

NOTES ON A PAPER OF POKHOZHAEV

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ABSTRACT. We prove a second order identity for the Kirchhoff equation which yields, in particular, a simple and direct proof of Pokhozhaev's second order conservation law when the nonlinearity has the special form $(C_1s + C_2)^{-2}$. As applications, we give: an estimate of order ε^{-4} for the lifespan T_ε of the solution of the Cauchy problem with initial data of size ε in Sobolev spaces when the nonlinearity is given by any C^2 function $m(s) > 0$; a necessary and sufficient condition for boundedness of a second order energy of the solutions.

1. Introduction

As it was proved by S.I. Pokhozhaev ([P1], [P2]), if the nonlinearity has the form

$$a(s) = \frac{1}{(C_1s + C_2)^2} \quad (C_1, C_2 \in \mathbb{R} \text{ not both zero})$$

and if u is a sufficiently regular solution in $\Omega \times [0, T)$ ($\Omega \subset \mathbb{R}^n$ a bounded, C^2 domain; $T > 0$) of the Kirchhoff equation

$$(1.1) \quad u_{tt} - a \left(\int_{\Omega} |\nabla u|^2 dx \right) \Delta u = 0, \quad u = 0 \text{ on } \partial\Omega \times [0, T),$$

then the second order functional

$$(1.2) \quad Iu(t) := (C_1s + C_2) \int_{\Omega} |\nabla u_t|^2 dx + \frac{1}{C_1s + C_2} \int_{\Omega} |\Delta u|^2 dx - C_1 \left(\int_{\Omega} \nabla u \cdot \nabla u_t dx \right)^2,$$

with

$$s = s(t) = \int_{\Omega} |\nabla u(x, t)|^2 dx,$$

remains constant in $[0, T)$.¹ See [P2, formula (2.3)].

This is the so-called Pokhozhaev's second order conservation law, associated with (1.1), whose proof was only sketched in [P2], referring to [P1] (in Russian) for more details. See also [MP] for a simpler explanation and [PA] for some applications to the global existence of low regularity solutions of (1.1). We show here that this conservation law is a consequence of a second order identity, formula (2.5) below, which holds for every C^2 nonlinearity m , as long as $m(s) > 0$. Despite its elementary nature, this identity does not seem to be well known. We also give two applications: (1) an estimate of order ε^{-4} for the life span T_ε of the solution of the Cauchy problem with initial data of size ε in Sobolev spaces, for every C^2 function $m(s) > 0$, extending the result of [BH1] where $m(s) = 1 + s$ and $\Omega = \mathbb{T}^n$. See also [BH2] where an estimate of order ε^{-6} for T_ε is given, but assuming in addition a rather strong condition on the initial data; (2) a necessary

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¹ It is clearly understood that we are assuming $C_1s(t) + C_2 \neq 0$ in $[0, T)$.

and sufficient condition for boundedness (or possible blow-up) of the second order energy $E(t)$; see (2.4) and Theorem 5.1 below.

2. A second order identity for general nonlinearity

For simplicity from now on we will assume $\Omega = \mathbb{R}^n$. If $\Omega \subset \mathbb{R}^n$ is a bounded, C^2 domain or if Ω is the n -dimensional torus \mathbb{T}^n , our arguments still work by adopting the usual boundary conditions.

So let us suppose $\Omega = \mathbb{R}^n$ and

$$(2.1) \quad u \in C^k([0, T]; H^{2-k}(\mathbb{R}^n)), \quad k = 0, 1, 2,$$

a given solution in $\mathbb{R}^n \times [0, T)$, $T > 0$, of the equation

$$(2.2) \quad u_{tt} - m \left(\int |\nabla u|^2 dx \right) \Delta u = 0,$$

with $m : J \rightarrow \mathbb{R}$ ($J \subset \mathbb{R}$ open interval) such that

$$(2.3) \quad m \in C^2(J), \quad m(s) > 0 \quad \text{for } s \in J, \quad \int |\nabla u|^2 dx \in J \quad \text{for } t \in [0, T).$$

Taking into account (2.1) and (2.3), we may define the *energy*

$$(2.4) \quad E(t) := \frac{1}{\sqrt{m}} \int |\nabla u_t|^2 dx + \sqrt{m} \int |\Delta u|^2 dx,$$

with $m = m(s)$ and

$$s = s(t) := \int |\nabla u|^2 dx \quad \text{for } t \in [0, T).$$

2.1. Second order identity. Under the previous assumptions, we can establish the following:

$$(2.5) \quad \frac{d}{dt} \left[E(t) - \frac{1}{4} \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) s'(t)^2 \right] = - \frac{1}{4} \frac{d^2}{ds^2} \left(\frac{1}{\sqrt{m}} \right) s'(t)^3,$$

where

$$\frac{d^k}{ds^k} \left(\frac{1}{\sqrt{m}} \right) = \frac{d^k}{ds^k} \left(\frac{1}{\sqrt{m}} \right) (s) \quad \text{for } k = 1, 2.$$

Proof. From the assumption (2.1) it easily follows that s is a C^2 function in $[0, T)$. In particular,

$$(2.6) \quad s'(t) = 2 \int \nabla u \cdot \nabla u_t dx$$

and, by (2.2),

$$(2.7) \quad s''(t) = 2 \int |\nabla u_t|^2 dx - 2m \int |\Delta u|^2 dx.$$

Now, we merely need to derive $E(t)$. The following calculation makes sense for a sufficiently regular solution, but it can be justified by density arguments (see, for instance, [Y, p. 23]). We find:

$$\begin{aligned} E'(t) &= \left(\frac{1}{\sqrt{m}} \right)' \int |\nabla u_t|^2 dx + (\sqrt{m})' \int |\Delta u|^2 dx + \frac{2}{\sqrt{m}} \int [\nabla u_t \nabla u_{tt} + m \Delta u \Delta u_t] dx \\ &= \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) s'(t) \int |\nabla u_t|^2 dx + \frac{d}{ds} (\sqrt{m}) s'(t) \int |\Delta u|^2 dx - \frac{2}{\sqrt{m}} \int [u_{tt} - m \Delta u] \Delta u_t dx \\ (2.8) \quad &= \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) s'(t) \left[\int |\nabla u_t|^2 dx - m \int |\Delta u|^2 dx \right], \end{aligned}$$

which immediately says that $E(t)$ is a C^1 function in $[0, T)$. Taking into account (2.7), we have

$$E'(t) = \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) s'(t) \frac{s''(t)}{2} = \frac{1}{4} \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) \frac{d}{dt} s'(t)^2,$$

and then the identity (2.5). \square

3. A simple proof of Pokhozhaev's second order conservation law

We first note that if $m \in C^2$ and $m(s) > 0$ in the open interval $J \subset \mathbb{R}$, then

$$\frac{d^2}{ds^2} \left(\frac{1}{\sqrt{m}} \right) \equiv 0 \text{ in } J \quad \Leftrightarrow \quad \frac{1}{\sqrt{m}} = C_1 s + C_2 \text{ in } J,$$

with $C_1, C_2 \in \mathbb{R}$ such that $C_1 s + C_2 > 0$ for $s \in J$. This means, in particular, that (2.5) gives

$$(3.1) \quad E(t) - \frac{1}{4} \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) s'(t)^2 = \text{const.} \quad \forall t \in [0, T),$$

if m has the special form $m(s) = (C_1 s + C_2)^{-2}$.

Then, writing explicitly (3.1) for this particular nonlinearity, we immediately obtain *Pokhozhaev's second order conservation law*, i.e.,

$$Iu(t) = \text{const.} \quad \forall t \in [0, T),$$

where Iu is the second order functional (1.2), with $\Omega = \mathbb{R}^n$.

4. An estimate of the life-span for small initial data

We consider now the Cauchy problem with initial data of size ε . That is,

$$(4.1) \quad \begin{cases} u_{tt} - m \left(\int |\nabla u|^2 dx \right) \Delta u = 0, & (x, t) \in \mathbb{R}^n \times \mathbb{R}_t^+ \\ u(x, 0) = \varepsilon u_0(x), \quad u_t(x, 0) = \varepsilon u_1(x) \end{cases}$$

for fixed u_0, u_1 (not both zero) and $\varepsilon > 0$ small enough (here $\mathbb{R}_t^+ = \{t \geq 0\}$).

Theorem 4.1. *Let $m \in C^2(\mathbb{R}^+)$, $m(s) > 0$ for all $s \in \mathbb{R}^+$, and let us suppose*

$$u_0 \in H^2(\mathbb{R}^n), \quad u_1 \in H^1(\mathbb{R}^n).$$

There exist then constants $C, \delta > 0$ such that the Cauchy problem (4.1) has a unique solution $u \in C^k([0, T_\varepsilon]; H^{2-k}(\mathbb{R}^n))$ ($k = 0, 1, 2$) with

$$T_\varepsilon > \frac{C}{\varepsilon^4}, \quad \text{if } 0 < \varepsilon \leq \delta.$$

Proof. Let us first recall the well known *first order conservation law*

$$(4.2) \quad \int |u_t|^2 dx + M \left(\int |\nabla u|^2 dx \right) = \varepsilon^2 \int |u_1|^2 dx + M \left(\varepsilon^2 \int |\nabla u_0|^2 dx \right),$$

independently of $t \geq 0$, and where $M : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is the continuous, strictly increasing function

$$(4.3) \quad M(s) = \int_0^s m(h) dh.$$

It is well known that (4.2), under the assumption that m is coercive at ∞ (i.e. $\int_0^\infty m(s) ds = +\infty$), implies that the Cauchy problem (4.1), with initial data $u(0, x) \in H^2(\mathbb{R}^n)$ and $u_t(0, x) \in H^1(\mathbb{R}^n)$,

has a unique local solution $u \in C^k([0, T]; H^{2-k}(\mathbb{R}^n))$ ($k = 0, 1, 2$) for some $T > 0$. See the general result of [AP, Theorem 2.1] and the bibliography therein. See also [Y].²

Coercivity at ∞ is used to prove the boundedness of $\int |\nabla u|^2 dx$. Here the assumption of small initial data allows us to merely assume $m(s) > 0$ because, in this case, the first order conservation law (4.2) is enough to obtain that $\int |\nabla u|^2 dx$ remains small. This, in turn, implies that $m(\int |\nabla u|^2 dx)$ remains greater than a positive constant (see (4.4)-(4.6) below). Let us also remark that [AP] would give an estimate of the lifespan T_ε of the solution of order ε^{-2} , while here the second order identity (2.5) allows us to easily get an estimate of T_ε of order ε^{-4} .

To prove it let us set

$$N(\varepsilon) := \varepsilon^2 \int |u_1|^2 dx + M \left(\varepsilon^2 \int |\nabla u_0|^2 dx \right).$$

From (4.2) it is clear that $\int |u_t|^2 dx$ is bounded above by $N(\varepsilon)$. To obtain an *a-priori* bound for $\int |\nabla u|^2 dx$ let us first remark that $\lim_{\varepsilon \rightarrow 0^+} N(\varepsilon) = 0$ and hence there exists $\varepsilon_0 > 0$ such that

$$N(\varepsilon) \leq N(\varepsilon_0) < \int_0^{+\infty} m(h) dh, \quad \forall 0 \leq \varepsilon \leq \varepsilon_0.$$

This implies that $M^{-1}(N(\varepsilon))$ is well defined for all $0 \leq \varepsilon \leq \varepsilon_0$ and

$$(4.4) \quad s(t) = \int |\nabla u|^2 dx \leq M^{-1}(N(\varepsilon)) \leq M^{-1}(N(\varepsilon_0)) < \infty, \quad \forall t \geq 0, 0 \leq \varepsilon \leq \varepsilon_0.$$

Since

$$(4.5) \quad 0 < m_0 \leq m(s) \leq m_1 \quad \text{if} \quad 0 \leq s \leq M^{-1}(N(\varepsilon_0))$$

for suitable constants m_0, m_1 , it easily follows from (4.2) that there exists $c_0 > 0$ such that

$$(4.6) \quad s(t) = \int |\nabla u|^2 dx \leq c_0 \varepsilon^2, \quad \forall t \geq 0, 0 \leq \varepsilon \leq \varepsilon_0.$$

From (2.6), Hölder's inequality, (4.6) and (2.4), we thus obtain, for $0 \leq \varepsilon \leq \varepsilon_0$:

$$(4.7) \quad \begin{aligned} s'(t)^2 &= 4 \left(\int \nabla u \cdot \nabla u_t dx \right)^2 \leq 4 \int |\nabla u|^2 dx \cdot \int |\nabla u_t|^2 dx \\ &\leq 4c_0 \varepsilon^2 \int |\nabla u_t|^2 dx \leq 4c_0 \varepsilon^2 \sqrt{m} E(t) \leq c_1 \varepsilon^2 E(t), \end{aligned}$$

with $c_1 := 4c_0 \sqrt{m_1}$.

Therefore, for $c_2 := \max_{0 \leq s \leq c_0 \varepsilon_0^2} \left| \frac{1}{4} \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) \right|$ and

$$(4.8) \quad F(t) := E(t) - \frac{1}{4} \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) s'(t)^2,$$

we have that

$$E(t) - c_2 c_1 \varepsilon^2 E(t) \leq F(t) \leq E(t) + c_2 c_1 \varepsilon^2 E(t)$$

and hence

$$(4.9) \quad \frac{E(t)}{2} \leq F(t) \leq \frac{3E(t)}{2}$$

²Theorem I of [Y] proves that the Cauchy problem for the dissipative Kirchhoff equation is locally well posed. But the proof can be easily adapted to the non-dissipative case (4.1).

provided $0 \leq \varepsilon \leq \delta$, with $\delta = \min \{\varepsilon_0, 1/\sqrt{2c_2c_1}\}$. Similarly, from (4.7) and (4.9) we get

$$(4.10) \quad \left| \frac{1}{4} \frac{d^2}{ds^2} \left(\frac{1}{\sqrt{m}} \right) s'(t)^3 \right| \leq 2c_4 \varepsilon^3 F(t)^{3/2} \quad (0 \leq \varepsilon \leq \delta)$$

for a suitable constant $c_4 > 0$. From (2.5) we thus obtain the differential inequality

$$(4.11) \quad F'(t) \leq 2c_4 \varepsilon^3 F(t)^{3/2}.$$

Since for $0 \leq \varepsilon \leq \delta$ we have also $F(0) \leq c_5 \varepsilon^2$, with $c_5 > 0$, integrating (4.11) we finally have

$$\sqrt{F(t)} \leq \frac{\sqrt{F(0)}}{1 - c_4 \varepsilon^3 \sqrt{F(0)} t} \leq \frac{\sqrt{c_5} \varepsilon}{1 - c_4 \sqrt{c_5} \varepsilon^4 t} \quad \text{for } 0 \leq t < \frac{1}{c_4 \sqrt{c_5} \varepsilon^4}.$$

This means that, for $0 < \varepsilon \leq \delta$,

$$F(t) \leq 4c_5 \varepsilon^2 \quad \text{if } 0 \leq t \leq \frac{1}{2c_4 \sqrt{c_5} \varepsilon^4} =: \frac{C}{\varepsilon^4}.$$

By [AP] problem (4.1) has a unique solution on $\mathbb{R}^n \times [0, T_\varepsilon]$, with $T_\varepsilon > C/\varepsilon^4$. \square

By the same computations we also have:

Corollary 4.2. *If we further assume that, for some $\alpha > 0$,*

$$(4.12) \quad \frac{d^2}{ds^2} \left(\frac{1}{\sqrt{m}} \right) = O(s^\alpha) \quad \text{as } s \rightarrow 0^+,$$

then we obtain the estimate: $T_\varepsilon > C/\varepsilon^{4+2\alpha}$ when $0 < \varepsilon \leq \delta$.

Proof. In fact (4.12), together with (4.6), implies (4.10) with $\varepsilon^{3+2\alpha}$ instead of ε^3 . \square

5. A condition for boundedness of the energy $E(t)$

Sufficient conditions for boundedness of the energy $E(t)$ are already known. Assuming, for instance, that m is a C^1 function with $m(s) > 0$ and coercive at ∞ , we have that

$$(5.1) \quad s(t) \leq M^{-1} \left[\int |u_t(x, 0)|^2 dx + M \left(\int |\nabla u(x, 0)|^2 dx \right) \right] := \bar{s},$$

and hence the condition that $V(t) := \int_0^t |s'(\tau)| d\tau$ is bounded for $t \in [0, T)$ is sufficient for boundedness of $E(t)$ because (2.8) implies

$$(5.2) \quad E'(t) \leq \sqrt{m} \left| \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) \right| |s'(t)| E(t),$$

which yields

$$(5.3) \quad E(t) \leq E(0) e^{cV(t)} \quad \text{for } t \in [0, T)$$

for $c := \max_{0 \leq s \leq \bar{s}} \left| \sqrt{m} \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) \right|$.

Here, applying the second order identity (2.5), we find a *necessary* and *sufficient* condition for the boundedness of $E(t)$. More precisely, let us consider the Cauchy problem

$$(5.4) \quad \begin{cases} u_{tt} - m \left(\int |\nabla u|^2 dx \right) \Delta u = 0, & (x, t) \in \mathbb{R}^n \times \mathbb{R}_t^+ \\ u(x, 0) = \phi_0(x), \quad u_t(x, 0) = \phi_1(x) \end{cases}$$

with $m \in C^2(\mathbb{R}^+)$, $m(s) > 0$, m coercive at ∞ , and initial data $\phi_0 \in H^2(\mathbb{R}^n)$, $\phi_1 \in H^1(\mathbb{R}^n)$.

If $u \in C^k([0, T]; H^{2-k}(\mathbb{R}^n))$ ($k = 0, 1, 2$) is the local solution of the Cauchy problem (5.4) in $\mathbb{R}^n \times [0, T)$, $T > 0$, then:

Theorem 5.1. *The energy $E(t)$ of the solution u is bounded in $[0, T)$ if and only if*

$$S(t) := \int_0^t \frac{d^2}{ds^2} \left(\frac{1}{\sqrt{m}} \right) s'(\tau)^3 d\tau$$

is bounded in $[0, T)$.

Proof. From (5.1) we have, for $0 \leq s \leq \bar{s}$,

$$0 < \Lambda_0 \leq m(s) \leq \Lambda_1 \quad \text{and} \quad \left| \frac{dm}{ds}(s) \right| \leq \Lambda_2,$$

for suitable constants $\Lambda_0, \Lambda_1, \Lambda_2$. Besides, from the second order identity (2.5) we get

$$(5.5) \quad \left[E - \frac{1}{4} \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) s'^2 \right]_0^t = -\frac{1}{4} \int_0^t \frac{d^2}{ds^2} \left(\frac{1}{\sqrt{m}} \right) s'(\tau)^3 d\tau.$$

Now, let us assume $E(t)$ bounded in $[0, T)$. Then, arguing as in (4.7),

$$(5.6) \quad s'(t)^2 \leq 4\bar{s}\sqrt{\Lambda_1}E(t) \quad \text{and} \quad \left| \frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) (s(t)) \right| \leq \frac{\Lambda_2}{2\Lambda_0^{3/2}} \quad \text{for } t \in [0, T),$$

and hence from (5.5) we immediately deduce that $S(t)$ remains bounded in $[0, T)$.

Conversely, let us suppose $S(t)$ bounded in $[0, T)$ and let $\bar{t} \in (0, T)$. If $s'(\bar{t}) = 0$, it is enough to observe that from (5.5) we have:

$$E(\bar{t}) = E(0) - \frac{1}{4} \left[\frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) s'^2 \right]_{t=0} - \frac{S(\bar{t})}{4}.$$

Otherwise, let us suppose $s'(\bar{t}) > 0$ (the arguments will be similar if $s'(\bar{t}) < 0$). Then, we define

$$t_0 = \min \left\{ t \in [0, \bar{t}] \mid s'(\tau) > 0 \text{ for } t < \tau \leq \bar{t} \right\}.$$

Clearly, at least one of the following must hold:

$$t_0 = 0 \quad \text{or} \quad s'(t_0) = 0.$$

If $t_0 = 0$, having $s'(t) > 0$ in $(0, \bar{t}]$, we find from (5.2)

$$E(\bar{t}) \leq E(0)e^{c \int_0^{\bar{t}} |s'| d\tau} = E(0)e^{c[s(\bar{t}) - s(0)]} \leq E(0)e^{c\bar{s}}.$$

where c is constant introduced after (5.3).

If $s'(t_0) = 0$, we have

$$E(\bar{t}) \leq E(t_0)e^{c \int_{t_0}^{\bar{t}} |s'| d\tau} \leq E(t_0)e^{c[s(\bar{t}) - s(t_0)]} \leq E(t_0)e^{c\bar{s}},$$

with

$$E(t_0) = E(0) - \frac{1}{4} \left[\frac{d}{ds} \left(\frac{1}{\sqrt{m}} \right) s'^2 \right]_{t=0} - \frac{S(t_0)}{4}$$

because of (5.5).

In conclusion, in all cases $E(\bar{t})$ is bounded by a constant which depends only on the initial data and the values of $S(t)$ in $[0, \bar{t}]$. \square

Remark 5.2. Note that if $\frac{dm}{ds}(s) \geq 0$ for all $s \in \mathbb{R}^+$ then formula (5.5) is enough to prove that $E(t)$ is bounded if and only if $S(t)$ is bounded, because of (5.6).

Moreover, as in Theorem 4.1, if the initial data are sufficiently small we can avoid the assumption that m is coercive at ∞ . Also, if $u_t|_{t=0} \equiv 0$, i.e. $\phi_1 \equiv 0$, then

$$M \left(\int |\nabla \phi_0|^2 dx \right) < \int_0^{+\infty} m(h) dh$$

and hence, arguing similarly as in (4.4), we have that $s(t)$ is bounded and again coercivity at ∞ can be avoided.

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