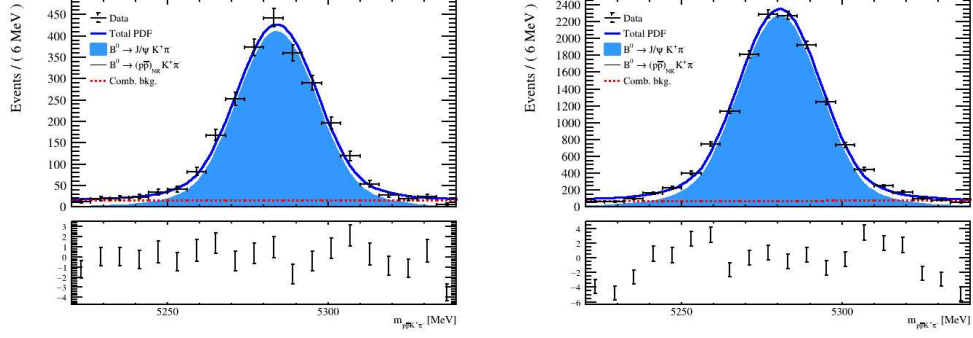


Figure M.11: Run1 (left) and Run2 (right) $m_{p\bar{p}}$ invariant mass distribution fit projection. Fit applied in the mass range $3082 < m_{p\bar{p}} < 3112$ MeV. The pull plot are also shown.

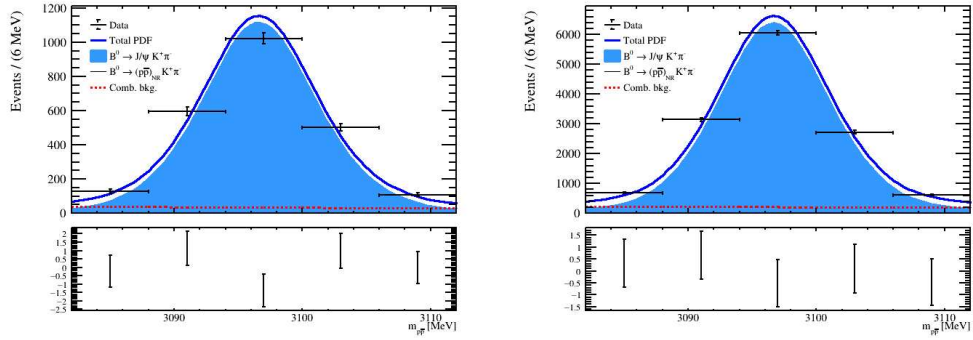


Figure M.12: Run1 (left) and Run2 (right) $m_{p\bar{p}K^+\pi^-}$ invariant mass distribution fit projection. Fit applied in the mass range $3082 < m_{p\bar{p}} < 3112$ MeV. The pull plot are also shown.

N. Systematic uncertainties of the fit fractions from the $K^* + T_{c\bar{c}}(4100)^-$ model
with $J^P = 1^-$

N Systematic uncertainties of the fit fractions from the $K^* + T_{c\bar{c}}(4100)^-$ model with $J^P = 1^-$

Component	Nominal value	Fixed yields	Fit bias	Efficiency	η_c
$K^*(892)$	0.491 ± 0.013	+0.005	± 0.011	± 0.004	-0.011
$K^*(1410)$	0.039 ± 0.009	-0.004	± 0.001	± 0.002	-0.016
LASS	0.316 ± 0.050	+0.018	± 0.011	± 0.005	-0.041
$K_2^*(1430)$	0.040 ± 0.009	-0.003	± 0.004	± 0.002	+0.001
$K^*(1680)$	0.015 ± 0.009	+0.005	± 0.009	± 0.002	+0.001
$K_0^*(1950)$	0.022 ± 0.005	+0.006	± 0.007	± 0.002	+0.001
$T_{c\bar{c}}(4100)^-$	0.011 ± 0.005	-0.001	± 0.001	± 0.002	-0.001
$K^*(892) - K^*(1410)$	-0.014 ± 0.011	+0.006	± 0.003	± 0.002	-0.002
$K^*(892) - K^*(1680)$	0.003 ± 0.009	+0.005	± 0.002	± 0.002	+0.002
$K^*(1410) - K^*(1680)$	-0.026 ± 0.012	+0.003	± 0.007	± 0.002	+0.025
LASS- $K_0^*(1950)$	0.051 ± 0.057	-0.032	± 0.019	± 0.004	$^{+0.015}_{-0.009}$
$K^*(892) - T_{c\bar{c}}(4100)^-$	0.011 ± 0.007	-0.003	± 0.004	± 0.001	-0.001
$K^*(1410) - T_{c\bar{c}}(4100)^-$	-0.007 ± 0.003	+0.002	± 0.004	± 0.001	+0.001
LASS- $T_{c\bar{c}}(4100)^-$	0.034 ± 0.016	-0.005	± 0.001	± 0.003	-0.002
$K_2^*(1430) - T_{c\bar{c}}(4100)^-$	0.001 ± 0.002	+0.001	± 0.002	± 0.001	+0.001
$K^*(1680) - T_{c\bar{c}}(4100)^-$	0.005 ± 0.003	0	± 0.001	± 0.001	$^{+0.001}_{-0.002}$
$K_0^*(1950) - T_{c\bar{c}}(4100)^-$	0.007 ± 0.003	-0.002	± 0.003	± 0.001	0

Table 18: Systematic uncertainties associated to the fit fractions calculated using the $K^* + T_{c\bar{c}}(4100)^-$ model. The nominal value from the amplitude fit is reported in the second column with its statistical uncertainty. Components labelled like $K^*(892) - K^*(1410)$ stand for the interference fit fraction between the two resonances.

N. Systematic uncertainties of the fit fractions from the $K^* + T_{c\bar{c}}(4100)^-$ model
with $J^P = 1^-$

Component	Fixed parameters
$K^*(892)$	+0.001
$K^*(1410)$	+0.017
LASS	+0.034
$K_2^*(1430)$	+0.001
$K^*(1680)$	+0.002
$K_0^*(1950)$	+0.010
$T_{c\bar{c}}(4100)^-$	+0.003
$K^*(892) - K^*(1410)$	+0.006
$K^*(892) - K^*(1680)$	+0.027
$K^*(1410) - K^*(1680)$	+0.021
LASS- $K_0^*(1950)$	+0.013
$K^*(892) - T_{c\bar{c}}(4100)^-$	+0.003
$K^*(1410) - T_{c\bar{c}}(4100)^-$	+0.002
LASS- $T_{c\bar{c}}(4100)^-$	+0.011
$K_2^*(1430) - T_{c\bar{c}}(4100)^-$	+0.001
$K^*(1680) - T_{c\bar{c}}(4100)^-$	+0.001
$K_0^*(1950) - T_{c\bar{c}}(4100)^-$	+0.001

Table 19: Systematic uncertainties associated to the fit fractions calculated using the $K^* + T_{c\bar{c}}(4100)^-$ model. The nominal value from the amplitude fit is reported in the second column with its statistical uncertainty. Components labelled like $K^*(892) - K^*(1410)$ stand for the interference fit fraction between the two resonances.

N. Systematic uncertainties of the fit fractions from the $K^* + T_{c\bar{c}}(4100)^-$ model
with $J^P = 1^-$

Component	Background parametrization	$K_0^*(1430)$	Efficiency veto
$K^*(892)$	± 0.013	-0.027	+0.017
$K^*(1410)$	± 0.018	+0.040	+0.022
LASS	± 0.038	+0.037	-0.001
$K_2^*(1430)$	± 0.015	-0.008	-0.017
$K^*(1680)$	± 0.018	-0.005	-0.003
$K_0^*(1950)$	± 0.083	-0.015	+0.004
$T_{c\bar{c}}(4100)^-$	± 0.009	-0.005	-0.001
$K^*(892) - K^*(1410)$	± 0.028	+0.096	-0.042
$K^*(892) - K^*(1680)$	± 0.013	-0.026	-0.022
$K^*(1410) - K^*(1680)$	± 0.022	-0.006	+0.053
LASS- $K_0^*(1950)$	± 0.290	-0.049	-0.010
$K^*(892) - T_{c\bar{c}}(4100)^-$	± 0.009	-0.011	+0.006
$K^*(1410) - T_{c\bar{c}}(4100)^-$	± 0.004	+0.007	-0.002
LASS- $T_{c\bar{c}}(4100)^-$	± 0.015	-0.013	+0.006
$K_2^*(1430) - T_{c\bar{c}}(4100)^-$	± 0.002	+0.001	+0.002
$K^*(1680) - T_{c\bar{c}}(4100)^-$	± 0.003	-0.005	-0.010
$K_0^*(1950) - T_{c\bar{c}}(4100)^-$	± 0.005	-0.008	-0.002

Table 20: Systematic uncertainties associated to the fit fractions calculated using the $K^* + T_{c\bar{c}}(4100)^-$ model. The nominal value from the amplitude fit is reported in the second column with its statistical uncertainty. Components labelled like $K^*(892) - K^*(1410)$ stand for the interference fit fraction between the two resonances.

N. Systematic uncertainties of the fit fractions from the $K^* + T_{c\bar{c}}(4100)^-$ model
with $J^P = 1^-$

Component	Nominal value	Total
$K^*(892)$	0.491 ± 0.013	$+0.025$ -0.034
$K^*(1410)$	0.039 ± 0.009	$+0.051$ -0.026
LASS	0.316 ± 0.050	$+0.72$ -0.057
$K_2^*(1430)$	0.040 ± 0.009	$+0.015$ -0.026
$K^*(1680)$	0.015 ± 0.009	$+0.021$ -0.022
$K_0^*(1950)$	0.022 ± 0.005	$+0.084$ -0.085
$T_{c\bar{c}}(4100)^-$	0.011 ± 0.005	$+0.010$ -0.011
$K^*(892) - K^*(1410)$	-0.014 ± 0.011	$+0.100$ -0.051
$K^*(892) - K^*(1680)$	0.003 ± 0.009	$+0.030$ -0.038
$K^*(1410) - K^*(1680)$	-0.026 ± 0.012	$+0.077$ -0.023
LASS- $K_0^*(1950)$	0.051 ± 0.057	$+0.291$ -0.296
$K^*(892) - T_{c\bar{c}}(4100)^-$	0.011 ± 0.007	$+0.012$ -0.015
$K^*(1410) - T_{c\bar{c}}(4100)^-$	-0.007 ± 0.003	$+0.009$ -0.006
LASS- $T_{c\bar{c}}(4100)^-$	0.034 ± 0.016	$+0.019$ -0.021
$K_2^*(1430) - T_{c\bar{c}}(4100)^-$	0.001 ± 0.002	$+0.004$ -0.003
$K^*(1680) - T_{c\bar{c}}(4100)^-$	0.005 ± 0.003	$+0.003$ -0.014
$K_0^*(1950) - T_{c\bar{c}}(4100)^-$	0.007 ± 0.003	$+0.006$ -0.010

Table 21: Summary of systematic uncertainties associated to the fit fractions calculated using the $K^* + T_{c\bar{c}}(4100)^-$ model. The nominal value from the amplitude fit is reported in the second column with its statistical uncertainty. Components labelled like $K^*(892) - K^*(1410)$ stand for the interference fit fraction between the two resonances.

O Effect of reweighting on the MC distributions

In this sections the comparison between data, MC and reweighted MC using the GBreweighter tool are shown.

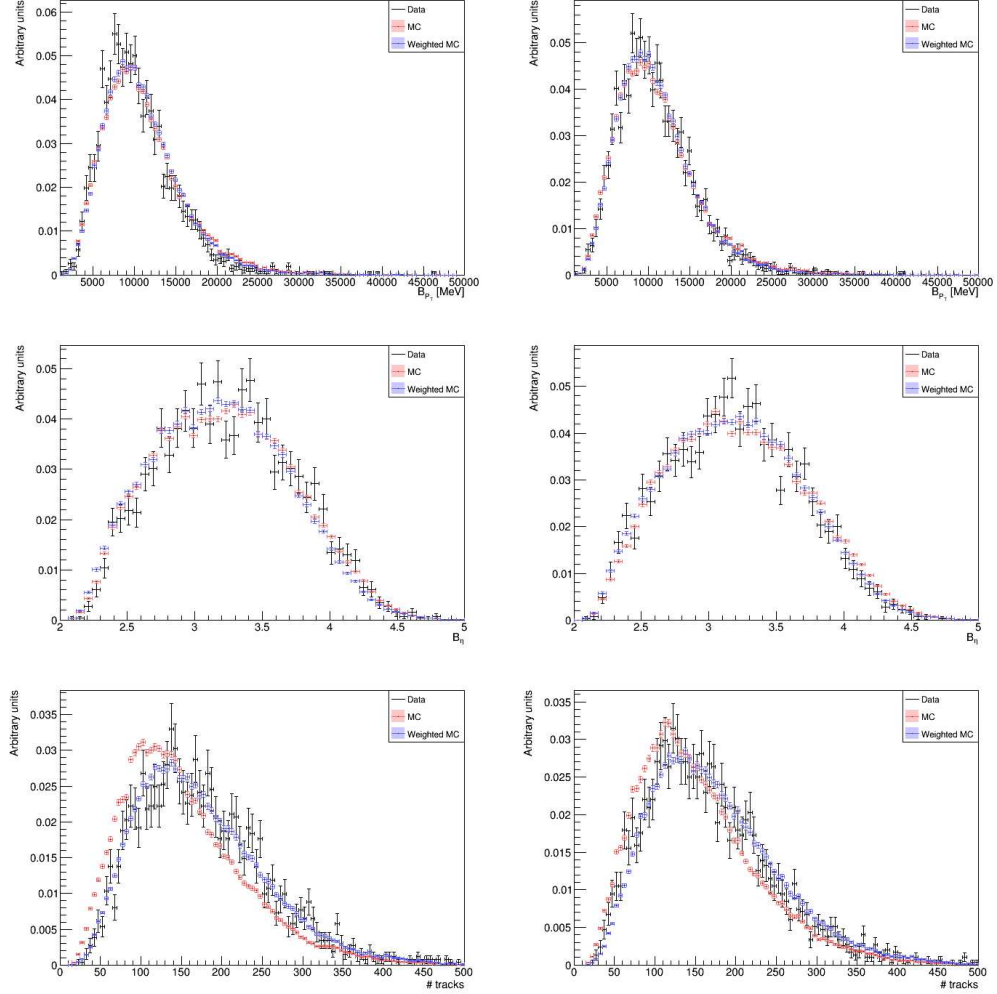


Figure O.1: Effect of the B kinematic reweight on the training variables for Run 1, left for the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay and right for the $B^0 \rightarrow J/\psi K^+ \pi^-$. The original MC distributions are shown in red, the data sample is shown in black, and the weighted MC distributions are shown in red.

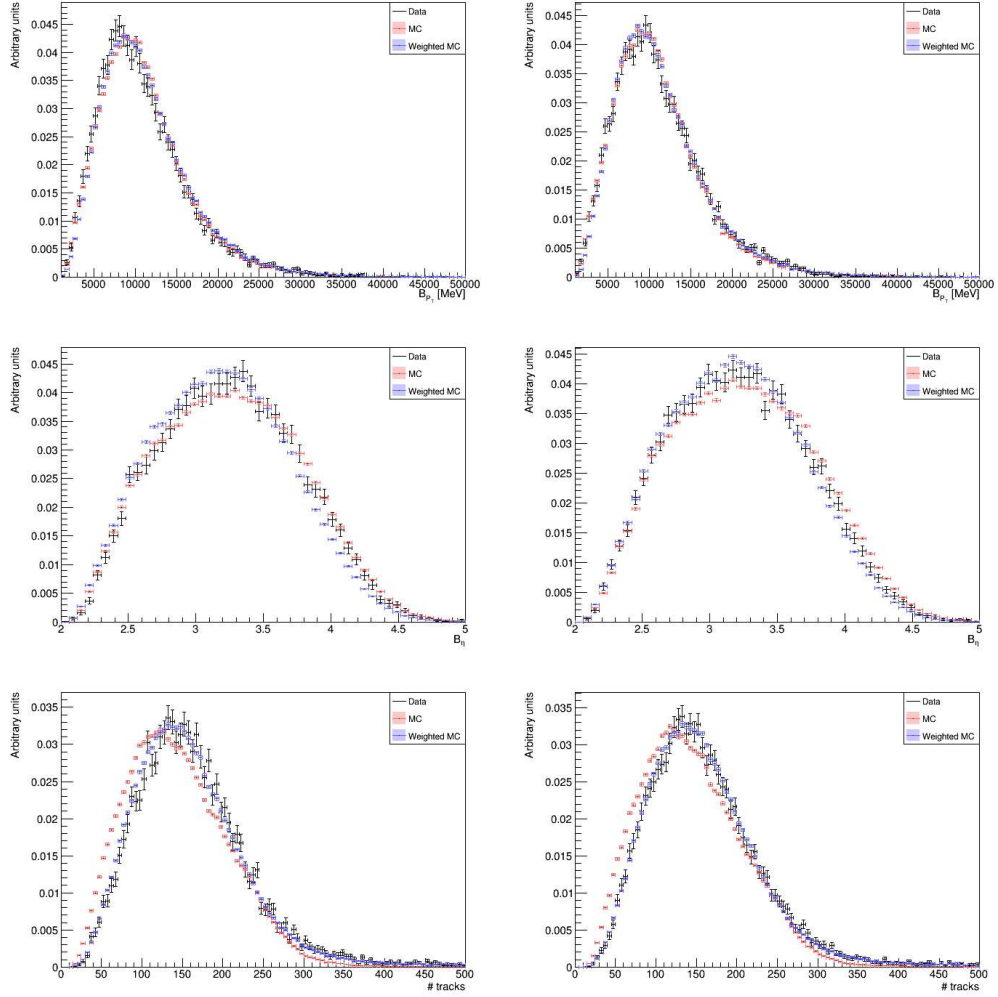


Figure O.2: Effect of the B kinematic reweight on the training variables for Run 2, left for the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay and right for the $B^0 \rightarrow J/\psi K^+ \pi^-$. The original MC distributions are shown in red, the data sample is shown in black, and the weighted MC distributions are shown in red.

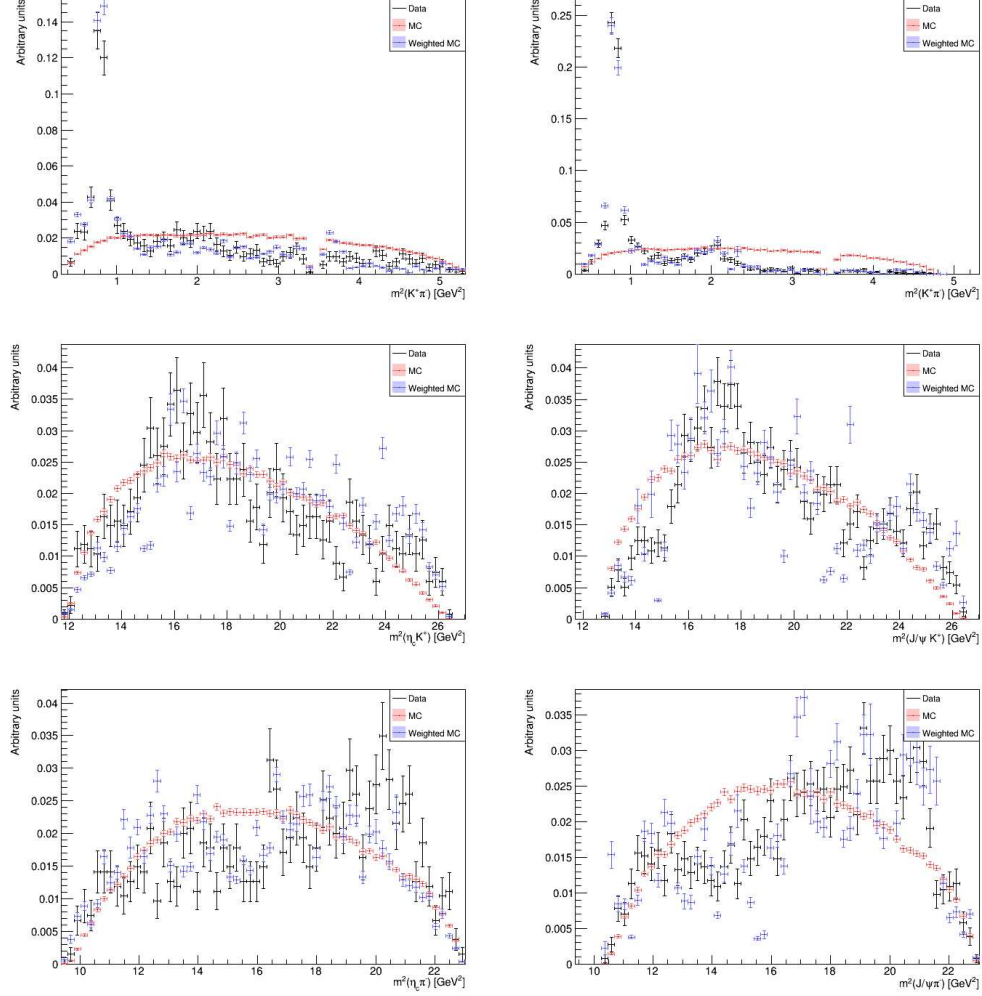


Figure O.3: Effect of the Dalitz plot reweight on the training variables for Run 1, left for the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay and right for the $B^0 \rightarrow J/\psi K^+ \pi^-$. The original MC distributions are shown in red, the data sample is shown in black, and the weighted MC distributions are shown in red.

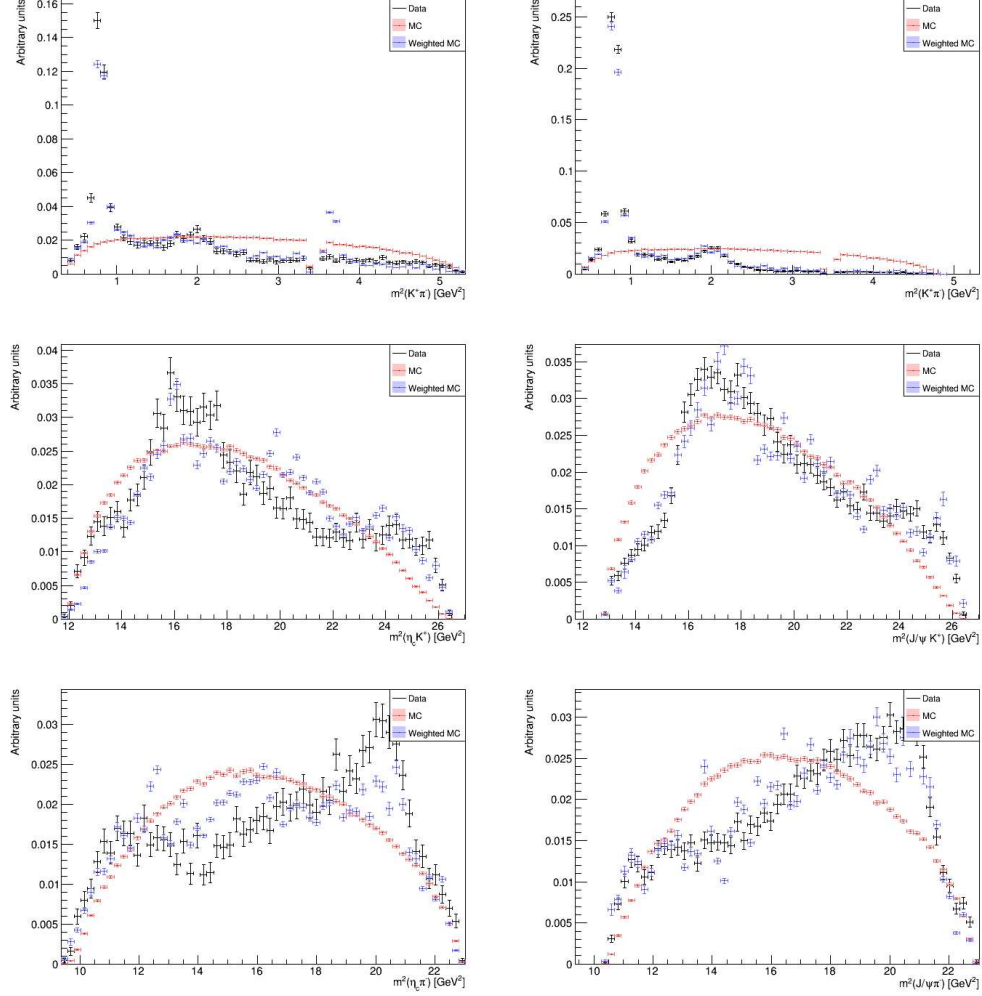


Figure O.4: Effect of the Dalitz plot reweight on the training variables for Run 2, left for the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay and right for the $B^0 \rightarrow J/\psi K^+ \pi^-$. The original MC distributions are shown in red, the data sample is shown in black, and the weighted MC distributions are shown in red.

Addition related to the 4DPHOTON tube characterisation

P Impact of the trackers on the beam properties

The impact of the trackers on the beam properties studied using the Geant4 simulation are reported. As described in Sec. 8.2.3, the beam is composed of protons with a momentum of $(0, 0, 180)$ GeV. As can be seen from Figure P.1, the momentum along the $\hat{x}$ or $\hat{y}$ axis of the protons when entering the radiator lens follow Gaussians distribution with $\sigma < 5$ MeV. The effect on the beam position is studied on a point-like beam, where the particle of each event is generated with coordinates $(0,0)$, Figure P.1 shows the distribution of the protons entering the radiator lens. As expected, the distribution are Gaussian with $\sigma \sim 17 \mu\text{m}$, comparable to the pixel size of the Timepix4. Considering these results, the impact of the tracker system is negligible on the beam properties.

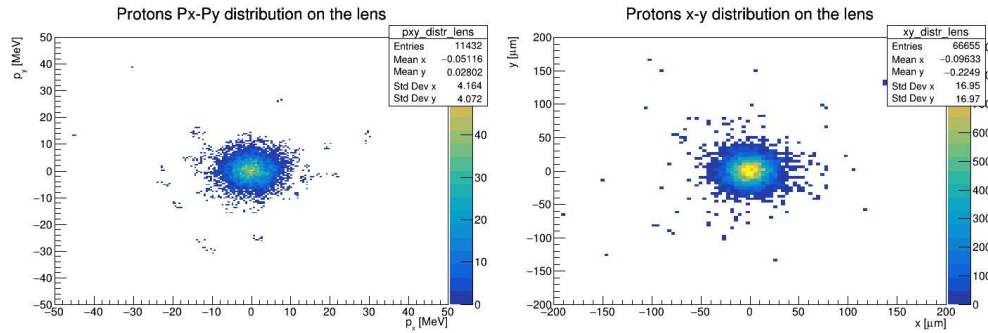


Figure P.1: (Left) x and y momentum distribution of the protons of the beam entering the radiator lens. (Right) x and y distribution of the protons of the beam entering the radiator lens.

Q Time resolution studies on the timing system

The studies to optimise the position of the timing system are reported. Two beam configuration have been tested. A point-like beam, with no divergence and same emission point for the protons and one gaussian smeared, in position with a $\sigma = 5$ mm and a divergence of $\sigma = 1$ mrad. Figure Q.1 shows the distribution of the resolution of the system as function of the distance from the beam and the angle with respect to the $\hat{z}$ axis. The performance with a gaussian beam are consistent along the tested range, while a minimum around 1.5 cm and an angle of 60° is found, this configuration is then used in the final testbeam setup. In this studies, the resolution is extracted as the σ from a gaussian fit applied to the peak of the distribution of a single radiator.

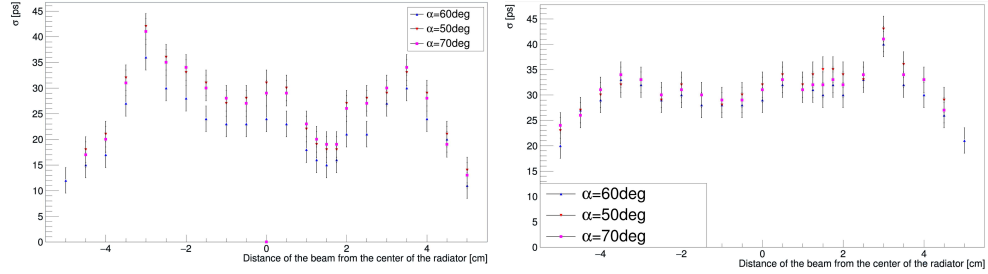


Figure Q.1: Time resolution of the timing system as a function of the distance and angle from the beam axis. (Left) Calculated from a point-like beam and (right) a gaussian smeared beam.

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