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Variance of fatigue damage in narrowband Gaussian random loadings: Exact solution and approximations

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Corresponding Author:	J. M. E. MARQUES University of Ferrara Ferrara, ITALY
First Author:	Denis Benasciutti
Order of Authors:	Denis Benasciutti
	Julian Marcell Enzveiler Marques
Abstract:	This technical note investigates some formulae used for estimating the variance of fatigue damage in Gaussian narrowband random loadings. A simple approximation is proposed and compared with the exact closed-form solution based on the Gaussian hypergeometric function. The approximation estimates the coefficient of variation with an error that is about 2% for an S-N inverse slope of m=4 and below 10% for m=6.

Cover Letter

DIPARTIMENTO DI INGEGNERIA
UNIVERSITA' DEGLI STUDI DI FERRARA
via Saragat, 1 - 44122 FERRARA
Tel. 0532 / 974800 - Fax 0532 / 974870

Julian Marcell Enzveiler Marques
Postdoctoral researcher - Engineering Science
Tel. +39-0532-974104
email: nzvjnm@unife.it

Prof. Dr. M. Vormwald
Editor of the
International Journal of Fatigue

Ferrara, 22 April 2021

Dear Editor,

please find herewith enclosed a technical note entitled "Variance of the fatigue damage in narrow-band Gaussian random loadings: Exact solution and approximations" by D. Benasciutti and J. M. E. Marques.

This technical note investigates the variance of fatigue damage in narrow-band Gaussian random loadings. A simple approximated formula is proposed and compared with the exact solution. This comparison may also be computed by a spreadsheet (Excel file), which is provided as a supplementary material. The approximated formula shows a good agreement with the exact solution.

I hope that this technical note is eligible for publication in the International Journal of Fatigue. I also confirm, on behalf of my co-author, that this technical note has not yet been published, nor submitted to other journals.

Best regards

Highlights:

- Estimating the variance of fatigue damage in narrowband Gaussian random loadings;
- A simple approximation formula for the variance of fatigue damage;
- The approximation is compared with the exact solution;

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Technical note

Variance of fatigue damage in narrowband Gaussian random loadings: Exact solution and approximations

Denis Benasciutti, Julian Marcell Enzveiler Marques*

Department of Engineering, University of Ferrara, via Saragat 1, 44122, Ferrara, Italy

Abstract

This technical note investigates some formulae used for estimating the variance of fatigue damage in Gaussian narrowband random loadings. A simple approximation is proposed and compared with the exact closed-form solution based on the Gaussian hypergeometric function. The approximation estimates the coefficient of variation with an error that is about 2% for an S-N inverse slope of m=4 and below 10% for m=6.

Keywords: random loading; narrowband loading; variance of damage; coefficient of variation.

1 Introduction

In a recent article in this journal [1], the authors surveyed four methods used for estimating the variance and coefficient of variation of damage $D(T)$ in a narrowband Gaussian random time-history $x(t)$ with duration T. The methods rely on the concept of random process $X(t)$, formed by an infinite ensemble of time-histories with infinite length. Since, in this infinite ensemble, $x(t)$ is one single time-history with finite duration, its damage $D(T)$ is a single random value of an infinite population of damage values characterising the infinite ensemble in $X(t)$. The random variable $D(T)$ follows a certain probability distribution, with expected value $E[D(T)]$ and variance $Var[D(T)]$.

Among the methods considered in [1], the two with fewer assumptions in the estimation of $Var[D(T)]$ were those of Madsen et al. [2] and of Low [3]. Both methods showed a comparable good accuracy when benchmarked against Monte Carlo simulations with various types of narrowband random loadings.

Recently, it has nevertheless been discovered that the formula used in [1] for the method of Madsen et al.'s, though being in theory only valid for m=2, was incorrectly applied also to values greater than two [4]. Despite this unfortunate circumstance, the simulations did not highlight exaggerated errors. This outcome made the authors conjecture that the "incorrect" use of the formula may be viewed as a viable approximation of the exact solution. The purpose of this technical note is to investigate in more detail this approximation and compare it with the exact solution.

2 Theoretical background

Let $X(t)$ be a stationary narrowband Gaussian random process, defined as the infinite ensemble of time-histories $x(t)$ with infinite time duration. Process $X(t)$ has mean value $\mu_X = E[X(t)]$, variance σ_X^2 and autocorrelation function $R_X(\tau) = E[X(t)X(t+\tau)]$. They define the autocorrelation coefficient function [5]:

$$\rho_X(\tau) = \frac{R_X(\tau) - \mu_X^2}{\sigma_X^2} \tag{1}$$

which is bounded as $-1 \leq \rho_X(\tau) \leq 1$. The process $X(t)$ can also be described in the frequency-domain by a one-sided power spectral density (PSD) $G_X(\omega)$, which corresponds to the Fourier transform of $R_X(\tau)$ [5]. The PSD admits the spectral moments [5]:

$$\lambda_j = \int_0^\infty \omega^j \, G_X(\omega) \, d\omega, \quad j = 0,1,2 \ldots \tag{2}$$

which define, for example, the variance $\lambda_0 = Var[X(t)]$ and mean upcrossing rate $\nu_0 = \sqrt{\lambda_2/\lambda_0}/2\pi$ of $X(t)$ [5]. From now on, the power spectrum $G_X(\omega)$ is assumed to be narrowband.

3 Expected value and variance of damage in narrowband process

For a random time-history $x(t)$ of duration T, the Palmgren-Miner fatigue damage is:

$$D(T) = \sum_{i=0}^{N-1} d_i = \frac{1}{K}\sum_{i=0}^{N-1} s_i^m \tag{3}$$

where N is the number of cycles counted in $x(t)$, $d_\mathrm{i} = \frac{s_\mathrm{i}^m}{K}$ is the damage of the i-th counted cycle with amplitude s_i, and computed from the S–N curve $s^m N_f = K$. Though, in a random time-history, fatigue cycles are identified by e.g. the rainflow counting, in a narrowband loading – where every positive peak is followed by a nearly symmetrical valley – cycles are formed by pairing a peak with the next adjacent valley.

Since, in a random time-history, N and s_i are random variables, $D(T)$ is also a random variable. It represents a single value out of an infinite population of damage values that correspond to the infinite ensemble of time-histories that define $X(t)$. The damage $D(T)$ follows a certain probability distribution with expected value $E[D(T)]$ and variance $Var[D(T)]$.

Following [1,3,6], the expected value and variance of fatigue damage in a narrowband process are:

$$E[D(T)] = \frac{N}{K}\left(\sqrt{2\lambda_0}\right)^m \Gamma\left(1+\frac{m}{2}\right)$$
$$\mathrm{Var}[D(T)] = \left[N + 2\sum_{l=1}^{N-1}(N-l)\rho_{d_0 d_l}(l)\right]\frac{(2\lambda_0)^m}{K^2}\left[\Gamma(1+m) - \Gamma^2\left(1+\frac{m}{2}\right)\right] \tag{4}$$

where $N \cong E[N] = \nu_0 T$ is the average number of cycles in time T. The previous formula of the variance involves the autocorrelation coefficient function of damage:

$$\rho_{d_0 d_l}(l) = \frac{R_{d_0 d_l}(l) - \mu_d^2}{\sigma_d^2} \tag{5}$$

in which $R_{d_0 d_l}(l) = E[d_0 d_l]$ is the autocorrelation function of damage, $\mu_d = E[d]$ the expected value and $\sigma_d^2 = \mathrm{Var}[d]$ the variance of damage of one cycle [3,6]:

$$E[d] = \frac{1}{K}\left(\sqrt{2\lambda_0}\right)^m \Gamma\left(1+\frac{m}{2}\right) \qquad Var[d] = \frac{(2\lambda_0)^m}{K^2}\left[\Gamma(1+m) - \Gamma^2\left(1+\frac{m}{2}\right)\right] \tag{6}$$

In the previous definitions, the quantity $d_0 = E[d]$ indicates the damage of the first cycle counted in $x(t)$. The coefficient $\rho_{d_0 d_l}(l)$ is bounded as $0 \leq \rho_{d_0 d_l}(l) \leq 1$. It measures the correlation between two damage values d_0 and d_l from two cycles separated by a time lag $\tau = l/\nu_0$. Since in a narrowband process a cycle is paired to each peak and two adjacent peaks are distant $1/\nu_0$ seconds, the N counted cycles are only defined at discrete times values l/ν_0, with l integer from 0 to $N-1$.

The ratio $\sqrt{Var[D(T)]}/E[D(T)]$ provides a dimensionless measure of variability and corresponds to the coefficient of variation (CoV) of the damage [3]:

$$C_D = \frac{1}{N}\sqrt{\left[N + 2\sum_{l=1}^{N-1}(N-l)\rho_{d_0 d_l}(l)\right]\left[\frac{\Gamma(1+m)}{\Gamma^2\left(1+\frac{m}{2}\right)} - 1\right]} \tag{7}$$

This general expression applies to any stationary Gaussian narrowband random process. The value of CoV is a function of the number of counted cycles N, of the S-N inverse slope m, and of the specific expression taken by the autocorrelation coefficient of the damage, $\rho_{d_0 d_l}(l)$.

4 Exact solution of the CoV of damage

For a narrowband process, $\rho_{d_0 d_l}(l)$ admits a closed form expression. The autocorrelation function of the damage can indeed be expressed by a Gaussian hypergeometric function [2, pag. 263]:

$$R_{d_0 d_l}(l) = \frac{(2\lambda_0)^m}{K^2} \; {}_2F_1\left(-\frac{m}{2}, -\frac{m}{2}; 1; \kappa_l^2\right) \Gamma^2\left(1+\frac{m}{2}\right) \tag{8}$$

where $\kappa_l^2 = \kappa(l)^2$ indicates the function $\kappa(\tau)^2$ computed at discrete time lags $\tau = l/\nu_0$, with l any positive integer. The quantity $\kappa(\tau)^2$ represents the autocorrelation coefficient function of the squared amplitude of the Crandall-Mark envelope [2, pag. 177]. For a narrowband process, it can be approximated as $\kappa(\tau)^2 \cong \rho_X^2(\tau) + (\rho_X'(\tau)/2\pi\nu_0)^2$, in which $\rho_X'(\tau)$ is the first derivative of $\rho_X(\tau)$.

After substituting in Eq. (5) the expressions of $R_{d_0 d_l}(l)$ and those in Eq. (6), one obtains the exact formula of the autocorrelation coefficient $\rho_{d_0 d_l}(l)$ (the term $\frac{(2\lambda_0)^m}{K^2}$ simplifies):

$$\rho_{d_0 d_l}(l) = \frac{\Gamma^2\left(1+\frac{m}{2}\right)\left[\,{}_2F_1\left(-\frac{m}{2}, -\frac{m}{2}; 1; \kappa_l^2\right) - 1\right]}{\left[\Gamma(1+m) - \Gamma^2\left(1+\frac{m}{2}\right)\right]} \tag{9}$$

This formula, when inserted into Eq. (7), provides the exact CoV of damage for a stationary Gaussian narrowband process, as a function of N and m.

If m is an even integer, a simpler solution yet exists. After writing the hypergeometric function in Eq. (9) as an infinite power series [7], one recognises that, for m even integer, the series becomes a finite summation that simplifies to a polynomial of degree m and with $m/2$ terms, see Appendix. The equations of $\rho_{d_0 d_l}(l)$ for the first four even integer m values are:

$$\rho_{d_0 d_l}(l) = \begin{cases} \kappa_l^2 & \text{if } m = 2 \\ \frac{4}{5}\kappa_l^2 + \frac{1}{5}\kappa_l^4 & \text{if } m = 4 \\ \frac{9}{19}\kappa_l^2 + \frac{9}{19}\kappa_l^4 + \frac{1}{19}\kappa_l^6 & \text{if } m = 6 \\ \frac{16}{69}\kappa_l^2 + \frac{36}{69}\kappa_l^4 + \frac{16}{69}\kappa_l^6 + \frac{1}{69}\kappa_l^8 & \text{if } m = 8 \end{cases} \tag{10}$$

The first three polynomials coincide with those also given in [3] (though derived in a different way), the only difference lying in the definition of κ_l^2 that was taken as $\kappa_l^2 = \rho_X^2(l)$ – but notice that [3] considered the Cramer-Leadbetter envelope.

Of more interest is to study the relative contribution of each term. For $m = 4$, the ratio of second to first term is $\kappa_l^2/4$, which shows that for $\kappa_l = 1$ the second term is one fourth of the first, and it becomes even smaller for $\kappa_l < 1$ (for example, 0.562 for $\kappa_l = 0.5$). This relative difference becomes more pronounced for larger m, see Figure 1: for $m = 6$ and 8, the first two terms in κ_l^2 and κ_l^4 contribute much more to $\rho_{d_0 d_l}(l)$ than the others with higher degree. Not always are the first and second terms the most significant, yet; for example, for $m = 10$ the major contribution comes from the second and third terms. Each case then needs to be studied in detail.

5 An approximation of the CoV of damage

Equation (10) shows that, for $m = 2$, the autocorrelation coefficient of damage is $\rho_{d_0 d_l}(l) = \kappa_l^2$. Nevertheless, in [1] the range of validity of this expression was incorrectly extended to values of m other than two [4]. Although theoretically not correct, this use of $\rho_{d_0 d_l}(l)$ yielded estimations that deviated not very much from the simulation results of [1]. This surprising outcome suggests that $\tilde{\rho}_{d_0 d_l}(l) \cong \kappa_l^2$ could well approximate the exact $\rho_{d_0 d_l}(l)$ also for values $m \geq 2$, in the range $2 \leq m \leq 6$. Recalling the definition of κ_l^2, the approximation may be written as:

$$\tilde{\rho}_{d_0 d_l}(l) \cong \rho_X^2(l) + \left(\frac{\rho_X'(l)}{2\pi\nu_0}\right)^2 \qquad \text{for } 2 \leq m \leq 6 \tag{11}$$

When inserted in Eq. (7), this approximation gives a corresponding approximated CoV of damage:

$$\tilde{C}_D \cong \frac{1}{N}\sqrt{\left\{N + 2\sum_{l=1}^{N-1}(N-l)\left[\rho_X^2(l) + \left(\frac{\rho_X'(l)}{2\pi\nu_0}\right)^2\right]\right\}\left[\frac{\Gamma(1+m)}{\Gamma^2\left(1+\frac{m}{2}\right)} - 1\right]} \quad \text{for } 2 \leq m \leq 6 \tag{12}$$

Compared to Eq. (7), the only difference is the use of $\tilde{\rho}_{d_0 d_l}(l)$ in place of $\rho_{d_0 d_l}(l)$ in Eq. (9). This difference simplifies the calculation of CoV, since it does not involve the hypergeometric function in Eq. (9) – instead, $\tilde{\rho}_{d_0 d_l}(l)$ can be computed for example in spreadsheets (as Excel).

The loss in accuracy from using the approximations $\tilde{\rho}_{d_0 d_l}(l)$ and $\tilde{C}_D$ is now investigated.

6 Benchmark example

This example refers to a narrowband process with rectangular power spectral density, centred at frequency $\omega_c = 2\pi f_c$ and with constant height h over the frequency range $2b$. This process has autocorrelation coefficient function $\rho_X(\tau) = \cos(2\pi f_c \tau)\frac{\sin(2\pi b\tau)}{2\pi b\tau}$ [5]. The mean upcrossing rate of the process is $\nu_0 \cong f_c$. A small ratio b/f_c (with values 0.05, 0.10, 0.15) assures that the power spectrum is narrow band; in the following, no restrictions are imposed on f_c. The S–N slope m takes on integer values from 2 to 6. The number of cycles N is 1000, 10000, 100000. For any given f_c and N, and since $\nu_0 \cong f_c$, the relationship $N \cong \nu_0 T$ gives the time length T of $x(t)$ that corresponds to N.

For this narrowband process, the approximation in Eq. (11) for $m = 2$ is:

$$\tilde{\rho}_{d_0 d_l}(l) \cong \rho_X^2(l) + \frac{1}{(2\pi l)^2}\left(\cos\left(\frac{2\pi b}{f_c} l\right) - \rho_X(l)\right)^2 \qquad (13)$$

Figure 2 compares the approximated $\tilde{\rho}_{d_0 d_l}(l)$ and the exact $\rho_{d_0 d_l}(l)$ autocorrelation coefficient for $m = 2, 4, 6$. The bottom box displays their ratio $r = \tilde{\rho}_{d_0 d_l}(l)/\rho_{d_0 d_l}(l)$. For all m, both $\tilde{\rho}_{d_0 d_l}(l)$ and $\rho_{d_0 d_l}(l)$ rapidly decay to zero for $l \leq 5$, and oscillate close to zero as l increases; for $l \geq 5$ they are lower than 0.1, for $l \geq 10$ they become even smaller and, for this reason, they are not plotted in the figure. At values $l = 5$ and its multiples, the function $\rho_{d_0 d_l}(l)$ is very small, but not zero. Based on Figure 2, it may be concluded that both $\tilde{\rho}_{d_0 d_l}(l)$ and $\rho_{d_0 d_l}(l)$ are entirely described by their trend in the region where l spans from 0 to 10 (note that this region is independent of ω_c).

Looking at lower box of Figure 2, one observes that, for $m = 2$, the ratio is unity and the approximated $\tilde{\rho}_{d_0 d_l}(l)$ matches the exact $\rho_{d_0 d_l}(l)$ for any l (and even for the larger l not shown in figure). But for $m = 4$ and 6, the approximation $\tilde{\rho}_{d_0 d_l}(l)$ overestimates $\rho_{d_0 d_l}(l)$; their ratio r increases with l, and for $l \geq 10$ it approaches a constant plateau (equal to about 2 for $m = 6$).

This discrepancy between the two autocorrelation coefficients seems not to cause the same degree of discrepancy between the CoVs of damage. Simulation results in [1], using the same type of narrowband power spectrum, showed that the approximated CoV computed by Eq. (12) overestimates simulations by only 2% for $m = 3$, and by at most 8% for $m = 6$ – an error much lower than the ratio $r = 2$ between autocorrelation coefficients.

These smaller errors in CoVs may be attributed to the contribution of each term $(N - l)\rho_{d_0 d_l}(l)$ in the summation of Eq. (7) or (12), which in turn closely depends on the particular decay rate of $\rho_{d_0 d_l}(l)$. When l increases and the ratio $\tilde{\rho}_{d_0 d_l}(l)/\rho_{d_0 d_l}(l)$ also increases (i.e., the approximation becomes too imprecise), the values of $\tilde{\rho}_{d_0 d_l}(l)$ rapidly decreases towards zero, so they contribute very little to the summation $\sum(N-l)\rho_{d_0 d_l}(l)$. This explanation suggests that only a limited number of terms in the summation is significant – and must be retained indeed – in the calculation of CoV.

This conclusion is confirmed by results showed in the following. For more convenience, Eq. (7) and (12) are rewritten in the form:

$$\tilde{C}_D^{(U)} \cong \frac{1}{\sqrt{N \cdot A(m)}} \sqrt{\left\{1 + 2\sum_{l=1}^{U}\left(1 - \frac{l}{N}\right)\rho_{d_0 d_l}^{(*)}(l)\right\}} \tag{14}$$

in which the upper limit U of the summation is made to vary from 1 to $N-1$. Parameter $A(m)$ is defined in Appendix. Symbol $\rho_{d_0 d_l}^{(*)}(l)$ is used to indicate either $\rho_{d_0 d_l}(l)$ (exact) or $\tilde{\rho}_{d_0 d_l}(l)$ (approximated).

For $U = N-1$, the formula coincides with the exact solution in Eq. (7) if $\rho_{d_0 d_l}(l)$ replaces $\rho_{d_0 d_l}^{(*)}(l)$, or it coincides with the approximation in Eq. (12) if $\tilde{\rho}_{d_0 d_l}(l)$ replaces $\rho_{d_0 d_l}^{(*)}(l)$. For $U < N-1$, the formula always introduces an approximation that depends on the number of terms U considered in the summation. The loss of accuracy due to the approximation in Eq. (12), instead, depends on the fact that $\rho_{d_0 d_l}(l)$ is replaced by $\tilde{\rho}_{d_0 d_l}(l)$.

6.1 Effect of the number of terms U on the exact solution

Figure 3 displays the relative error $\varepsilon = \tilde{C}_D^{(U)}/C_D - 1$ as a function of U, by using $\rho_{d_0 d_l}(l)$ in Eq. (14) to compute $\tilde{C}_D^{(U)}$. The three lines for different values of N are very close, though not exactly overlapped; this means that the approximation in $\tilde{C}_D^{(U)}$ is almost independent of the number of counted cycles. This outcome may be explained by considering the behaviour of the summation: at small l, the first few terms simplify as $\left(1 - \frac{l}{N}\right) \approx 1$ regardless of N; at increasing l, the terms become more dependent on N, but they multiply $\rho_{d_0 d_l}^{(*)}(l)$ that is almost zero for $l > 10$.

In Figure 3, for a given value of m, the lines shift upward as b/f_c increases (i.e., as the random process becomes less narrow band), the change being more pronounced at small U. For all the three values of b/f_c, the lines follow practically the same trend for each value of m, except for almost unnoticeable differences in the region close to the origin. The figure makes clear that it is sufficient to sum very few terms (about $U = 10$) for the approximated CoV to approach the exact one, with an error of less than 10%.

6.2 Effect of the approximated autocorrelation

Figure 4 displays the same results as Figure 3, though obtained with the approximation $\tilde{\rho}_{d_0 d_l}(l)$ used in place of $\rho_{d_0 d_l}(l)$. This introduces an additional error, although the trends remain almost identical to the previous ones. While a change of N has almost no effect (the lines are again practically overlapped), an increase in the PSD bandwidth makes the curves shift upward, that is, a less narrowband process requires a lower number of terms U for the summation to converge. As already observed in [1], the approximation decreases its accuracy as m becomes larger, but the error remains lower than 10%.

Appendix

The autocorrelation coefficient in Eq.(9) can be written also as:

$$\rho_{d_0 d_l}(l) = A(m) \cdot \left[{}_2F_1\left(-\frac{m}{2}, -\frac{m}{2}; 1; \kappa_l^2\right) - 1\right] \quad , \quad A(m) = \frac{\Gamma^2\left(1+\frac{m}{2}\right)}{\left[\Gamma(1+m) - \Gamma^2\left(1+\frac{m}{2}\right)\right]} \tag{15}$$

where $A(m)$ is a parameter that only depends on m. Since the autocorrelation is less than unity, $\kappa_l^2 \leq 1$, the hypergeometric function becomes a power series [7, pag. 384]:

$$\rho_{d_0 d_l}(l) = A(m) \left[\left(\sum_{j=0}^{\infty} \frac{(-m/2)_j \, (-m/2)_j}{(1)_j} \frac{\kappa_l^{2j}}{j!}\right) - 1\right] \tag{16}$$

in which $(y)_j$ is the Pochhammer symbol for the rising factorial [7]. Since it is $j! = \Gamma(1+j)$, $(1)_j = \Gamma(1+j)$, and also $(-m/2)_0 = 1$ for the first term, the infinite series simplifies as:

$$\rho_{d_0 d_l}(l) = A(m) \sum_{j=1}^{\infty} \frac{(-m/2)_j \, (-m/2)_j}{\Gamma^2(1+j)} \kappa_l^{2j} \tag{17}$$

It can be demonstrated that $(-m/2)_j = (-1)^j (m/2)^{\underline{j}}$, where $(y)^{\underline{j}}$ denotes the falling factorial [7]. Since also the falling factorial can be expressed by a gamma function [7], the previous power series turns into:

$$\rho_{d_0 d_l}(l) = A(m) \left(\sum_{j=1}^{\infty} \frac{\Gamma^2\left(1+\frac{m}{2}\right)}{\Gamma^2(1+j)\Gamma^2\left(1+\frac{m}{2}-j\right)} \kappa_l^{2j}\right) \tag{18}$$

If m is an even integer, the series terminates at $m/2$ and becomes a polynomial [7, pag. 385]:

$$\rho_{d_0 d_l}(l) = A(m)\left(\sum_{j=1}^{m/2} \frac{\Gamma^2\left(1+\frac{m}{2}\right)}{\Gamma^2(1+j)\,\Gamma^2\left(1+\frac{m}{2}-j\right)}\kappa_l^{2j}\right) \tag{19}$$

The expression for the first four even integers for m values are reported in Eq. (10).

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Supplementary material

A spreadsheet (Excel file) is provided with calculation of relative difference between approximated and exact CoV of damage.

Figures

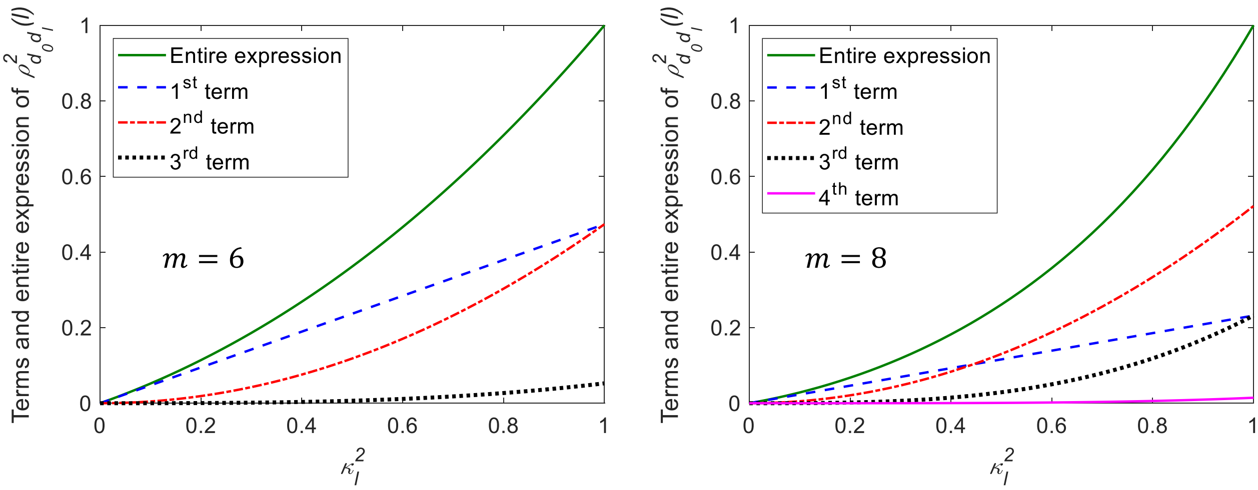

Figure 1. Comparison between all terms and entire expression of $\rho_{d_0 d_l}(l)$ for $m = 6$ and 8.

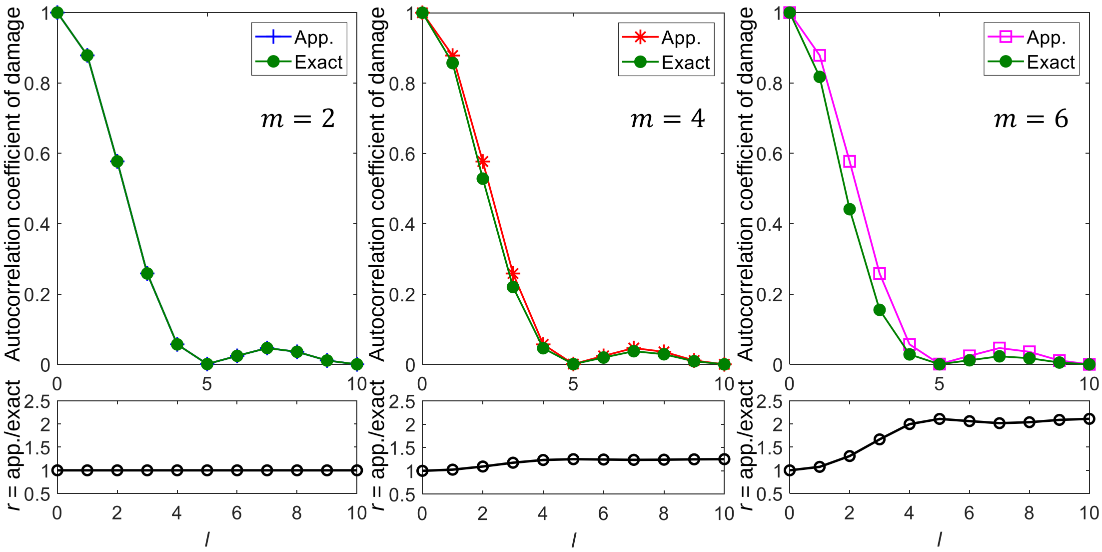

Figure 2. Autocorrelation coefficient of damage: approximated $\tilde{\rho}_{d_0 d_l}(l)$ versus exact $\rho_{d_0 d_l}(l)$, for three inverse slopes $m = 2, 4, 6$. Bottom figures: ratio $r = \tilde{\rho}_{d_0 d_l}(l)/\rho_{d_0 d_l}(l)$. Straght lines between markers are only for graphical purposes.

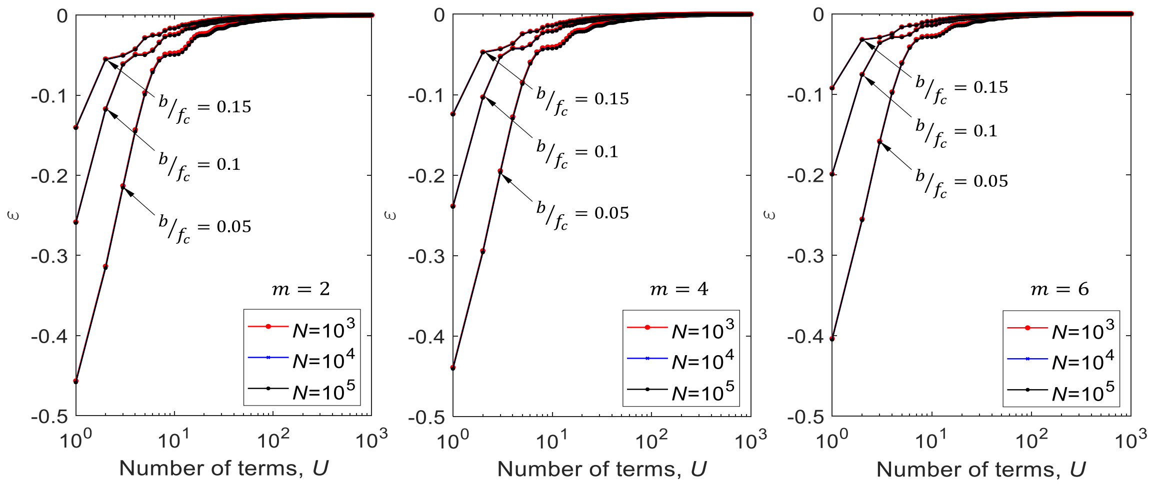

Figure 3. Relative difference $1 - \tilde{C}_D^{(U)}/C_D$ as a function of the number of terms U in the summation of Eq. (14), for different ratios b/f_c and S-N slopes $m = 2, 4, 6$.

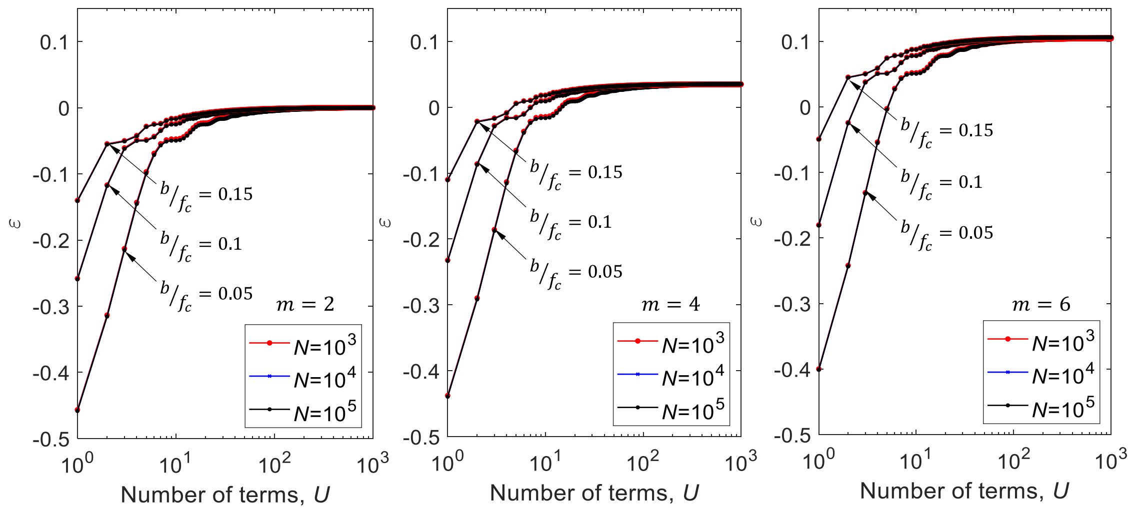

Figure 4. Relative difference $1 - \tilde{C}_D/C_D$ between approximated and exact CoV of damage, as a function of the number of terms U in the summation of Eq. (14), for different ratios b/f_c and S-N slopes $m = 2, 4, 6$.

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Supplementary Material
BENA MARQUES_Supplementary material_Technical note.xlsx

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: