



# Proceedings of the fourteenth Congress of the European Society for Research in Mathematics Education

**Editors:** Marianna Bosch Casabò, Susana Carreira, Giorgio Bolondi, Michael Gaidoschik and Camilla Spagnolo  
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| ... <i>Mickael Da Ronch</i>                                                                                                                             | 1742 |
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| ... <i>Valérie Batteau, Jana Trgalová and Sébastien Jolivet</i> . . . . .                                                     | 1886 |
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| ... <i>Bernadett Szakácsné Győrey and Katalin Gosztonyi</i> . . . . .                                                         | 1888 |
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| Mathematics education in Belgium inspired by Decroly's pedagogy                                                                          |      |
| ... <i>Dirk De Bock and Wendy Goemans</i> . . . . .                                                                                      | 1899 |
| Papy's minicomputer                                                                                                                      |      |
| ... <i>Wendy Goemans and Dirk De Bock</i> . . . . .                                                                                      | 1907 |
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| ... <i>Maria Célia Leme da Silva, Ana Paula Jahn, Marizete Nink Carvalho and Thiago Pedro Pinto</i> . . . .                              | 1915 |
| Mathematics for all, but what mathematics? Comparing the overlap of three coexisting curricula in the mid-20 <sup>th</sup> century Spain |      |
| ... <i>Antonio M. Oller-Marcén</i> . . . . .                                                                                             | 1923 |
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| ... <i>Hilde Opsal and Bjørn Smestad</i> . . . . .                                                                                       | 1931 |
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| ... <i>José Gerardo Piña-Aguirre and Rosa María Farfán Márquez</i> . . . . .                                                             | 1939 |
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| ... <i>Barbara Schmidt-Thieme and Tanja Hamann</i> . . . . .                                                                             | 1955 |
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| . . . <i>Christiane Benz, Axel Schulz and Sebastian Wartha</i> . . . . .                                                                                                 | 2000 |
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| . . . <i>Camilla Björklund and Hanna Palmér</i> . . . . .                                                                                                                | 2008 |
| Collaborating on researching mathematical awareness in kindergarten class                                                                                                |      |
| . . . <i>Mette Amalie Bundgaard</i> . . . . .                                                                                                                            | 2016 |
| Individual Differences in Parental Support for Home-Numeracy in Pre-schoolers with Intellectual Disabilities: A Cluster Analytic Approach                                |      |
| . . . <i>Garyfalia Charitaki, Anastasia Alevriadou and Maria Kypriotaki</i> . . . . .                                                                                    | 2024 |
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| . . . <i>Jessica Elofsson, Camilla Björklund and Joanne Mulligan</i> . . . . .                                                                                           | 2048 |
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| . . . <i>Ineta Helmane, Malgożata Raščevska, Dita Nīmanīte, Solvita Umbrāško, Andra Vabale, Ieva Bite, Baiba Trinīte, Gundega Tomele and Egija Laganovska,</i> . . . . . | 2064 |
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# Pre-service teachers' professional development and mathematics large-scale assessment

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*This research aims at investigating the knowledge and beliefs of a sample of Italian secondary school mathematics pre-service teachers (PSTs) regarding the INVALSI national large-scale tests. The units of analysis are the PSTs' answers to an online questionnaire, in which some items taken from the past INVALSI national tests are proposed and discussed individually by the participants, and the collected data are framed with the Mathematics Teacher Specialised Knowledge (MTSK) model (Carrillo-Yañez et al., 2018). The main purpose of the research is to identify the educational needs of the study sample, to then use them as a starting point for the construction of a model for teachers' professional development based on an innovative bottom-up approach. Results show evidence of beliefs and many different aspects of PSTs' knowledge, categorised in the Pedagogical Content Knowledge (PCK) domain of the MTSK model.*

*Keywords: Pre-service teacher education, Large-Scale Assessment (LSA), MTSK model.*

## Rationale

A new field of research has been emerging in recent years, aimed at investigating how to integrate results, methods, theoretical frameworks and findings from Large-Scale Assessment (LSA) into the actions of teachers, teacher educators and researchers, both in the school and classroom context and in the broader one of educational institutions (De Lange, 2007; Meinck et al., 2017). My research belongs to this framework and fits within the strand of research linking LSA with teacher professional development (Campbell & Levin, 2009, Ferretti & Santi, 2023). Several studies in this area have been carried out from macro-phenomena that emerged at the Italian national level about INVALSI LSA (Ferretti et al., 2022); starting from what is already known, the focus of my research is to investigate Italian pre-service teachers' (PSTs) knowledge and beliefs regarding national LSA to identify their educational needs, inspired by studies already conducted in teacher education contexts (Montes et al., 2019), with a view to contributing to the development of a new model of teacher education. My study is, indeed, part of a larger project, the Italian National Research Project “*Mathematics standardized assessment as a tool for teachers' professional development*”, which involves several researchers from different Italian Universities and whose aim is to create a model for teachers' professional development, consisting of theoretical and operational tools, starting from the results collected from the Italian national assessment tests INVALSI. The INVALSI tests are administered nationally by the National Institute for the Evaluation of the Education and Training System; the administration involves students at all school levels (primary school grades 2 and 5, low secondary school grade 8, and high secondary school grades 10 and 13). The framework within which they are developed is consistent with the Italian Ministerial Curricular Guidelines and with the main results of research in mathematics education. All the information and data collected from the INVALSI tests are disseminated and made public on a regular basis and each school receives its own data processed and compared with different benchmarks. Having such information available, it is possible to use the data

provided by INVALSI as interpretative tools for some aspects of the mathematics teaching-learning processes specific to the Italian context, also and above all with a view to teacher professional development. In contrast to the traditional (top-down) impact of LSA, the innovative idea of the project in which my research is framed is to provide these instruments for a bottom-up approach, in which the results are used directly by pre- and in-service teachers, also with a view to integrate data from LSA with a formative perspective.

Thus, the research question this study aims to address is: *What are the knowledge and beliefs of pre-service Italian mathematics teachers regarding the national INVALSI standardized assessment tests?*

### **Theoretical framework**

The main purpose of the Italian National Research Project to which my study belongs is creating a model for the teachers' professional development, inspiring the one outlined in Ferretti et al. (2020). The model aims to enhance specialized knowledge for teaching mathematics. To ensure an informed and conscious construction of the model and to meet the real training needs, a questionnaire was created. The design of the questionnaire and the analysis of the data are framed within the Mathematics Teacher Specialized Knowledge (MTSK) model (Carrillo-Yañez et al., 2018). In the MTSK model, mathematics teacher's knowledge consists of the two domains of Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK), and the two areas are interrelated by the teachers' beliefs, both about mathematics and its teaching-learning processes. The main aim of the research phase addressed in this paper is investigating the knowledge and beliefs of Italian teachers regarding the mathematics teaching and learning process, rather than focusing strictly on the mathematical knowledge, so the data analyzed in this study are mainly framed within the subdomains of PCK. They are: *Knowledge of Mathematics Teaching - KMT* (concerning theoretical knowledge of mathematics teaching, i.e. strategies, resources, teaching theories, etc.); *Knowledge of Features of Learning Mathematics - KFLM* (knowing, in general, how students interact with mathematics while also considering affective aspects); *Knowledge of Mathematics Learning Standards - KMLS* (knowledge of the national indications for curricula, for any school grade).

### **Methods**

The research tool used in the first phase of the study is a pilot questionnaire, declined into different versions containing similar questions but formulated according to the group of participants involved (pre- and in-service teachers) and the school level in which they are teaching/will be teaching (focusing on grade 5, 8 and 10). It contains questions on several INVALSI mathematics items, taken from the GESTINV database ([www.gestinv.it](http://www.gestinv.it)) and chosen because they are indicative of national educational macro-phenomena, namely items where the percentage of correct answers is very low and the distractor choices can be framed within well-known constructs in mathematics education. There are also questions that can be defined as “transversal”, aimed specifically at investigating the participants' perceived purpose and usefulness with respect to INVALSI tests and the impact of these tests on their teaching practice (or future teaching practice). The questionnaire was administered online by using a Google Form: in this way, it was possible to reach a larger and more varied sample of pre- and in-service teachers; regarding the PSTs, this was also important because it allowed us to reach participants from different universities, that were not influenced by the same mathematics

didactics courses attended. Moreover, the choice of this tool was also useful in the data analysis phase; in fact, in addition to the open questions, participants were also asked several closed-ended questions on a Likert scale: using the form, it is possible to have an immediate analysis of the participants' response rates.

Table 1 is an example of a part of the questionnaire for secondary school PSTs, whose responses will be discussed in the following sections.

**Table 1: Question posed to PSTs and related to the item 15 of the 2011 INVALSI test, grade 10, proposed in the questionnaire**

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Consider the following INVALSI item proposed to the secondary classes of the Secondary School.</p> <p><b>D15. Dividere un numero per 0,2 è lo stesso che moltiplicarlo per</b></p> <p><input type="checkbox"/> A. <math>\frac{1}{5}</math></p> <p><input type="checkbox"/> B. <math>\frac{1}{2}</math></p> <p><input type="checkbox"/> C. 2</p> <p><input type="checkbox"/> D. 5</p> <p>Translation (by the author): “Dividing a number by 0.2 is the same as multiplying it by...”</p> |
| 1. In your opinion, what does this item predominantly evaluate?                                                                                                                                                                                                                                                                                                                                                                                                                            |
| 2. The correct option is option D. What do you think has been the most popular wrong option nationwide? Why?                                                                                                                                                                                                                                                                                                                                                                               |
| 3. On a scale of 1 to 4, how consistent do you think the item is with the National Curricular Guidelines for High Schools and the Guidelines for Technical and Professional Institutes?<br>(1 = not at all, 4 = completely)                                                                                                                                                                                                                                                                |
| 4. Imagine you are teaching in a grade 10 class. On a scale of 1 to 10, how difficult do you think this item is for students?<br>(1 = very easy, 10 = very difficult)                                                                                                                                                                                                                                                                                                                      |
| 5. Imagine you are teaching in a grade 10 class. On a scale of 1 to 4, how influential do you think the wording of this item is in determining the difficulty of the item for students? Motivate your answer.<br>(1 = not at all, 4 = completely)                                                                                                                                                                                                                                          |
| 6. Imagine you are teaching in a grade 10 class. On a scale of 1 to 4, how appropriate do you think the item is for assessing student learning?<br>(1 = not at all, 4 = completely)                                                                                                                                                                                                                                                                                                        |

7. Would you use this item in the classroom? If yes, why and how? If not, why?

INVALSI item 15 was chosen because close to the Italian school curriculum: as stated in the National Guidelines, among the specific learning objectives, it is mentioned that “The student will develop their skills in calculation [...] with rational numbers both in fraction form and in decimal representation” (MIUR, 2010). Hence, the theoretical knowledge required for the correct resolution of this item involves knowledge of the concept of rational numbers and the required competence is about grasping and managing the various semiotic representation of rational numbers (Duval, 2006). In addition, it is a closed-answer item and, thanks to the GESTINV database, national percentages are available and these show that the item represents a macro phenomenon at national level (as described in the previous paragraph). The percentage of correct answers is very low (only 24.4% of students choose the correct option D) and the most chosen distractor is option A, with 46.1%, followed by option B with 20.2%. It can be interpreted that the students who answered A only focused on the numerical part of the problem and thus thought they only had to carry a treatment (in the sense of Duval) between the decimal representation of the number and its representation as a fraction of integers, without considering the operation verbally requested in the assignment. Those who have chosen option B also acted in the same way, but they also made a mistake in treatment.

Questions asked to PSTs allow us to collect information about the PCK sub-domains of MTSK model. Question 1 is aimed at revealing PSTs’ cognition about the mathematical knowledge and competence that a student must have acquired to answer the item correctly; thus, highlighting if and what aspects of the KMT emerge from the sample. The purpose of question 2, on the other hand, is to explore which, if any, didactic constructs are used to interpret possible mistakes made by students and, in this case, we are within the KMT subdomain. Question 3 investigates another subdomain of PCK, i.e. KMLS; PSTs are asked whether the item is consistent with the National Guidelines or not, so we want to find out if they are informed about the national curricula. Questions 4 and 5 are still intended to bring out different aspects of KFLM; in particular, by asking for reasons about the choice of the score given on the Likert scale in question 5, it is also highlighted any beliefs of the PSTs concerning the students’ learning processes. The last two questions are aimed at investigating what other aspects of the KMLS arise and, at the same time, what are the PSTs’ beliefs about assessment and about their possible future teaching practices.

The teachers’ answers were analysed differently depending on the type of request. Regarding the data collected from the open-ended questions, such as question 1, the units of analysis were divided into categories, to distinguish the different argument given by the PSTs to justify their answers. This approach allowed for an investigation of the aspects that actually emerged in relation to the considered knowledge domain. On the other hand, in the case of closed-ended questions with a Likert scale, such as question 3, the percentages collected and processed directly from the Forms platform were examined to deduce information regarding the knowledge domain of reference.

A total of 20 PSTs (all students enrolled in the master’s degree program in Mathematics) completed the questionnaire for high secondary school and their answers are analysed in the next section.

## Data collection and analysis

The answers to question 1 can be grouped into different categories depending on students' mathematical competences and knowledge PSTs consider necessary to be able to solve the problem. Most of the participants state that several skills (e.g. being able to carry out the treatment between the decimal and fractional representation of the number) and several knowledge (e.g. being able to carry out operations with rational numbers, with particular reference to division) are required. A few claim that the question mainly assesses only the ability to perform operations with rational numbers, while others identify the criticality only in the management of representations. Responses of this type can reveal an awareness of the theory of semiotic representation registers (Duval, 2006), even though no participant explicitly mentions it in their answers. In addition, consciousness of the intuitive “partitive division” model, which leads students to think that the quotient must be smaller than the dividend (Fischbein et al., 1985), emerges in several answers. An example is the following extract from a PST: “This item aims to make the student realize that a quotient is not necessarily less than the dividend, as one is used to with, for example, divisions between natural numbers.” Even in these cases, however, there is no explicit reference to the didactic construct. It can be therefore identified in this data, as assumed a priori, some elements that can be framed into the KMT sub-domain of PCK. It comprehends knowledge (both personal and institutional) specific to the process of teaching mathematics, which stems primarily from research in mathematics education; these include aforementioned constructs implicitly highlighted in the PSTs' answers.

Regarding question 2, it is immediately noticeable that the majority of respondents (70%) state that the most chosen wrong answer among students is exactly the one chosen by the national sample (option A, selected by 46.1% of the students). The reasons given are various, but the main ones can be categorized into (in brackets, the number of PSTs who gave that motivation):

- Inattentive reading/distraction mistakes (2)
- Problems with the wording of the text: the treatment takes place between the two representations, but the operation is not completed because the words related to the division and multiplication operations are written in letters, while the students “give priority” only to the part of request where the numerical data appears (3)
- Misconception linked to the multiplication that increases (2)
- Difficulties in performing the operation correctly (6)

It is observed that, although most of the answers refer to mistakes strictly related to the procedure, other factors are linked to students' erroneous conceptions and/or the wording of the text. In the first case, we can frame those responses as evidence of the KMT subdomain within the MTSK model, because (as in the case of the previous question) the PSTs do not explicitly mention the mathematics teaching theories they are referring to, but they still show that they are aware of criticalities expressed by them (for instance, when they talk about “misconceptions”, in the sense of Fischbein). On the other hand, assertions regarding the “problematic” formulation of the text can be framed within the KFLM subdomain. This type of knowledge, indeed, includes the ability to recognize which features of a specific content may be advantageous or disadvantageous to learning. From the PSTs', it emerges that the item's formulation is perceived as unfavourable for students' understanding of the task. There

is also a small percentage of PSTs (15%) who claims that the most chosen distractor is B, since when computing between decimal and fraction there is confusion between the numerical parts of the two representations (2 is present in both “0.2” and “1/2”). These responses can also be classified as KFLM due to the fact that they also refer explicitly to where students have difficulties and how they might act to solve the item, considering the possible obstacles.

Almost all the PSTs (95%) consider the item consistent with the National Curricular Guidelines for High Schools and the Guidelines for Technical and Professional Institutes; 25% of these consider themselves “in agreement” and 70% “very in agreement”. This is evidence of a highly developed knowledge within the KMLS domain, which includes knowledge of what students should be able to know and do upon reaching a specific grade level, in accordance with the curriculum established in the given context. In our case, the PSTs show to know what the learning goals of grade 10 are.

Furthermore, the item is generally perceived as difficult for students: that 70% of PSTs choose 6 or more on the Likert scale. Additionally, 65% of them answer question 5 saying that the item’s wording may have had a significant effect on its difficulty. The main reason given is that the words “dividing” and “multiplying” express two opposite concepts but are present in the same prompt, thus confusing the students. Compared to what was previously analyzed in the responses to question 2, this finding seems inconsistent. In fact, initially, few PSTs considered the wording of the text as a possible cause of mistake (only 3 among those who identified the most chosen distractor at the national level, and none of the others). After reflecting on question 5, however, the percentage of participants who contemplate it as a factor of difficulty, and consequently the likelihood of making mistakes, rose significantly. These responses can also be framed within the KFLM subdomain, as the PSTs demonstrated an awareness of how students might approach an item based on its phrasing, and how this factor could influence their learning process.

In addition, 80% of PSTs consider the item suitable for evaluating students’ learning; this is probably because, being considered very consistent with the indications given at national level for school curricula, it can be considered a useful tool for confirming or not the achievement of the learning purposes foreseen by them. There is hence a sort of belief in the PSTs, which can be expressed as: “if a mathematical question/problem is consistent with national curricula then it can be useful in terms of assessment, regardless of its intrinsic characteristics” (which, in this case, are considered problematic for students by PSTs themselves, as seen in the previous answers). In agreement with this last observation, almost the same 80% says that they would use that item in the classroom: of those who would not use it, 3 out of 4 are those who did not consider it suitable for assessment, and therefore remain consistent with their idea. The main reasons listed for using the item in class are (in brackets, the number of PSTs who gave that motivation):

- overcoming possible misconceptions (3)
- identifying and correcting any lack and/or consolidating previous knowledge (5)
- reflecting on the reasoning needed for the solution (4)
- mastering the management of a number’s representations (2).

Most PSTs would let students work individually on the item and then engage them in a collective discussion; alternatively, it would be used to design laboratory activities and/or group work. Others

would also set up an individual class activity on it and/or with the use of technology. Those who claim that they would not use the item during their future teaching practices, on the other hand, express as their main motivation its formulation, which makes it confusing for the students, and unsuitable for evaluating their learning processes. For example, a PST stated: “*I don’t like to formulate questions that have tricky discursive texts, like double negations or things like that. That is, if I want to test whether they can translate a fraction into decimal, I ask only that; if instead I want to test if they can invert a fraction, I give them a numerically formulated text.*”. Since the respondents to our questionnaire are PSTs who have mostly never had any class experience, their answer to these last questions about classroom practices can be categorised as beliefs they have on them.

## Discussion and conclusion

The INVALSI tests can be a key tool for investigating the knowledge and beliefs of mathematics teachers: having the data from the national sample available, it is possible to use them to induce pre- and in-service teachers to think about some crucial aspects related to LSA resources. This is what was done in this research, in which findings suggest that a large part of the knowledge that emerged can be framed in the three subdomains of the PCK described by Carillo-Yanez et al. (2018), as hypothesized during the design phase of the questionnaire. At the same time, several beliefs about students' behaviour during the performance of mathematical tasks (e.g. inattention) and good teaching practices that a teacher should adopt during assessment (e.g. being as explicit as possible about the proposed requests) also arise.

In these terms, it is possible to identify if and what strengths and weaknesses stand out from the PSTs' answers, which can then be used as a starting point for the implementation of the teacher training programme addressed by the Italian National Research Project. For example, from the analysis conducted, it can be observed how important the wording of a mathematical question is considered (implicitly or not) for its correct understanding and performance by the students. Initially, few PSTs identified this as the main cause of students' mistakes, but when their attention was directed to the explicit question 5, the percentage of them claiming that the wording is deliberately misleading and/or can create confusion increased significantly. This must be considered during the design of the training model, in the sense that it would be appropriate to carry out activities aimed at enhancing this awareness, so that it develops as an established future teaching practice of mathematics teachers.

In future studies the intention is also to make a comparison between the responses of PSTs and in-service teachers to the same questionnaire. The aim is to analyse any similarities and/or differences: firstly, to see if and how the classroom experience can influence the development of the different knowledge areas of the MTSK model and, secondly, to identify further educational needs to be met by the teacher professional development model.

## References

- Campbell, C., & Levin, B. (2009). Using data to support educational improvement. *Educational Assessment, Evaluation and Accountability (formerly: Journal of Personnel Evaluation in Education)*, 21, 47–65. <https://doi.org/10.1007/s11092-008-9063-x>

- Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Nielka, R., Pablo, F., Álvaro, A. G., Ribeiro, M., Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education*, 20(3), 236–253. <https://doi.org/10.1080/14794802.2018.1479981>
- De Lange, J. (2007). Large-scale assessment and mathematics education. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning*, 2, 1111–1144. National Council of Teachers of Mathematics.
- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational studies in mathematics*, 61(1), 103–131. <https://doi.org/10.1007/s10649-006-0400-z>
- Ferretti, F., Gambini, A., & Santi, G. (2020). The Gestinv Database: A Tool for Enhancing Teachers Professional Development within a Community of Inquiry. In H. Borko and D. Potari (Eds.), *Proceedings of the Twenty-fifth ICMI Study School Teachers of mathematics working and learning in collaborative groups* (pp.621–628). ICMI.
- Ferretti, F. & Santi, G. (2023). Pre-service teachers' subjectification: a dialogue between LSA and mathematics education. *For the learning of Mathematics*, 43(2), 19–21.
- Ferretti, F., Santi, G. & Bolondi, G. (2022). Interpreting difficulties in the learning of algebraic inequalities, as an emerging macro-phenomenon in Large Scale Assessment. *Research in Mathematics Education*, 24(3), 367–389. <https://doi.org/10.1080/14794802.2021.2010236>
- Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for research in mathematics education*, 16(1), 3–17. <https://doi.org/10.5951/jresmetheduc.16.1.0003>
- Gestinv 3.0. Archivio interattivo delle prove INVALSI [Interactive archive of INVALSI tests]. <http://www.gestinv.it>
- Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of mathematics teacher education*, 9(2), 187–211. <https://doi.org/10.1007/s10857-005-1223-z>
- Meinck, S., Neuschmidt, O., & Taneva, M. (2017). Workshop Theme: “Use of Educational Large-Scale Assessment Data for Research on Mathematics Didactics”. In Kaiser, G. (eds) *Proceedings of the 13th International Congress on Mathematical Education*. ICME-13 Monographs. Springer, Cham. [https://doi.org/10.1007/978-3-319-62597-3\\_132](https://doi.org/10.1007/978-3-319-62597-3_132)
- Montes, M., Carrillo, J., Contreras, L. C., Liñán García, M. D. M., & Barrera Castarnado, V. J. (2019). Estructurando la formación inicial de profesores de matemáticas: Una propuesta desde el modelo MTSK [Structuring mathematics teacher education: A proposal framed by the MTSK model]. *Investigación sobre el profesor de matemáticas: Práctica de aula, conocimiento, competencia y desarrollo profesional*, 157–176. Ediciones Universidad Salamanca.
- MIUR (2010). National Curricular Guidelines for High Schools. [https://www.indire.it/lucabas/lkmw\\_file/licei2010/indicazioni\\_nuovo\\_impaginato/decreto\\_indicazioni\\_nazionali.pdf](https://www.indire.it/lucabas/lkmw_file/licei2010/indicazioni_nuovo_impaginato/decreto_indicazioni_nazionali.pdf)