Annals of Operations Research
https://doi.org/10.1007/s10479-025-06557-0

ORIGINAL RESEARCH

Game-based modeling of delayed risk contagion in cryptocurrency exchanges

Mauro Aliano[1] · Stefania Ragni[1]

Received: 20 June 2024 / Accepted: 26 February 2025

Abstract

During the last years, financial market contagion has become a critical concern for policy-makers and investors, particularly with respect to the financial stability of cryptocurrency platforms. This paper explores the contagion effect among crypto exchanges employing the Susceptible–Infected–Recovered (SIR) model with time delay and investigates possible cooperative strategies. The SIR dynamical system is integrated with the replicator equation of evolutionary game theory to study the interplay between the spread of risk and the propensity of cryptocurrency platforms to become cooperative under the pressure of financial contagion. Different equilibrium points which correspond to both pure and mixed cooperative strategies characterize the resulting model. We carry out a theoretical analysis of the problem by studying the asymptotic behavior in the steady state. In addition, using extensive cryptocurrency market data from 2017 to 2023, we identify the key factors driving contagion and assess the dynamics of cooperative versus non-cooperative behavior. Our findings point out that cooperative strategies are essential to ensure financial stability, particularly in the long term, as they mitigate systemic risks and foster resilience. These results provide critical insights for policy makers and investors, offering actionable strategies to enhance the robustness of crypto markets and address the growing challenges of financial contagion in the digital asset ecosystem.

Keywords Evolutionary game · Cooperative actions · SIR model · Risk contagion · Spillover effect · Cryptocurrency

1 Introduction

Crypto platforms have experienced unprecedented growth in recent years, largely fueled by their expanding market capitalization (see Samueal, 2023). However, concerns about exploitative practices have also grown in conjunction with this expansion, especially in regard to those that impact the Global South (see Howson, 2020). In response to these negative

✉ Stefania Ragni
stefania.ragni@unife.it

Mauro Aliano
mauro.aliano@unife.it

[1] Department of Economics and Management, University of Ferrara, Via Voltapaletto 11, 44121 Ferrara, Italy

Published online: 19 March 2025

impact, scholars and practitioners alike have proposed alternative governance structures, such as platform cooperatives, as viable solutions to counterbalance such inequalities. At the heart of these platforms, blockchain technology underpins the financial sector by fostering trust and reducing the reliance on traditional intermediaries through consensus-based verification mechanisms (see Renduchintala et al., 2022). Although cryptocurrencies such as Bitcoin and Ethereum have been heralded as decentralized systems, developers and academics have shown concerns about the real level of decentralization of these systems (see Barbereau et al., 2022).
The COVID-19 pandemic's outbreak has further transformed both the cryptocurrency and the broader financial markets. During that volatile period, sentiment analysis has pointed out the effectiveness of advanced predictive models like CNN-LSTM (see Washington, 2023). At the same time, the pandemic brought attention to the short-term benefits of investing in cryptocurrencies, as they often outperform conventional financial instruments for risk-seeking investors (see Waspada et al., 2022). However, regulators noticed this rapid growth and, as a result, more stringent regulatory frameworks for crypto-assets have been developed. These frameworks require service providers to obtain formal authorization before commencing operations (see Wronka, 2024), with the objective of improving transparency and accountability within crypto exchanges. These evolving regulations have, in turn, directly influenced investors' decision-making processes and their willingness to engage in cryptocurrency markets (see Prasetyo & Kurniasari, 2023).

Even though cryptocurrency platforms are still showing a considerable growth and potential, there are still a number of important issues that need to be fixed. Key issues include exploitative practices, questions surrounding true decentralization, the evolving regulatory landscape, and external disruptions such as the COVID-19 pandemic. Building a more trustworthy and sustainable cryptocurrency system requires addressing these challenges with a combination of more regulation, alternative governance strategies, and improved transparency.

1.1 Financial risk contagion in cryptocurrency exchanges

Contagion is crucial for the so-called "systemic risk" in the financial literature, where a big cascade of crises could be determined by both endogenous and exogenous factors. In particular, if a financial intermediary approaches a crisis or precrisis state, it may cause a crisis or precrisis situation for other intermediaries, beginning with an outbreak that has a cascading impact (see Haldane & May, 2011). Interdependencies within financial networks can be considered as a mechanism of contagion transmission and are one of the most significant factors influencing the spread of defaults (see Roukny et al., 2013). In this framework, due to the worries that have arisen recently regarding financial service providers which operate outside of standard schemes in decentralized finance, we focus on the spreading of risk in relation to cryptocurrency exchanges. The issue of financial risk inside cryptocurrency markets is extensively discussed in the literature. For instance, Aramonte et al. (2021) points out that decentralized finance is a new type of intermediation in cryptocurrency markets that has certain weaknesses which could threaten financial stability; actually, the need of governance lets a certain degree of centralization be inevitable. High leverage, liquidity mismatches, and the absence of shock absorbers like banks can make these vulnerabilities extremely serious. Concerns regarding customer safety also exist, and they include issues such as operational platform failures and cyberattacks, volatility and usage of leverage (see Bains et al., 2022).

In this framework, the issue of managing a potential evolution of risk contagion among cryptocurrency exchanges can be addressed by applying a dynamic compartmental strategy based on the Susceptible–Infected–Recovered (SIR) classification. This mathematical approach was introduces in the study of epidemiological models as helpful tool for assessing how an infection develops from its first outbreak onward (see Kendall 1956 and Kermack and McKendrick 1927). The modeling approach based on SIR dynamics has the great potential to also be applicable to other contexts other than epidemiology. Actually, based on the analogy existing between ecosystems and financial systems, this approach has also proved useful for describing risk contagion in economics and finance: Cao and Zhu (2012) and Fanelli and Maddalena (2020) apply SIR methodology to crisis contagion thorough banking networks; Garas et al. (2010) exploits SIR perspective to analyze financial crises among different nations; Zhao et al. (2021) describes credit risk contagion of Internet peer-to-peer lending platforms by the epidemiological approach.
We are inspired by the literature and describe risk contagion phenomenon by applying a compartmental model based on a specific time delay differential system, which has been proposed by Kyrychko and Blyuss (2005) in a biological context to simulate the dynamics of disease spread in the SIR perspective. The inclusion of temporal delay, which accounts for latency or immunity and enhances the realism of infection transmission dynamics, is one of its key features. In this regard, the model is revisited within the framework of finance in order to explain the evolution of contagion with respect to crash risk at low and high levels inside cryptocurrency markets. Although this SIR model is not a novelty, the application to cryptocurrency markets may be considered as worthy of attention due to the growing interest in financial services out of classical schemes. The key idea is that low risk cryptocurrency platforms are susceptible to contagion upon interaction with high risk platforms within the infected compartment. After contagion, some of the infected platforms recover and obtain financial immunity for a period of length $\tau > 0$. From a mathematical point of view, the whole dynamics is temporally delayed by this parameter τ. On the other hand, from a financial point of view, several stakeholders-stockholders, financial intermediaries, customers, and regulatory bodies-take action to temporarily prevent future financial troubles following an initial or recurring financial distress event. For example, in the wake of a financial crisis, consumers are probably going to keep a closer eye on things, and regulators would tighten up their supervision, so they are kind of like financial watchdogs. Furthermore, following financial difficulties, reputation-building is essential as it reinforces market resilience. This is similar to developing immunity, as it acts as a safeguard against future financial hardships. The time delay parameter τ models the duration of the steps performed to stop financial instability from happening again. In this respect, immunity is not lifelong in the SIR dynamics but it ends after period τ: while some recovered cryptocurrency platforms might exit the market, others might revert to be susceptible of contagion again.

1.2 Cooperation and evolutionary games

In the framework of risk spreading by contagion, we investigate the collaborative initiatives among cryptocurrency platforms; in particular, we focus on the cooperative efforts endorsed by international authorities. This cooperation is centered around implementing policies designed to prevent abuse and protect investors, with the aim of mitigating risks associated with financial instability and crises. The Financial Stability Board emphasized the need for cross-border collaboration in a 2022 report due to the inherently global nature of cryptocurrencies, presenting substantial regulatory, supervisory and enforcement challenges. Divergent regulatory classifications across different jurisdictions and the inadequacy of current cross-border frameworks to handle the complexities inherent in crypto-assets are the causes of these challenges. The same cryptocurrency may be classified differently in different jurisdictions due to inconsistent regulatory classifications, which may result in regulatory arbitrage or evasion. This disparity leads certain entities to structure their operations in ways that exploit the variances in regulatory requirements. Moreover, current cross-border regulatory cooperation frameworks have been primarily tailored for traditional financial institutions and activities, and they frequently lack the adaptability required to accommodate the unique characteristics of crypto-assets. A reevaluation of these frameworks is essential to determine their effectiveness in facilitating information sharing and coordination in the regulation, supervision, and enforcement of crypto-assets, especially when activities span various sectors and jurisdictions. International authorities have started working together on stablecoins, especially because of their importance to small economies in the Global South. The applications of specific stablecoin arrangements are diverse and continually evolving. The Financial Stability Board's February 2022 report points out several common use cases for stablecoins, including: i) serving as a bridge between traditional fiat currencies and volatile digital assets; ii) acting as collateral in derivative transactions involving crypto-assets; and iii) facilitating various activities such as trading, lending, and borrowing, particularly within decentralized finance (DeFi) ecosystems. In addition, in 2022, the Bank for International Settlements' Committee on Payments and Market Infrastructures (CPMI) and the International Organization of Securities Commissions (IOSCO) has issued guidance indicating that stablecoin arrangements should adhere to international standards for payment, clearing, and settlement systems. This guidance applies the principle of "same risk, same regulation" to stablecoins, by extending international standards to cover systemically important stablecoin arrangements. Platform collaboration would serve two purposes: it would mitigate spillover effects and help safeguard interconnected entities.

As a novel issue in this framework, we provide a model that integrates the standard SIR system with the replicator equation to describe how cooperation changes over time in the field of Evolutionary Game Theory (see Weibull, 1995). The two models are joined by assuming that the parameters of the game payoff matrix depend on the strength of the risk spread at any time. It means that the higher is the gravity of crisis, the higher is the propensity to cooperate. The primary conclusions of the study center the evidence that long-term cooperation lowers systemic risks and promotes trust, which are two factors essential for a stable financial environment. The degree of cooperative behavior tends to rise in response to the intensity of contagion diffusion in the short-term.

1.3 Outline of the analysis

The remaining of this paper is organized as follows. The mathematical framework of the SIR model for risk contagion is described in Sect. 2. Particular focus is placed on how risk contagion may influence the behavior of cryptocurrency platforms in Sect. 2.1, where the replicator dynamics is described. As the next step, Sect. 2.2 introduces the risk infection game model obtained by coupling the SIR dynamics and the replicator equation. In Sect. 3 we focus on the analysis of the game based model in terms of steady state behavior. In particular, we focus on dynamics that are affected by the existence of a mixed-strategy steady state which is feasible. Actually, we are interested in scenarios where the mixed-strategy equilibrium is viable and stable and hence observable in the practice. In this framework, we provide the conditions for mixed-strategy equilibrium viability together with its stability analysis in Sects. 3.1 and 3.2, respectively. Furthermore, in Sect. 4 we simulate the dynamics of risk contagion under a cooperation action among the governance tokens in a specific data sample by employing a classical numerical procedure for approximating the solution of a differential system with constant delay. Finally, we draw our concluding remarks in Sect. 5.

2 The contagion dynamics

Three distinct compartments are considered inside a given market: the class of susceptible platforms with density $S(t)$, the class of infected platforms with density $I(t)$, and the class of recovered with density $R(t)$, at each time $t \geq 0$. Then, total density is equal to $N(t) = S(t) + I(t) + R(t)$ at any time t. Since some cryptocurrency platforms may exit the market, $N(t)$ decreases with the elimination rate from the market, denoted by γ with $0 < \gamma < 1$, while it increases over time under the assumption that more susceptible platforms will enter the market at a specific growth rate $b > 0$.
On one hand, the risk spread is modelled by a bilinear incidence term $a\, S(t)I(t)$, where $a > 0$ represents the removal rate as a result of contagion and measures the interconnections among the exchanges. In this respect, the contagion grows linearly according to both susceptible and infected densities. On the other hand, a portion of infected platforms may be recovered from high risk: they become able to keep their risk at a low level, so that they are no longer infectious for a time period of length $\tau > 0$. In this regard, τ is a form of financial immunity, whereas parameter δ, with $0 < \delta < 1$, denotes the recovery rate from the high risk to the low one. When τ expires, some cryptocurrency platforms that have recovered may revert susceptible to infection once more: they correspond to the portion $\delta e^{-\gamma\tau} I(t - \tau)$.

Under the previous statements, the following SIR dynamics models risk transmission through the market:

$$\dot{S}(t) = b - \gamma S(t) - a\, S(t)\, I(t) + \delta e^{-\gamma\tau} I(t - \tau), \tag{1}$$
$$\dot{I}(t) = a\, S(t)\, I(t) - (\gamma + \delta) I(t), \tag{2}$$
$$\dot{R}(t) = \delta I(t) - \gamma R(t) - \delta e^{-\gamma\tau} I(t - \tau), \tag{3}$$

completed by the initial value for the recovered compartment corresponding to

$$R(0) = \delta \int_{-\tau}^{0} e^{\gamma s} I(s)\, ds, \tag{4}$$

and the following initial conditions prescribed for susceptible and recovered classes:

$$S(0) = S_0 > 0, \tag{5}$$
$$I(s) = I_0(s) \geq 0, \quad \text{for all } s \in [-\tau, 0], \quad \text{with } I_0(0) > 0, \tag{6}$$

where the history function $I_0(\cdot)$ is continuous in $[-\tau, 0]$. Under these assumptions, there exists a unique solution of the previous differential system (see Hale, 1977). It can be proved that, as the dynamics starts from positive initial values, then $S(t)$, $I(t)$ and $R(t)$ remain positive at any time $t > 0$ (see Kyrychko & Blyuss, 2005). In addition, the total density $N(t)$ can be integrated according to the following relationship:

$$\dot{N}(t) \leq b - \gamma N(t), \qquad t \geq 0.$$

It follows that there exists a constant value

$$M = \max\{N_0 := N(0); b/\gamma\},$$

such that $N(t) \leq M$ at each $t \geq 0$. This bound on $N(t)$ is inherited by the different compartments so that

$$0 \leq S(t) \leq M, \qquad 0 \leq I(t) \leq M, \qquad 0 \leq R(t) \leq M, \tag{7}$$

for any time $t \geq 0$. This finding implies that contagion does not diverge but instead it remains bounded from above; indeed, the density related to each compartment does not exceed the upper bound M.

As a further remark, once the infected dynamics $I(t)$ is known, then the recovered one can be integrated to get

$$R(t) = \delta \int_{t-\tau}^{t} e^{\gamma(s-t)} I(s)\, ds; \tag{8}$$

therefore, equation (3) can be omitted without loss of generality and the model reduces to the first two equations. As a result, it is possible to only focus on system (1)-(2), which is completed by initial conditions (5)-(6).

2.1 How risk contagion influences cryptocurrency platforms' behavior

The adoption of restrictions and control mechanisms set forth by international laws may encourage cryptocurrency platforms to embrace more ethical conduct, especially with regard to customers. Unfortunately, when making decisions under pressure, such as in the early phases of a pandemic or when there is a risk of infection, the necessary actions may arouse selfish inclinations in both individuals and economic operators, such as the impulse to take advantage of others and the aversion to being duped. This framework inspires the investigation of assessing cooperative and non-cooperative actions in light of a new legal context. As an analogy with the study developed in Madeo and Mocenni (2021) where the SIR epidemic model is involved in the framework of an evolutionary game to study the interplay between the COVID-19 spreading and the propensity of people to become cooperative, we model a possible cooperation action for a more ethical behavior inside the cryptocurrency platforms' market.

In this respect, we assume that every cryptocurrency platform has the option to choose to cooperate or not by implementing only one of the two strategies. Then, we denote the share of cooperators by $x \in [0, 1]$, whereas the share of non-cooperators is defined by $z = 1 - x$. In the field of Game Theory (see Weibull, 1995), the decision-making process is determined by a payoff matrix that establishes the outcome of the interactions between pairs of players which are randomly selected. We define this payoff matrix as:

$$B = \begin{pmatrix} 1 & \bar{\mathcal{S}} \\ \mathcal{T} & 0 \end{pmatrix}$$

where Player 1's payoffs for cooperation are listed in the first row and his reward for defection or non-cooperation is shown in the second. In an analogous way, Player 2's decisions are reflected in the columns of the payoff matrix. More precisely, $\bar{\mathcal{S}}$ represents the "sucker's payoff," or the fear of being betrayed by others, and $\mathcal{T}$ stands for the temptation to defect. For example, Player 1 receives 1 if both players cooperate, $\mathcal{T}$ if he defects and the other player cooperates, $\bar{\mathcal{S}}$ if he cooperates and the other player defects, and 0 if neither player cooperates.

In a given population of platforms that is assumed to be large and well-mixed.[1], the dynamical evolution of cooperative and non-cooperative strategies can be described by the Replicator Equation (see Weibull 1995, Hofbauer and Sigmund 2003), which corresponds to the following system

$$\dot{x} = x(\pi_C - \bar{\pi}), \tag{9}$$

$$\dot{z} = z(\pi_N - \bar{\pi}), \tag{10}$$

where π_C and π_N are the average payoffs collected by the share of cooperative and non-cooperative platforms, respectively, such that

$$\begin{pmatrix} \pi_C \\ \pi_N \end{pmatrix} = B \cdot \begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} x + \bar{\mathcal{S}}z \\ \mathcal{T}x \end{pmatrix},$$

while $\bar{\pi}$ represents the average payoff of the whole cryptocurrency market and is defined as

$$\bar{\pi} = x\pi_C + z\pi_N = x^2 + (\mathcal{T} + \bar{\mathcal{S}})xz.$$

In a dynamic context, it is worthwhile to notice that the levels of both Infected and Susceptible may provide a reasonable estimate of platforms' perceptions of the risk contagion strength; as a consequence, it is reasonable to suppose that the ruling parameters $\mathcal{T}$ and $\bar{\mathcal{S}}$ may vary according to the ratio $(I - S)/M$ which may work as a "game-switching" term. Actually, we assume that matrix B is dependent of both I and S and define the net payoffs for defection and cooperation as

$$\bar{\mathcal{S}} = \frac{I - S}{M}, \quad \mathcal{T} = 1 - \bar{\mathcal{S}};$$

therefore, the resulting payoff matrix

$$B(S, I) = \begin{pmatrix} 1 & \frac{I-S}{M} \\ 1 - \frac{I-S}{M} & 0 \end{pmatrix}. \tag{11}$$

According to the value of ratio $(I - S)/M$, the game switches from being a Prisoner's Dilemma to a Harmony game when the infected level I exceeds the susceptible one S. Indeed, for $I - S < 0$, $B(S, I)$ represents a Prisoner's Dilemma game due to the fact that $\mathcal{T} > 1$ and $\bar{\mathcal{S}} < 0$. That is to say, defection is the "natural" choice of action when the contagion is sufficiently low, as the implementation of any containment measures is not justified by a perceived weakness in the risk spread. In this instance, limitations and control mechanisms naturally wane until the current crisis intensifies again to the point where infection is so high that $I - S > 0$. Then, it holds that $\mathcal{T} < 1$ and $\bar{\mathcal{S}} > 0$ so that $B(S, I)$ turns to a Harmony

[1] The assumption that the population of cryptocurrency platforms is large and well-mixed is supported by several studies highlighting the extensive nature and interconnectedness of the cryptocurrency market. For instance, the cryptocurrency market has experienced significant growth, with thousands of active cryptocurrencies contributing to a diverse ecosystem that exhibits characteristics of a large population (see Abbasi et al., 2021) This extensive market size allows for the application of statistical models that assume a well-mixed environment, where interactions among platforms can be analyzed without significant bias from structural limitations (see Bogdan, 2023). Furthermore, research indicates that the behaviors of traders within this market, such as herding and feedback trading, reflect dynamics which are typical of large, well-mixed populations (see Bouri et al. 2019 and King and Koutmos 2021). The high volatility and liquidity of cryptocurrencies also suggest that price movements are influenced by collective behaviors across a broad spectrum of platforms, reinforcing the notion of a well-mixed population (see Dutta & Bouri, 2022).

game. In this respect, the payoff matrix can be interpreted as incentives for doing what seems the right thing in the right situation in the framework of a "perfect right dynamics".
In the various fields of application related to behavioral epidemiology, a similar but different approach based on a dynamic structure of the payoff matrix is proposed by Madeo and Mocenni (2022) and Báez-Sánchez (2018): they rely on the capability of networked populations to self-regulate the behavior when a higher common good should be preserved because of individual awareness. Based on the analogy between financial systems and ecosystems, in our framework individuals correspond with cryptocurrency platforms which aim to preserve the higher common good given by preventing abuse and safeguarding investors to mitigate the risks of financial instability.

Under the assumption that the payoff matrix is defined in (11), it is not so difficult to verify that the Replicator Equations (9)–(10) reduces to

$$\dot{x} = x(1-x)\,\frac{I-S}{M}, \tag{12}$$

as the equation with respect to z is omitted since relationship $z = 1 - x$ can be exploited for evaluating the dynamics of the share of non-cooperators.

2.2 The risk contagion game-based model

As already mentioned, cooperation entails the adoption of best practices which are meant to lessen the transmission of risk, like facilitating various activities such as trading, lending, borrowing, and acting as collateral within DeFi ecosystem. As a result, the behavior of platforms influences the contagion rate a in the SIR dynamics. Actually, cooperation lowers the contagion rate, while defection increases it. In this framework, we assume that contagion rate a linearly depends on the cooperation share x according to the rule

$$a(x) = a_0(1-x), \tag{13}$$

where a_0 represents the "natural" rate of contagion. Precisely, a_0 captures the way throughout one platform influences each other in absence of cooperation. This phenomenon can be identified as an interconnection among the platforms in the market and parameter a_0 is measured by the net-spillover index according to Diebold and Yilmaz (2012), Diebold and Yilmaz (2014), Diebold and Yilmaz (2016).
We notice that determining the contagion rate based on (13) yields that no new infected platforms would arise if the whole cryptocurrency market adopted cooperative conduct; indeed, $a(x) > 0$ for $0 \le x < 1$ and $a(1) = 0$.

Based on the SIR dynamics (1)–(2)–(3) with contagion rate defined by (13) and on the Replicator Equation (12), we propose the following delayed model

$$\dot{S}(t) = b - \gamma S(t) - a_0(1-x(t))\,S(t)\,I(t) + \delta e^{-\gamma\tau} I(t-\tau), \tag{14}$$

$$\dot{I}(t) = a_0(1-x(t))\,S(t)\,I(t) - (\gamma+\delta) I(t), \tag{15}$$

$$\dot{R}(t) = \delta I(t) - \gamma R(t) - \delta e^{-\gamma\tau} I(t-\tau), \tag{16}$$

$$\dot{x}(t) = x(t)(1-x(t))\,\frac{I(t)-S(t)}{M}, \tag{17}$$

equipped with suitable initial conditions (4)–(5)–(6) joint with $x(0) = x_0$ setting $x_0 \in [0, 1]$. A schematic representation of the model is shown in Fig. 1.
Under the previous assumption that $I_0(\cdot)$ in (6) is continuous over the lag interval $[-\tau, 0]$, there exists a unique solution of the previous delayed differential system (see Hale, 1977).

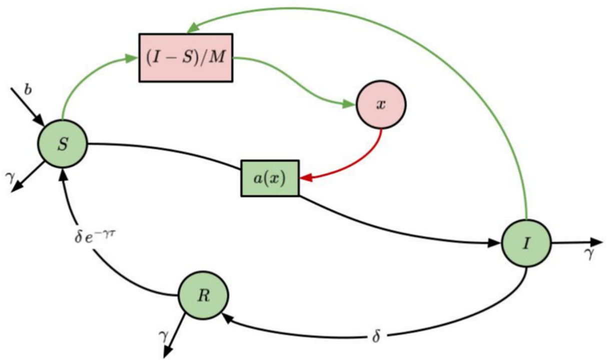

Fig. 1 Schematic representation of the game-based model. The SIR model and the Replicator Equation are coupled by two feedback mechanisms: (1) the densities S and I influence the cooperation dynamics by changing the ratio $(I-S)/M$ (green arrows); (2) the cooperation x influences the contagion rate $a(x)$ inside the spreading of risk (red arrow). (Color figure online)

Moreover, it can be proved that the bounds stated in (7) still hold under the same argument discussed in the previous Sect. 2. In this respect, we notice that the "game-switching" term is bounded such that $-1 \le (I(t)-S(t))/M \le 1$ for any time. According to the properties of the Replicator Equation, it holds that $0 \le x(t) \le 1$ for each $t \ge 0$.

3 Analysis of the delayed game model

As already stated in the previous Sect. 2, once the infected dynamics is evaluated, $R(t)$ can be directly integrated according to (8); for this reason, equation (16) can be omitted. Therefore, we focus on the analysis of the reduced coupled model (14)–(15)–(17). It is characterized by four steady states:

- $E_1^* = (b/\gamma, 0, 1)$ which represents the pure game strategy of always pursuing the cooperation strategy.
- $E_2^* = (b/\gamma, 0, 0)$ which represents the pure game strategy of always pursuing the non-cooperation strategy in the case where risk spread tends to disappear from the market.
- $E_3^* = (S_3^*, I_3^*, 0)$, with $S_3^* = (\delta+\gamma)/a_0$ and $I_3^* = (b-\gamma S_3^*)/(\gamma+\delta-\delta e^{-\gamma\tau})$; this equilibrium point represents the pure game strategy of always pursuing non-cooperation strategy in the case when risk spread tends to be endemic in the market.
- $E_4^* = (S_4^*, I_4^*, x_4^*)$, with $S_4^* = I_4^* = b/(2\gamma+\delta-\delta e^{-\gamma\tau})$ and $x_4^* = 1-(\delta+\gamma)/(a_0\, S_4^*)$; this equilibrium point corresponds to a mixed-strategy in the game.

A crucial role in the dynamics is played by the so-called "basic reproduction number"

$$\rho_0 = \frac{a_0 b}{\gamma(\delta+\gamma)}, \tag{18}$$

which has a cutoff value that marks the boundary of a region where a mixed-strategy equilibrium exists and there are no stable pure strategy equilibria. In the sequel, we refer to the viability of the mixed-strategy equilibrium as the condition that defines this region.

3.1 Viability of the mixed-strategy equilibrium

The mixed-strategy equilibrium E_4^* is considered viable if it is feasible (i.e. $0 < x_4^* < 1$), when the pure strategy equilibria E_1^*, E_2^* and E_3^* are unstable (see Wettergren, 2023). Since the mixed-strategy is not observed when the dynamics are attracted by the pure strategies, the attention is focused on scenarios in which the pure strategy equilibria are asymptotically stable. In this respect, we find that long-term stable dynamics that do not approach one of the pure strategies require the existence of a mixed-strategy equilibrium which is feasible. Actually, the behavior of the pure game strategies in the steady state can be addressed in terms of local asymptotical stability and is influenced by the level of the parameter ρ_0, as already mentioned.

In this framework, we start from assessing the stability of the pure game strategies by employing the classical tools of delay differential system analysis. We notice that E_1^* and E_2^* are always feasible; in addition, their steady state behavior is characterized by the following results.

Proposition 1 *The steady state E_1^* is unstable for any combination of the parameter ρ_0 and the time delay $\tau \geq 0$.*

Proof We linearize system (14)–(15)–(17) near the steady state E_1^* and focus on the characteristic equation with respect to λ defined as

$$\det(J_0(E_1^*) + e^{-\lambda\tau} J_\tau(E_1^*) - \lambda J) = 0, \tag{19}$$

where J is the identity matrix and

$$J_0(E_1^*) = \begin{pmatrix} -\gamma & 0 & 0 \\ 0 & -\delta - \gamma & 0 \\ 0 & 0 & b/(\gamma M) \end{pmatrix}, \quad J_\tau(E_1^*) = \begin{pmatrix} 0 & \delta e^{-\gamma\tau} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Equation (19) is solved by real eigenvalues corresponding to $\lambda_1 = -\gamma$, $\lambda_2 = -\delta - \gamma$ and $\lambda_3 = b/(\gamma M)$. We have $\lambda_1 < 0$ and $\lambda_2 < 0$, but $\lambda_3 > 0$; this condition implies that E_1^* is unstable in correspondence with any values of ρ_0 and τ. □

Proposition 2 *The following statements hold:*

- *if $\rho_0 < 1$, then the risk-free equilibrium E_2^* is locally asymptotically stable, E_1^* is unstable and the other equilibrium points are unfeasible;*
- *in the opposite case when $\rho_0 > 1$, then E_2^* is unstable.*

Proof The linearization of system (14)–(15)–(17) near E_2^* yields the following characteristic equation with respect to λ:

$$\det(J_0(E_2^*) + e^{-\lambda\tau} J_\tau(E_2^*) - \lambda J) = 0, \tag{20}$$

where J is the identity matrix, $J_\tau(E_2^*) = J_\tau(E_1^*)$ and

$$J_0(E_2^*) = \begin{pmatrix} -\gamma & -a_0 b/\gamma & 0 \\ 0 & (\delta + \gamma)(\rho_0 - 1) & 0 \\ 0 & 0 & -b/(\gamma M) \end{pmatrix}.$$

The solution of equation (20) corresponds to the real eigenvalues $\lambda_1 = -\gamma$, $\lambda_2 = (\delta + \gamma)(\rho_0 - 1)$ and $\lambda_3 = -b/(\gamma M)$.

Under the assumption that $\rho_0 < 1$, we get $\lambda_1 < 0$, $\lambda_2 < 0$ and $\lambda_3 < 0$. It follows that E_2^* is locally asymptotically stable. In addition, on one hand we notice that the equilibrium points E_3^* and E_4^* are not feasible, due to $I_3^* < 0$ and $x_4^* < 0$. On the other hand, E_1^* is always unstable due to Proposition 1.
In the opposite case when $\rho_0 > 1$, we get $\lambda_1 < 0$, $\lambda_3 < 0$ and $\lambda_2 > 0$. It follows that E_2^* is unstable. Then, the proof is completed. □

We remark that the first two equilibrium points E_1^* and E_2^* are characterized as risk-free, indicating that the system can reach a stable state without ongoing risk. It means that at these points, neither the susceptible nor the infected platforms present a threat, likely due to effective control mechanisms or eradication of the problem being modeled. In this respect, according to the previous Proposition 1, we may state that it is not realistic to provide incentives for any kind of cooperative activity to prevent the spread of risk in the absence of real long-term risk. On the other hand, Proposition 2 states that the absence of long-term cooperation is justified only if the force of infection is low and ρ_0 is below the threshold 1, since the equilibrium E_2^* is risk-free.

The following result deals with the stability of the endemic equilibrium point E_3^* in the absence of cooperation. It exploits the polynomial function defined by

$$p(\omega) = \omega^2 - 2\delta\omega + \gamma^2,$$

for any $\omega \in \mathbb{R}$.

Proposition 3 *The following statements hold:*

- *If* $1 < \rho_0 < 2 + (\delta - \delta e^{-\gamma\tau})/\gamma$, *then* E_1^* *and* E_2^* *are unstable, while* E_4^* *is unfeasible; moreover, the risk-free equilibrium* E_3^* *is locally asymptotically stable in correspondence with the fixed time delay* $\tau \geq 0$ *under the further assumption that the following condition is satisfied:*

$$p_0 := p(a_0 I_3^*) \geq 0. \tag{21}$$

- *In the other case when* $\rho_0 > 2 + (\delta - \delta e^{-\gamma\tau})/\gamma$, *then* E_1^*, E_2^* *and* E_3^* *are unstable.*

Proof By following the same approach as in the previous propositions, the linearization of system (14)–(15)–(17) is carried out near E_3^*. As a result, the characteristic equation with respect to λ is defined as

$$\det(J_0(E_3^*) + e^{-\lambda\tau} J_\tau(E_3^*) - \lambda J) = 0,$$

where J represents again the identity matrix, $J_\tau(E_3^*) = J_\tau(E_1^*)$ and

$$J_0(E_3^*) = \begin{pmatrix} -\gamma - a_0 I_3^* & -\delta - \gamma & a_0 S_3^* I_3^* \\ a_0 I_3^* & 0 & -a_0 S_3^* I_3^* \\ 0 & 0 & (I_3^* - S_3^*)/M \end{pmatrix}.$$

An eigenvalue corresponds to

$$\bar{\lambda} = \frac{I_3^* - S_3^*}{M} = \frac{\gamma(\delta + \gamma)}{M a_0(\gamma + \delta - \delta e^{-\gamma\tau})} \left(\rho_0 - \left(2 + \frac{\delta - \delta e^{-\gamma\tau}}{\gamma}\right)\right),$$

while the other two eigenvalues solve the following equation

$$\lambda^2 + (\gamma + a_0 I_3^*)\lambda + (\gamma + \delta - \delta e^{-(\lambda+\gamma)\tau}) a_0 I_3^* = 0. \tag{22}$$

On one hand, we suppose that $\rho_0 > 2 + (\delta - \delta e^{-\gamma\tau})/\gamma$ which also yields $\rho_0 > 1$. Under this assumption, E_1^* and E_2^* are unstable according to the results proved in Propositions 1 and 2. As an additional consequence, condition $\rho_0 > 2 + (\delta - \delta e^{-\gamma\tau})/\gamma$ assures that $\bar{\lambda} > 0$ which yields E_3^* is unstable.
On the other hand, we suppose that $1 < \rho_0 < 2 + (\delta - \delta e^{-\gamma\tau})/\gamma$. Again, this assumption implies E_1^* and E_2^* are unstable, as in the previous case. Furthermore, the same assumption yields $\bar{\lambda} < 0$; therefore, the location of the roots of equation (22) in the plane affects the stability for steady state E_3^*. In particular, the stability of this equilibrium changes as the root values move from the left half of the plane into the right half. With the aim of investigating this possibility, first we notice that when $\tau = 0$, equation (22) reduces to

$$\lambda^2 + (\gamma + (\delta + \gamma)(\rho_0 - 1))\lambda + \gamma(\delta + \gamma)(\rho_0 - 1) = 0;$$

then, it admits roots with negative real parts, since its coefficients are positive. Hence, condition $1 < \rho_0 < 2 + (\delta - \delta e^{-\gamma\tau})/\gamma$ implies that E_3^* is locally stable when $\tau = 0$. In the general case where $\tau > 0$, we argue about the existence of purely imaginary eigenvalues; therefore, we apply the substitution $\lambda = iv$ $(v > 0)$ to (22) and obtain

$$-v^2 + (\gamma + a_0 I_3^*)iv + (\gamma + \delta)a_0 I_3^* - a_0 I_3^* \delta e^{-\gamma\tau} e^{-iv\tau} = 0.$$

Separating the real and imaginary components leads to the following system of equations

$$\begin{aligned} -v^2 + (\gamma + \delta)a_0 I_3^* &= a_0 I_3^* \delta e^{-\gamma\tau} \cos(v\tau), \\ (\gamma + a_0 I_3^*)v &= -a_0 I_3^* \delta e^{-\gamma\tau} \sin(v\tau), \end{aligned}$$

which are squared and added in order to obtain

$$v^4 + p_0 v^2 + (a_0 I_3^*)^2 \left(\gamma^2 + 2\gamma\delta + \delta^2(1 - e^{-2\gamma\tau})\right) = 0, \tag{23}$$

where we have $p_0 := p(a_0 I_3^*)$. We notice that the sign of coefficient p_0 defined by condition (21) is crucial since it assures that equation (23) has positive coefficients and cannot be solved. It implies that the eigenvalues corresponding to E_3^* do not move from the left half of the plane into the right half, hence E_3^* is locally stable. Thus the proof is complete. □

These results are essential for examining the contagion behavior around the pure strategy equilibrium E_3^* as well as the viability of the mixed-strategy equilibrium. Actually, regarding steady state E_3^*, we assume that

$$1 < \rho_0 < 2 + (\delta - \delta e^{-\gamma\tau})/\gamma, \tag{24}$$

for $\tau \geq 0$ and notice that the discriminant $\Delta = 4(\delta^2 - \gamma^2)$ of the polynomial function $p(\cdot)$ determines the sign of the term p_0. Indeed, if $\gamma \geq \delta$, then $\Delta \leq 0$ and $p(\omega) \geq 0$ for any value $\omega \in \mathbb{R}$. This yields condition (21) is satisfied in correspondence with any value of time delay $\tau \geq 0$. Therefore, condition (24) and $\gamma \geq \delta$ ensure that E_3^* is locally asymptotically stable for any τ, due to Proposition 3. In the opposite case $\gamma < \delta$, we get $\Delta > 0$ and $p(\omega) \geq 0$ for any $\omega \in (-\infty, \omega_1] \cup [\omega_2, +\infty)$ with

$$\omega_1 = \delta - \sqrt{\delta^2 - \gamma^2},$$

and

$$\omega_2 = \delta + \sqrt{\delta^2 - \gamma^2}.$$

We focus on both conditions $a_0 I_3^* = \omega_1$ or $a_0 I_3^* = \omega_2$ which correspond to

$$e^{-\gamma\tau} = 1 + \frac{\gamma}{\delta} - \frac{\gamma(\delta+\gamma)}{\omega_1}(\rho_0 - 1), \tag{25}$$

and

$$e^{-\gamma\tau} = 1 + \frac{\gamma}{\delta} - \frac{\gamma(\delta+\gamma)}{\omega_2}(\rho_0 - 1), \tag{26}$$

respectively. In this respect, it is worth noting that the time delays defined by equations (25) or (26) together with conditions (24) and $\gamma < \delta$ represent critical values that may define a transition between stability and instability of steady state E_3^* giving rise to to Hopf bifurcations in the dynamics plane.

On the other hand, the viability of the mixed-strategy equilibrium is affected by the results stated in the previous propositions. Indeed, it has been proved how the threshold $\rho_0 > 2 + (\delta - \delta e^{-\gamma\tau})/\gamma$ for $\tau \geq 0$ implies that all steady states E_1^*, E_2^* and E_3^* are asymptotically unstable, moreover the mixed-strategy steady state is admissible. This implication can be reversed by a straightforward argument. Thus, the following result is obtained.

Proposition 4 *The mixed-strategy equilibrium E_4^* is viable for the dynamics related to a fixed time delay $\tau \geq 0$ if and only if the following condition holds:*

$$\rho_0 > 2 + (\delta - \delta e^{-\gamma\tau})/\gamma. \tag{27}$$

3.2 Stability analysis of the mixed-strategy equilibrium

Assuming a scenario where (27) holds so that the mixed-strategy equilibrium is viable with respect to the fixed time delay $\tau \geq 0$, we focus on a stability analysis around the equilibrium point E_4^* with the aim of determining conditions which ensure that the mixed-strategy steady state is stable and hence observable in practice.

In this respect, by the same approach as in Sect. 3.1, system (14)–(15)–(17) is linearized near E_4^*; thus, the characteristic equation with respect to λ is

$$\det(J_0(E_4^*) + e^{-\lambda\tau} J_\tau(E_4^*) - \lambda J) = 0,$$

where J is again the identity matrix, $J_\tau(E_4^*) = J_\tau(E_1^*)$ and

$$J_0(E_4^*) = \begin{pmatrix} -2\gamma - \delta & -\delta - \gamma & a_0 S_4^* I_4^* \\ \delta + \gamma & 0 & -a_0 S_4^* I_4^* \\ -x_4^*(1 - x_4^*)/M & x_4^*(1 - x_4^*)/M & 0 \end{pmatrix}.$$

Since $a_0(1 - x_4^*)S_4^* I_4^* = (\delta + \gamma)I_4^*$ and $S_4^* = I_4^*$, the previous characteristic equation is equivalent to

$$\lambda^3 + (2\gamma + \delta)\lambda^2 + A\lambda + B = \delta e^{-(\gamma+\lambda)\tau}(\delta + \gamma)\left(\frac{x_4^* S_4^*}{M} + \lambda\right), \tag{28}$$

where we set

$$A = (\delta + \gamma)\left(2\frac{x_4^* S_4^*}{M} + \delta + \gamma\right), \qquad B = (2\gamma + \delta)(\delta + \gamma)\frac{x_4^* S_4^*}{M}.$$

Under assumption (27), we notice that $x_4^* S_4^*$ corresponds to the following term

$$x_4^* S_4^* = \frac{\gamma(\delta + \gamma)}{a_0(2\gamma + \delta - \delta e^{-\gamma\tau})}\left(\rho_0 - \left(2 + \frac{\delta - \delta e^{-\gamma\tau}}{\gamma}\right)\right) > 0.$$

The stability of the mixed-strategy equilibrium is described by the location of the roots of the characteristic equation (28) in the plane. Actually, E_4^* is locally asymptotically stable in the case when the eigenvalues lie on the left side of the plane. Some sufficient conditions for the stability of the mixed-strategy steady state are provided by the following result. In particular, eigenvalues' location depends on the following polynomial functions:

$$\psi(\omega) = 2\gamma^2 - \delta^2 - 2\omega,$$

and

$$\varphi(\omega) = \omega^2 - (2\gamma^2 - \delta^2)\omega + (\delta + \gamma)^4,$$

defined for $\omega \in \mathbb{R}$

Proposition 5 *Suppose that* (27) *and* $2\gamma^2 - \delta^2 \geq 0$ *hold. The mixed-strategy equilibrium* E_4^* *is locally asymptotically stable in correspondence with the fixed time delay* $\tau \geq 0$ *under the further assumption that the following conditions are both satisfied:*

$$\psi(x_4^* S_4^*/M) \geq 0, \tag{29}$$

and

$$\varphi(x_4^* S_4^*/M) \geq \delta^2(\delta + \gamma)^2 e^{-2\gamma\tau}. \tag{30}$$

Proof We focus on the roots of equation (28) and investigate their location in the plane. With this aim, first we note that when $\tau = 0$, equation (28) reduces to

$$\lambda^3 + (2\gamma + \delta)\lambda^2 + (\delta + \gamma)\left(\frac{\delta + \gamma}{a_0}(\rho_0 - 2) + \gamma\right)\lambda + \gamma\frac{(\delta + \gamma)^2}{a_0}(\rho_0 - 2) = 0,$$

that is equivalent to

$$(\lambda + \gamma)\left(\lambda^2 + (\delta + \gamma)\lambda + \frac{(\delta + \gamma)^2}{a_0}(\rho_0 - 2)\right) = 0.$$

It follows that an eigenvalue corresponds to $\bar{\lambda} = -\gamma < 0$, while the other two eigenvalues solve the following equation

$$\lambda^2 + (\delta + \gamma)\lambda + \frac{(\delta + \gamma)^2}{a_0}(\rho_0 - 2) = 0,$$

which admits roots with negative real parts under assumption (27), since its coefficients are positive. Therefore, condition (27) implies that E_4^* is locally stable when $\tau = 0$. In the general case where $\tau > 0$, it is possible to verify that $\bar{\lambda} = -\gamma < 0$ is a root of equation (28). Hence $\bar{\lambda} = -\gamma < 0$ once more represents an eigenvalue in correspondence with $\tau > 0$. As a result, the stability of E_4^* is determined by the location of the other eigenvalues, which are the other roots of the characteristic equation. Actually, stability changes as eigenvalues move from the left half of the plane into the right half. Then, we argue about the existence of other purely imaginary eigenvalues; hence, we insert $\lambda = iv$ $(v > 0)$ into (28) and obtain

$$-iv^3 - (2\gamma + \delta)v^2 + Aiv + B = \delta e^{-\gamma\tau}(\delta + \gamma)e^{-iv\tau}\left(\frac{x_4^* S_4^*}{M} + iv\right).$$

After separating the real and imaginary components, the following system of equations is obtained

$$
\begin{aligned}
B-(2\gamma+\delta)v^2 &= \delta e^{-\gamma\tau}(\delta+\gamma)\left(\frac{x_4^*S_4^*}{M}\cos(v\tau)+v\sin(v\tau)\right),\\
Av-v^3 &= \delta e^{-\gamma\tau}(\delta+\gamma)\left(v\cos(v\tau)-\frac{x_4^*S_4^*}{M}\sin(v\tau)\right).
\end{aligned}
$$

Both sides of the previous relationships can be squared and added, thus we get

$$
v^6+C_1v^4+C_2v^2+C_3=0, \tag{31}
$$

where we set

$$
\begin{aligned}
C_1 &= (2\gamma+\delta)^2-2A=\psi(2(\delta+\gamma)x_4^*E_4^*/M),\\
C_2 &= A^2-2B(2\gamma+\delta)-\delta^2(\delta+\gamma)^2e^{-2\gamma\tau}=\varphi(2(\delta+\gamma)x_4^*E_4^*)-\delta^2(\delta+\gamma)^2e^{-2\gamma\tau},\\
C_3 &= B^2-\delta^2(\delta+\gamma)^2e^{-2\gamma\tau}(x_4^*S_4^*/M)^2.
\end{aligned}
$$

In addition to the fact that $C_3 > 0$, conditions (29) and (30) yield $C_1 \geq 0$ and $C_2 \geq 0$ as well. It implies that equation (31) does not admit any solution since all of its coefficients are positive. It follows that no eigenvalue corresponding to E_4^* moves from the left half of the plane to the right half, hence E_4^* is locally stable. In this way, the proof is completed. □

We notice that the time delays at which the assumptions of the previous Proposition are not verified and equation (31) admits possible solutions represent critical values that may define a transition between stability and instability of the mixed-strategy at the steady state giving rise to Hopf bifurcations.

4 A simulation of cooperation dynamics

We examine governance tokens that have recently become associated with the most popular cryptocurrency trading platforms. Our selection is based on trading volume as of early 2023,[2] focusing on the most capitalized platforms while acknowledging that the crypto market is vast, encompassing numerous platforms. Using the Yahoo Finance data source, we retrieved daily unbalanced values of each currency from 2017 to 2023. Our database is a hand-curated compilation of publicly available data, arising from Yahoo Finance, where token prices are expressed in US dollars. Our simulation is initiated with these values. In our analysis, we explore potential connections across platforms and estimate the factors included in the SIR model by measuring spillover impacts, identifying risks, and assessing public attention to understand how risk may spread throughout the market. The dynamics of risk are projected into the future, focusing on long-term contagion in a possible stable steady state.

First we use the Granger causality framework to explore the theory of interconnection among cryptocurrency platform tokens. The studies provided in Diebold and Yilmaz (2012), Diebold and Yilmaz (2014), Diebold and Yilmaz (2016) describe an approach that is useful to determine the way information flows among platforms. It enables us to evaluate the potential impact of modifications to the predictive distribution of one group of time series variables, referred to as cause variables (CV), on another group, referred to as effect variables (EV). This analysis's primary goal is to determine how the EV projections affect the mean squared error. As established in the literature, ensuring that the time series of the variables are stationary is crucial to our analysis. If stationarity is lost, appropriate adjustments to the statistics must

[2] See https://coinmarketcap.com The website has been visited on March 13, 2024.

be made. In order to study Granger causality among the return series, we start with the dataset and employ the vector autoregression (VAR) model fitting technique, depending on the Akaike Information Criterion (AIC). We find the latency that yields the lowest AIC score after testing the VAR model with lags ranging from 1 to 4 days. To ascertain if one variable functions as a Granger cause of another, we then run Granger causality tests on every variable and equation inside the VAR framework. This is achieved by evaluating the null hypothesis using a Chi-square test that is based on a leave-one-out methodology. The findings of this causality analysis support our initial hypothesis about interconnectedness by showing that the governance tokens of the platforms under investigation have substantial relationships with one another. To quantify the influence of spillover on risks and returns, we estimate the degree of interconnection by establishing spillover indices. The analysis developed in Diebold and Yilmaz (2012), Diebold and Yilmaz (2014), Diebold and Yilmaz (2016) refers to an extended breakdown of the forecast-error variance of the VAR model as a means of achieving this. Derived from the Diebold and Yilmaz technique, the net spillover index allows us to estimate the interconnectivity parameter pertinent to the previously stated SIR dynamics. This research validates that there is a substantial level of interconnectedness among the platform governance tokens that are being considered, which validates our initial hunch about interconnectedness. The impact of spillover effects on risks and returns is then examined in order to assess the degree of this interconnectivity. Specifically, the spillover index, which is derived from the fluctuations in governance token prices, showcases the interconnectivity and the ability of several platforms to influence one another.

This spillover index is evaluated as $a_0 = 0.5554$ which corresponds with the interconnection parameter involved in the coupled system (14)–(15)–(17) describing the cooperation dynamics.

We calculate risk indicators by considering the residual from regressing daily token returns in an expanded index model, as suggested by Dumitrescu and Zakriya (2022) and Habib et al. (2018), after estimating token-specific daily returns according to the following estimate

$$r_{j,t} = \alpha_j + \beta_{a,j} r_{m,t-2} + \beta_{b,j} r_{m,t-1} + \beta_{c,j} r_{m,t} + \beta_{d,j} r_m + \beta_{e,j} r_{m,t+1} + \beta_{f,j} r_{m,t+2} + \varepsilon_{j,t},$$

where $r_{j,t}$ represents the daily return of the token i-th on day t, while $r_{m,t}$ corresponds to the market index return for the same period. We use the S&P Cryptocurrency Broad Digital Market Index as the reference, given its global scope and suitability in the absence of a more specific benchmark. In addition, we account for market fluctuations by incorporating both forward and lagged returns, extending the model to include returns from one and two days prior, as well as two days ahead, following the methodology of Hutton et al. (2009). To address the issue of skewed residuals in token returns, we adjust them by defining $W_{j,t} = \ln(1+\varepsilon_{j,t})$ to correct daily returns for the skewed residuals $\varepsilon_{j,t}$, which helps to normalize daily return distributions.

The down-to-up volatility measure (the so called DUVOL) of the crash likelihood is the indicator of crash risk. Token-specific daily returns are divided into two groups for each token j throughout a fiscal-year period t: “down” days, or when returns are below the annual mean, and “up” days, or when returns are above the monthly mean. For each of these two categories, the standard deviation of daily returns that are token-specific is computed independently. The natural logarithm of the ratio between the standard deviation on “down” days and the standard deviation on “up” days is known as DUVOL:

$$DUVOL_{j,t} = \ln\left((n-1) \frac{\sum_{down} W_{j,t}^2}{ndown - 1} \sum_{up} W_{j,t}^2 \right).$$

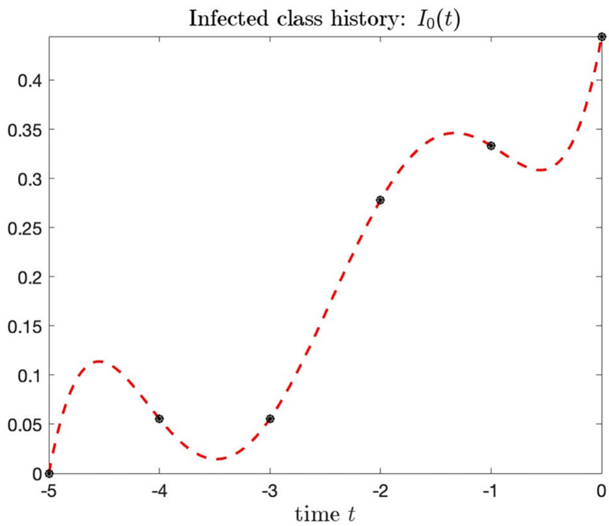

Fig. 2 The dashed red line corresponds to the plot of history function $I_0(\cdot)$ in the whole sample period $[-5, 0]$. Black bullets represent the nodal values which are obtained by relating the entries of the vector $(8,\ 6,\ 5,\ 1,\ 1,\ 0)$ to the initial number of companies $N_0 = 18$ operating at $t = 0$. (Color figure online)

While we assert that "Susceptible" tokens are associated with platforms that are quoted in the market, we also need to provide a standard for determining which tokens are "Infected" and when they are "Recovered". In this regard, we anticipate that infection will be assessed based on DUVOL. More specifically, we use the risk assessment method that estimates the Negative Conditional Skewness of the stock return variance (see Dumitrescu and Zakriya 2022 and Habib et al. 2018).

In this framework, on one hand we notice that risk identification allows for setting the parameters involved in the dynamics: indeed, we find $b = 0.1444$, $\gamma = 0.1277$ and $\delta = 0.044$. On the other hand, the identification of infected platforms allows for constructing the history function $I_0(t)$ involved in the initial condition (6). It is evaluated in the lag period $[-5, 0]$, that is the time horizon we are considering, which runs from August 2017 to January 2023. More precisely, the number of infected cryptocurrency platforms in the data sample are collected in the vector $\mathbf{v}_I = (8,\ 6,\ 5,\ 1,\ 1,\ 0)$ whose entries represent the number of high risk platforms measured at each time node $t = -5, -4, \ldots, 0$. According to the data, the dynamics starts with $N_0 = 18$ platforms at $t = 0$, which corresponds to the last date January 2023 in the sample; then, each entry of $\mathbf{v}_I$ is normalized with respect to N_0 in order to obtain the nodal values for I_0. The built-in MATLAB function `polyfit` is applied to asses a polynomial representing the best fit (in a least-squares sense) for the normalized nodal values provided by $\mathbf{v}_I./N_0$. The degree of the best fit polynomial is set at $p = 5$: actually, we have empirically verified that this choice allows us to fit the data by following their trend well as it is shown in Fig. 2, where we plot the resulting history function I_0. Moreover, according to the data, the dynamics starts at $S_0 = 0.53$.

As a further remark, we notice that time delay is not measured by the available data; anyway, different values are considered for τ with the aim of simulating risk dynamics under different assumptions on the length of financial immunity. According to the data, we find that the minimum period between one infection and the next one of the same platform corresponds to three years, throughout the sample; thus, we argue that τ does not exceed the threshold 3.

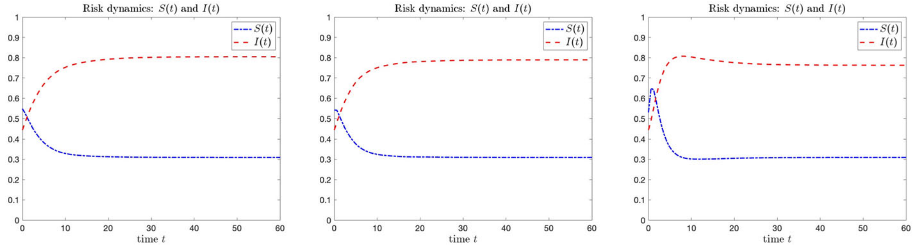

Fig. 3 Risk contagion dynamics in absence of cooperation, i.e. without game. The dynamics is obtained in correspondence with different lengths for temporary financial immunity: $\tau = 0.5$ on the left, $\tau = 1$ in the middle, $\tau = 2$ on the right. The solid blue line describes susceptible dynamics, the dashed red line corresponds to infected dynamics. (Color figure online)

Crypto crashes on platforms have increased in recent years; therefore, it is natural to suppose that in the absence of policy and regulatory intervention, the immunity period is short with respect to this threshold. In this framework, given that technology has let trading in a market happen quickly, it seems sense to compare the dynamics of very brief periods of immunity and longer intervals. Under this argument, we account for three different scenarios. Firstly, we assume that temporary immunity is very short and set $\tau = 0.5$; then we consider a longer time delay and choose $\tau = 1$. The third scenario assumes a very long financial immunity so that we set $\tau = 2$.

We remark that the risk spread dynamics is nonlinear and it cannot be written in closed-form, hence an approximation must be determined: we carry out a numerical simulation by exploiting the built-in function `dde23` in MATLAB environment.
As a first step, risk contagion is simulated in absence of cooperation. In this respect, the Susceptible-Infected system (1)–(2) without game is numerically integrated and its approximated dynamics is shown in Fig. 3 where, in correspondence with the different values for $\tau = 0.5$, $\tau = 1$ and $\tau = 2$, the results are provided on the left, in the middle and on the right, respectively. We notice that model (1)–(2) without game admits the risk-free equilibrium $\hat{E}_0 = (b/\gamma, 0)$ and one more non-zero steady state $\hat{E}_\tau = \left(\hat{S}_\tau, \hat{I}_\tau\right)$ with $\hat{S}_\tau = (\delta + \gamma)/a_0$ and $\hat{I}_\tau = (b - \gamma\hat{S}_\tau)/(\gamma + \delta - \delta e^{-\gamma\tau})$. This non-trivial equilibrium $\hat{E}_\tau$ represents an endemic or not-free-risk steady state. Due to the values of the parameters involved in the simulation, the basic reproduction number defined by (18) exceeds threshold 1, i.e. $\rho_0 > 1$; as a consequence, the not-free-risk steady state $\hat{E}_\tau$ is feasible. Moreover, it is possible to verify that $\hat{E}_\tau$ is asymptotically stable in correspondence with any time delay $\tau \geq 0$, according to the results provided in Aliano et al. (2024) due to the fact that $\delta < \gamma$ so that $(\gamma + \delta)/2 < \gamma$.

As expected, not-free-risk equilibrium endemically attracts Susceptible-Infected trajectories in each case related to the different time delays under consideration. It suggests that risk infection persistently exists over time for the long run. The infection tends to be so widespread that the density of the infected platforms is more than twice as high as the density of the susceptible ones.

As next step, our focus consists of simulating the effect of cooperation in the risk spreading related to Susceptible-Infected dynamics completed by Replicator equation. With this aim, we suppose that the cooperation action begins at a relatively low level, i.e. $x_0 = 0.005$. This assumption enables us to understand how the propensity for cooperation may impact the dynamics of contagion when the platforms initially exhibit no cooperative behavior but they are under threat from infection. In this framework, the coupled model (14)–(15)–(17) is

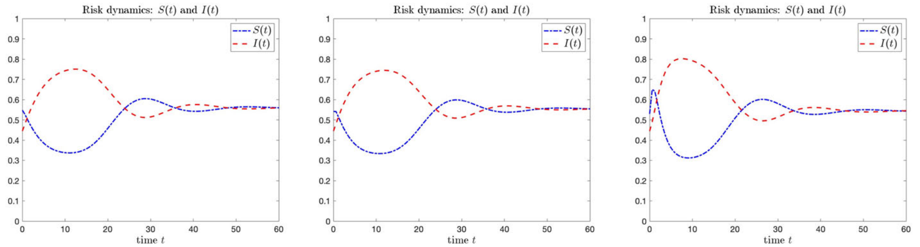

Fig. 4 Risk contagion dynamics in presence of cooperation. The dynamics is obtained in correspondence with different lengths for temporary financial immunity: $\tau = 0.5$ on the left, $\tau = 1$ in the middle, $\tau = 2$ on the right. The solid blue line describes susceptible dynamics, the dashed red line corresponds to infected dynamics. (Color figure online)

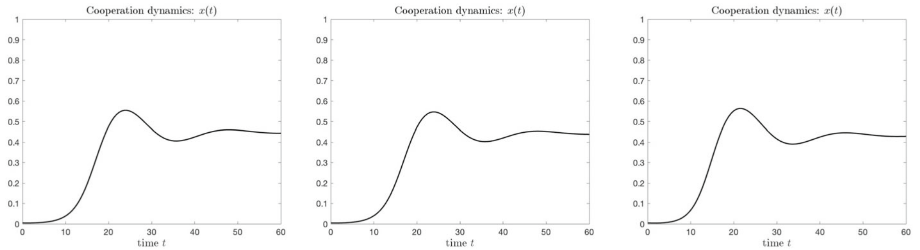

Fig. 5 Dynamics of propensity to cooperate $x(t)$. The plots are obtained by employing different lengths for temporary financial immunity: $\tau = 0.5$ on the left, $\tau = 1$ in the middle, $\tau = 2$ on the right

numerically solved. Then, the corresponding dynamics given in terms of densities $S(t)$ and $I(t)$ is plotted in Fig. 4; furthermore, the temporal evolution of cooperation among platforms related to the different values for $\tau = 0.5$, $\tau = 1$ and $\tau = 2$ is shown in Fig. 5.

It is worthwhile to mention that, in this real case, viability condition (27) holds in correspondence with every financial immunity period τ under consideration. As a result, all equilibrium points are feasible even if E_1^*, E_2^* and E_3^* are unstable. The results in Figs. 4 and 5 point out that the mixed-strategy equilibrium E_4^* endemically attracts the trajectories of SIR solution and cooperative dynamics in each case related to the different time delays under consideration. From a financial perspective, it implies again that risk infection remains endemically in the market among cryptocurrency platforms for the long run; however, in this scenario including games, platforms opt to collaborate together in order to control the spread of risk and preserve equal levels of susceptible and infected densities. In this framework, in contrast to the results in Fig. 3, the game effect mitigates the strength of the infection and is mainly evident at the steady state level since $\hat{S}_\tau < S_4^* = I_4^* < \hat{I}_\tau$: this is due to the cooperative activity among platforms in terms of adoption of best practices like facilitating various activities such as trading, lending, borrowing, and acting as collateral within the DeFi ecosystem, as already mentioned.

The central role that financial immunity plays is pointed out if we focus on the short-term dynamics shown in Fig. 3. In particular, financial immunity is significant especially when its level is set to $\tau = 2$. In this case, there is a beneficial condition since the density of susceptible platforms grows faster than the one of infected platforms. The plot on the right of Fig. 4, where the financial immunity period is once more $\tau = 2$, displays a similar dynamics. Anyway, cooperation becomes essential over time in the long run as it yields an equilibrium where the densities of susceptible and infected platforms overlap to the same level. In the short term,

financial immunity acts as a stabilizing factor. Actually, it lets the susceptible number grow faster than the infected one; as a result, financial risk can be contained, thus further spread of financial distress is prevented. Maintaining this level of immunity can mitigate immediate disruptions within the system, offering a buffer against sudden crises.
However, cooperation over time becomes the driving force in achieving a sustainable equilibrium. The system keeps a balance through collective efforts, with a larger density of susceptible platforms with respect to the infected one. While short-term immunity provides immediate relief, long-term stability relies on cooperation among various actors, such as institutions, markets, and regulators. The analysis suggests that both financial immunity in the short term and cooperation in the long term are essential components in achieving enduring financial stability.

The plots point out that cooperative conduct is essential for maintaining financial stability, particularly in the long run. Over extended periods, the data demonstrate a strong connection between sustained cooperative behavior among financial institutions and overall economic resilience. This long-term cooperation contributes to a stable financial environment by fostering trust and reducing systemic risks. In the short term, the plots show that the level of cooperative behavior tends to increase in response to the severity of contagion diffusion. During periods of heightened financial stress or crises, such as during a rapid spread of economic shocks across markets, entities within the financial system are more likely to engage in collaborative efforts to mitigate risks and stabilize the situation. This reactive increase in cooperation is driven by the immediate need to contain the spread of financial distress and prevent a cascading effect that could lead to widespread economic turmoil. In the financial sector, cooperation can take many forms, including information sharing, coordinated regulatory measures, and joint efforts to mitigate risks.

5 Concluding remarks

Predictive modeling of the temporal spread of risk among the exchanges can be a useful tool for understanding the dynamics of risk contagion in a cryptocurrency market at the long run. In this respect, the starting point is the analogy between risk spread in financial markets and an epidemic outbreak through ecosystems. Actually, in the framework of SIR dynamics, we employ a suitable time delay differential system for risk diffusion among the most popular trading cryptocurrency platforms, by modelling contagion evolution in terms of crash risk with low and high levels. SIR models are frequently employed in biomathematics, but their use in the financial setting is new and worthy of attention, especially with regard to Decentralized Finance. As an original issue, we integrate the risk spread dynamics and evolutionary games. Actually, the SIR system is coupled with the replicator equation with the aim of describing the propensity of cryptocurrency platforms to cooperate and adopting the measures taken to control the risk spread. A stability analysis of the resulting model equilibria has been carried out. Moreover, due to the fact that the overall model is nonlinear, then its solution is not available in closed form; as a consequence, it is necessary to provide an approximation and simulate the dynamics.

Our theoretical and numerical findings highlight that the steady-state solutions have significant policy implications for financial stability, particularly in the context of cryptocurrency platforms. When the basic reproduction number related to the dynamics is very high, then the system tends to settle into an endemic risky equilibrium, indicating persistent financial instability. This scenario is particularly concerning for risky cryptocurrency platforms, which can contribute to broader financial instability within the cryptocurrency market. For international policymakers, it is crucial to focus on reducing the basic reproduction number to very low levels, even in situations involving cooperative activities between different financial policymakers. Supervisory authorities can play a pivotal role in mitigating these risks. By implementing stringent oversight measures and leveraging clearing house mechanisms, they can help to alleviate and reduce the reproduction rate of the risk spread. This intervention is necessary to prevent the financial ecosystem from entering and maintaining a state of persistent risk.

While financial immunity is a specific activity related to the economic actor (i.e., in our case, the cryptocurrency platform) and thus has a more microeconomic connotation, cooperation has a broader macroeconomic scope as it involves the market structure as a whole. This distinction highlights the different levels at which financial immunity and cooperation operate. The concept of financial immunity pertains to the ability of individual actors, such as firms or platforms, to withstand shocks and be resilient. It involves measures that safeguard the specific entity from market volatility or financial contagion, reflecting a microeconomic perspective. On the other hand, cooperation is a systemic approach that requires the interaction of multiple market participants and stakeholders. It encompasses all market participants and goes beyond individual actors in its pursuit of establishing stability through shared efforts. This broad macroeconomic perspective guarantees that the market may reach equilibrium and maintain long-term health by implying that, although financial immunity is essential for individual actors, market stability as a whole depends on collaboration.

Cooperation is a crucial factor in shaping policies within the cryptocurrency market platforms for long-term sustainability. International cooperation among regulatory bodies is essential to formulate comprehensive policies that effectively tackle global financial challenges. By fostering collaborative behavior among stakeholders, the financial system can enhance its resilience to shocks and better navigate the complexities inherent in the global cryptocurrency economy. Policy implications in the crypto market platforms extend beyond regulatory frameworks. Updating accounting guidance on cryptocurrencies can contribute to a more robust market by providing informative approaches to measurement and reporting (see Hubbard, 2023). Additionally, assessing "crypto-friendliness" in policymaking can evaluate the extent to which policies accommodate blockchain development, influencing the overall ecosystem (cfr. Novak (2019)). In conclusion, policy implications in the crypto market platforms underscore the importance of cooperation, regulatory updates, environmental considerations, risk management avenues, and sustainable practices. Aligning policies with these key aspects can help policymakers foster a more resilient, transparent, and sustainable cryptocurrency ecosystem.

Funding Open access funding provided by Università degli Studi di Ferrara within the CRUI-CARE Agreement.

Declarations

Conflict of interest The authors have no conflict of interest to declare that are relevant to the content of this article.

References

Abbasi, G., Tiew, L., Jin-quan, T., Goh, Y., & Ramayah, T. (2021). The adoption of cryptocurrency as a disruptive force: Deep learning-based dual stage structural equation modelling and artificial neural network analysis. *Plos One, 16*(3), e0247582.

Aliano, M., Cananà, L., Ciano, T., Ragni, S., & Ferrara, M. (2024). On the dynamics of a SIR model for a financial risk contagion. *Quality & Quantity*. https://doi.org/10.1007/s11135-024-02009-2

Aramonte, S., Huang, W., & Schrimpf, A. (2021). DeFi risks and the decentralisation illusion. *BIS Quarterly Review*.

Báez-Sánchez, A. D. (2018). A mathematical model for behavioral epidemiology: A numerical approach. *Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, 6*(1).

Bains, P., Ismail, A.F.M., & Sugimoto, N. (2022). Regulating the crypto ecosystem: The case of unbacked crypto assets. FinTech Notes No 2022/007, International Monetary Fund.

Barbereau, T., Smethurst, R., Papageorgiou, O., Rieger, A., & Fridgen, G. (2022). DeFi, not so decentralized: The measured distribution of voting rights. In *Conference: Hawaii International Conference on System Sciences*.

Bogdan, S. (2023). Impact of liquidity and investors sentiment on herd behavior in cryptocurrency market. *International Journal of Financial Studies, 11*(3), 1–17.

Bouri, E., Gupta, R., & Roubaud, D. (2019). Herding behaviour in cryptocurrencies. *Finance Research Letters, 29*, 216–221.

Cao, H. H., & Zhu, J. M. (2012). Research on banking crisis contagion dynamics based on the complex network of system engineering. *Systems Engineering Procedia, 5*, 156–161.

Diebold, F. X., & Yilmaz, K. (2012). Better to give than to receive: Predictive directional measurement of volatility spillovers. *International Journal of Forecasting, 28*, 5–66.

Diebold, F. X., & Yilmaz, K. (2014). On the network topology of variance decompositions: Measuring the connectedness of financial firms. *Journal of Econometrics, 182*, 119–134.

Diebold, F. X., & Yilmaz, K. (2016). Trans-Atlantic equity volatility connectedness: U.S. and European financial institutions. *Journal of Financial Economics, 14*, 81–127.

Dumitrescu, A., & Zakriya, M. (2022). Governance, information flow, and stock returns. *Journal of Corporate Finance, 72*, 102168.

Dutta, A., & Bouri, E. (2022). Outliers and time-varying jumps in the cryptocurrency markets. *Journal of Risk and Financial Management, 15*(3), 128.

Fanelli, V., & Maddalena, L. (2020). A nonlinear dynamic model for credit risk contagion. *Mathematics and Computers in Simulation, 170*, 45–58.

Garas, A., Argyrakis, P., Rozenblat, C., Tomassini, M., & Havlin, S. (2010). Worldwide spreading of economic crisis. *New Journal of Physics, 12*(11), 30–43.

Habib, A., Hasan, M. M., & Jiang, H. (2018). Stock price crash risk: Review of the empirical literature. *Accounting & Finance, 58*, 211–251.

Haldane, A. G., & May, R. M. (2011). Systemic risk in banking ecosystems. *Nature, 469*, 351–355.

Hale, J. (1977). *Theory of functional differential equations*. Springer-Verlag.

Hofbauer, J., & Sigmund, K. (2003). Evolutionary game dynamics. *Bulletin of the American Mathematical Society, 40*, 479–519.

Howson, P. (2020). Climate crises and crypto-colonialism: Conjuring value on the blockchain frontiers of the global south. *Frontiers in Blockchain, 3*.

Hubbard, B. (2023). Decrypting crypto: Implications of potential financial accounting treatments of cryptocurrency. *Accounting Research Journal, 36*(4/5), 369–383.

Hutton, A. P., Marcus, A., & Tehranian, H. (2009). Opaque financial reports, R2, and crash risk. *Journal of Financial Economics, 94*(1), 67–86.

Kendall, D. G. (1956). Deterministic and stochastic epidemics in closed populations. In *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability: Contributions to Biology and Problems of Health* (Vol. 4, pp. 149–165).

Kermack, W. O., & McKendrick, A. G. (1927). A Contribution to the Mathematical Theory of Epidemics. In *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character* (Vol. 115, No. 772, pp. 700–721).

King, T., & Koutmos, D. (2021). Herding and feedback trading in cryptocurrency markets. *Annals of Operations Research, 300*(1), 79–96.
Kyrychko, Y. N., & Blyuss, K. B. (2005). Global properties of a delayed SIR model with temporary immunity and nonlinear incidence rate. *Nonlinear Analysis: Real World Applications, 6*(3), 495–507.
Madeo, D., & Mocenni, C. (2021). Evolutionary game theoretic insights on the SIRS model of the COVID-19 pandemic. *IFAC-PapersOnLine, 54*(17), 1–6.
Madeo, D., & Mocenni, C. (2022). Identification and control of game-based epidemic models. *Games, 13*(1), 10.
Novak, M. (2019). Crypto-friendliness: Understanding blockchain public policy. *Journal of Entrepreneurship and Public Policy, 9*(2), 165–184.
Prasetyo, T., & Kurniasari, F. (2023). The influence of subjective norms, financial literacy, trust, and government regulation on behavioral intention to invest in crypto assets. *Economics and Finance, 11*, 16–34.
Renduchintala, T., Alfauri, H., Yang, Z., Pietro, R., & Jain, R. (2022). A survey of blockchain applications in the fintech sector. *Journal of Open Innovation Technology Market and Complexity, 8*(4), 185.
Roukny, T., Bersini, H., Pirotte, H., Caldarelli, G., & Battiston, S. (2013). Default cascades in complex networks: Topology and systemic risk. *Scientific Reports, 3*, 2759.
Samueal, S. (2023). Blockchain in banking: A study on central bank digital currency. *Multidisciplinary Science Journal, 5*, 2023ss0302.
Washington, P. (2023). Analyzing influence of COVID-19 on crypto & financial markets and sentiment analysis using deep ensemble model. *Plos One, 18*(9), e0286541.
Waspada, I., Salim, D., & Krisnawati, A. (2022). Horizon of cryptocurrency before vs during COVID-19. *Investment Management and Financial Innovations, 20*(1), 14–25.
Weibull, J. (1995). *Evolutionary game theory*. MIT Press.
Wettergren, T. A. (2023). Replicator dynamics of evolutionary games with different delays on costs and benefits. *Applied Mathematics and Computation, 458*, 128228.
Wronka, C. (2024). Crypto-asset regulatory landscape: A comparative analysis of the crypto-asset regulation in the UK and Germany. *Journal of Asset Management*.
Zhao, C., Li, M., Wang, J., & Ma, S. (2021). The mechanism of credit risk contagion among internet P2P lending platforms based on a SEIR model with time-lag. *Research in International Business and Finance, 57*, 101407.