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Numerical insights on the structural assessment of historical masonry stellar vaults: the case of Santa Maria del Monte in Cagliari

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Abstract The aim of this paper is to present an in-depth numerical investigation on the statics of historical masonry stellar vaults, a special class of masonry ribbed vaults whose three-dimensional geometry features a star-shaped projection on the horizontal plane. In particular, the mechanical behavior of the masonry stellar vault belonging to the church of Santa Maria del Monte in Cagliari (Italy) is analyzed and illustrated as an especially meaningful

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case study. This church, which was built during the second half of the 16-th century, is a beautiful example of Gothic-Catalan style and its ribbed stellar vault is one of the most representative of this type in the town of Cagliari. The geometrical outline of the vault has been obtained through laser scanning techniques and a procedure of *reverse engineering*. Starting from a three-dimensional representation of its geometry, the ultimate load bearing capacity of the stellar vault can be accurately estimated through a recently developed, NURBS-based upper bound limit analysis scheme. A comparison with incremental non-linear analyses carried out with the commercial finite element code DIANA is presented. Furthermore, the paper also includes a sensitivity study aimed at investigating the role of ribs on the ultimate load bearing capacity of the structure.

Keywords Historical masonry vaults · stellar vaults · GA-NURBS limit analysis · finite elements · incremental non-linear analysis

1 Introduction

A stellar vault is a specific kind of Gothic ribbed vaults. It can be considered as an evolution of the traditional cross vault, characterized by a more complex geometry and by a system of supporting ribs. The name is due to the particular shape of its projection on a horizontal plane, which usually looks like a star (Kulig and Romaniak, 2007). Stellar vaults have been spread in the Mediterranean area between the 14-th and the 17-th century with the late-Gothic architecture. Several examples of this typology can be found in Sardinia, where stellar vaults are often used as ceilings for the presbytery, namely the most important area of the church. These structures can be considered among the most important works in the history of architecture and constitute a precious masterpiece of the cultural heritage that deserves being protected. However, their preservation requires a thorough knowledge of many

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perspectives, starting from constructive ones up to their structural behavior. Gothic builders made great progress in the structural field, with the continuous search for optimal solutions, the progressive reduction of masses and the attempts to differentiate the static role of the various elements of the construction.

The structural behavior of Gothic vaults has been the subject of scientific debate for a long time (Huerta Fernández, 2009): in this field different theories have been proposed explaining the structural role of ribs in cross vaults (Di Pasquale, 1996), (Viollet-le-Duc, 1854–1868), (Abraham, 1934), (Sabouret, 1928). Because of the lack of specific methods of analysis, the static behavior of such structures is still unclear. In addition to the typical problems of historical masonry building modelling, in this case further difficulties arise, due to the complexity of geometry and the intrinsically three-dimensional character of the problem. The considerable progress recently achieved by numerical modeling techniques, based on the finite element method, and the possibility of producing accurate geometric surveys thanks to laser scanning techniques, make the static behavior of the complex masonry vault an interesting topic of research.

In this work an investigation on the structural behavior of masonry stellar vaults is proposed: attention is focused on a case study, namely the stellar vault of the church of Santa Maria del Monte in Cagliari, Italy (Freddi and Salinas, 1959), which was built in the second half of the 16-th century in a Gothic-Catalan style (Manconi, 2015). A series of investigations about this vault have been performed by laser scanning techniques in combination with a *reverse engineering* procedure. A similar application of laser scanner techniques on the evaluation of the geometry of historical masonry buildings can be found in (Carini and Genna, 2012). The geometric outline has allowed the reconstruction of three-dimensional models in a CAD environment. Mechanical parameters of masonry have been deduced through a series of *in situ* experimental tests and by adopting the prescriptions provided

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by the Italian Building codes (NTC 2008, 2008), (Circolare n. 617, 2009). As it is well known, the strong heterogeneity of mechanical characteristics of masonry, which includes a very low tensile strength in comparison with a relatively high resistance in compression, makes linear elastic approaches unsuitable (Heyman, 1995), (Como, 2013). Therefore, in this work the structural behavior of this stellar vault has been studied through incremental non-linear analyses and limit analysis.

Incremental non-linear analyses have been performed through the FEM-based software DIANA (DIANA, 2015). Different constitutive laws for masonry behavior in tension have been considered. Recently, similar non-linear analyses have been performed on geometrically complex ribbed masonry vaults in order to evaluate the structural safety in terms of vertical load (Compán et al., 2017).

Limit analysis is the best alternative to non-linear incremental analyses for the evaluation of the ultimate load bearing capacity of masonry vaults. In the last years different methods have been applied to the study of Gothic vaults: among others, a limit analysis based on a lower-bound approach, which is the so-called Thrust Network Method (Block and Ochsendorf, 2007), (Block and Lachauer, 2014), and techniques based on the Discrete Element Method (Lengyel, 2017). Other approches based on the upper-bound theorem have been adopted in the study of seismic vulnerability of cross vaults (Gaetani et al., 2017). However, in these latter methods different possible collapse mechanisms must be considered in order to avoid an excessive overestimation of the collapse load multiplier. In this paper a new approach recently proposed in (Chiozzi et al., 2016b), (Chiozzi et al., 2017a), (Chiozzi et al., 2017b), (Chiozzi et al., 2018a), (Chiozzi et al., 2018b) has been applied to the limit analysis of stellar vaults: this procedure is based on the use of a NURBS (Not Rational Uniform B-Spline) model for the representation of the geometry, an upper-bound approach and a genetic algorithm. The use of NURBS surfaces instead of finite elements allows the

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representation of the exact geometry of the stellar vaults with few elements. Finally, the collapse mechanism associated to the minimum load multiplier is determined using a genetic algorithm.

It has to be noticed that, despite of its simplicity, Heyman's model may be inappropriate for vaults with the role of roofing elements, which are often characterized by high values of the span-to-thickness ratio and by the absence of backfill: the reader is referred to (Ramaglia et al., 2016) for a detailed research on the most suitable approaches for slender vaults. For these reasons, in the aforementioned model, a non-zero tensile strength and a finite compressive strength has been assumed for masonry.

The paper is organized as follows. Section 2 contains some historical details about ribbed stellar vaults. Section 3 includes a description of Santa Maria del Monte stellar vault and of the procedure which allowed to obtain the 3D models from the geometric outline of the structure. Finally, in Section 4 the details of the performed numerical analyses are presented, along with a thorough discussion of the obtained results. In the same Section also a sensitivity analysis of the ultimate load bearing capacity of the vault as a function of ribs' thickness is included. Finally, conclusions are drawn in Section 5.

2 Stellar vaults: generalities

During the late-Gothic period of 13-th and 14-th centuries, different compositions of ribbed cross vaults have been experimented by architects and builders. Through realization of more decorates and complex versions, stellar vaults (i.e. star-shaped vaults) appeared, in which secondary ribs were added to further subdivide the span. These particular vaults owe their name to the projection on the horizontal plane of the ribs system, which is usually a star-shaped figure.

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The additional elements can be distinguished in main (*tiercerons*) and secondary ribs (*liernes*), respectively ribs which start from the base of the vault and others which have the role of connections. On the horizontal plane, usually star-shaped vault's pattern can be obtained by subdividing the angle into four equal parts (Fig. 1(a)); therefore, main additional ribs are directed along the bisector between projections of perimeter and diagonal arches. A simple graphic method based on the circumscribed circle of the map and the two symmetry axes can be used to define main ribs on the horizontal plane (Casu, 2013) (see Fig. 1(b)). Through the addition of secondary ribs, different geometric shape can be obtained (Fig. 1(c)). Generally, pendulous buds are located under ribs intersections as decorations.

The first stellar vaults have been found in France: one of the most meaningful case is located in the Amiens cathedral (1220-1269) between the central nave and the transept (Willis, 1842), (Fitchen, 1961), (Bechmann, 1981). During the late-Gothic period this type has been spread in the Mediterranean and especially in Spain; moreover several cases are located in Poland and Hungary (Kulig and Romaniak, 2007), (Ther et al., 2010). In Sardinia the late-Gothic building typology appeared in the 14-th century with the greatest diffusion between the 15-th and the 17-th (see Fig. 2). Here, under the Crown of Aragona (1324–1479) and the Spanish Kingdom (1479–1714), the development of Gothic culture allowed the comparison of repeatable codified forms, which were used in both civil and religious architecture. Six cases of stellar vaults observed in Cagliari are shown in Fig. 3. As a representative case of this particular type, a detailed analysis of the stellar vault of church of Santa Maria del Monte is presented here.

Design, construction and measurement practical techniques on Gothic structures have been described for the first time in the *Portraiture book* or *Livre de portraiture* by Villard de Honnecourt (Lassus, 1858), that is composed by a series of boards compiled approximately in 1230 of which only 66 survived until now. The manuscript is an important proof of role

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and experience for specialists who lived in that historical period. Rules about building are the main topic developed in the book: about the design of vaults, the so-called three arches rule has been reported, as demonstration of the use of standard building methods.

About the late-Gothic vaults also the Renaissance treatises by Alonso de Vandelvira, Hernàn Ruiz and Rodrigo Gil de Hontañòn, (Huerta Fernández, 2006), are truly important. In addition to other topics, specific references to ribbed cross vaults are contained in these works, not only from the point of view of the design but also from that of estimating the size of the ribs.

From these works, it is clear that medieval building techniques were based on geometric rules, which are a direct consequence of structural behavior of masonry buildings. Indeed this method is still considered as a fundamental one in the study of historical masonry buildings and it has been proposed several times by Heyman and Como, (Heyman, 1995), (Como, 2013). Buildings were usually made of cut stone. However sometimes vault's webs were made of bricks, which are lighter and easier to lay, whereas stones were used for ribs only. Otherwise, in secondary cases the vault could be built entirely with bricks with special pieces for the ribs (corner bricks).

Like for the case of cross vault, the building of the stellar vaults requires production and laying of centrings near ribs and perimeter arches; at the center of the vault diagonal centrings stand on a pillar which is lifted up to the intersection point (Fig. 4).

This technique requires ribs to have adequate dimensions to act as support for vaults; usually, they are contoured and made in two different parts: one which is protruding and another whose top is hidden by the vault, as shown in Fig. 5. Guidelines for estimating the size of the ribs, with relation to the dimension of the vault, are reported in several historical essays. As an example, Breymann (Breymann, 1849) suggested a minimum of 13 cm and of 27 cm respectively for the width and the depth of a rib corresponding to a span equal to 5 m.

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From the historical point of view, the role of ribs in cross vaults was highly discussed during the first half of the 20-th century. An interesting review about this topic has been proposed by Di Pasquale (Di Pasquale, 1996).

According to Viollet-le-Duc (Viollet-le-Duc, 1854–1868) ribs work as a sort of elastic centrings, made by voussoirs that follow structural adjustments and support masonry above them. Viollet pointed out the difference between building and structure, i.e. filling and supporting elements. The supporting interpretation for ribs, proposed by the French architect, has been called into question some years later by Abraham (Abraham, 1934) and Sabouret (Sabouret, 1928) who suggested, starting by analyses of the great Gothic cathedrals which had been severely damaged during World War First, that vaults could sustain themselves even without ribs.

The study of static scheme of cross vaults is truly complex (Milani et al., 2014); stress are concentrated along diagonals even without protruding ribs. Maybe this fact was already known by Roman constructors, who used to reinforce cross intersections with arches located at the intrados of the vault. In addition, provided that stones used for ribs have usually better mechanical properties than those used for webs (even because of cutting mode and arrangement), this further contribution due to mechanical characteristics of the material should be considered.

However, the same static interpretation proposed for diagonal ribs in cross vaults (Como, 2013) cannot be applied always to additional ribs introduced, for example, in stellar vaults.

3 The stellar vault of Santa Maria del Monte: description and modeling

The church of Santa Maria del Monte (Freddi and Salinas, 1959) (Fig. 6), which is located in Cagliari in the historic district of Castello, was built in the second half of the 16-th century

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in a Gothic-Catalan style as the seat of the *Arciconfraternita del Sacro Monte di Pietà*. Several changes and renovations have been applied to the church, which is now used as the headquarter of the *Sovereign Military Order of Malta* (Manconi, 2015) in Sardinia.

The church is composed by a single nave without transept and with presbytery in the shape of a chapel (*capilla mayor*). The chapel stood out for the squared plan, tighter than the nave, and especially for the star-shaped vault, with ogive ribs connected by *tiercerons* and *liernes* and with five pendulous buds, Fig. 7(a), typical of the late-Gothic architectural style of Sardinia.

The very complex and peculiar geometric scheme of this vault, and the utmost importance of precise geometric data for analyzing these structures, required a survey carried out through laser scanning techniques, Fig. 7(b). The obtained point cloud has subsequently been simplified using a Poisson-disk sampling algorithm (Corsini et al., 2012). In this way the geometric characteristics of the structure have been determined with high accuracy, especially small local imperfections and asymmetry (in comparison with the standard pattern provided by architectural essays) have been precisely detected.

Generally speaking, the whole vault does not present large deformations. A greater cure in construction can be noticed in ribs than in the vault webs, especially along diagonals. Projections in the horizontal plane are all straight, however the *tiercerons* scheme is a bit different in comparison with the standard design: indeed, they are not directed along the bisector of the angle described by the ogive and the perimeter arch. Arches can be described generally as circular polycentric, forming, especially along the perimeter, little pronounced lancet arches (see Fig. 7(a)). Faint variations in elevation have been found in the additional ribs.

There are several hypotheses to explain these variations. It is reasonable to think that Gothic-Catalan constructors, the so-called *picapedras*, had their personal design procedure,

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improved over the years and maybe somehow different from the standard method; anyway, a span to rise ratio equal to 2 suggests their will to define lancet arches. Otherwise, another possible explanation lies in the use of existing centrings for the construction of ogive arches, which were therefore adapted to their current needs.

Starting from the detected point cloud, a three-dimensional NURBS (non-uniform rational B-splines) model of the whole vault has been obtained by using a solid modeling software (Rhinoceros®) through a process of *reverse engineering*. The model is composed of NURBS surfaces obtained through a refinement of the point cloud (see Fig. 8). The NURBS model has allowed the realization of a second model, to be used for analyses based on the Finite Element Method (FEM), directly in the CAD environment, in which the stellar vault is represented as a three-dimensional solid. In this latter model also the thickness of each element has been represented, as it can be seen in Fig. 9.

Diagonal and perimeter arches start 3.5 m in elevation above the floor and the height of the keystone is about 8 m. The horizontal projection corresponds to a square whose side is 6.3 m. These measures are the result of the adopted (and previously explained) scheme. Further simplifications have been necessarily considered for the ribs: indeed the molded sections have been represented as rectangular ones, see Fig. 10 for the assumed geometry. The thickness of the vault webs could not be measured directly, thus according to reference information (Viollet-le-Duc, 1854-1868 (Viollet-le-Duc, 1854–1868); Ungewitter and Mohrmann, 1890 (Ungewitter and Mohrmann, 1890); Breymann, (Breymann, 1849); Como, (Como, 2013)) a thickness of 15 cm has been assumed.

The vault has been supposed constrained only at the base according to the hypothesis of very low connection with lateral walls, which are located only on two sides (see Fig. 7(a)).

The vault is entirely built with cut stone extracted from local quarries. The vaults' structure appears generally regular with brick-rows orthogonally aligned with the perimeter arches; joints are actually made with lime mortar and have an average thickness equal to 2 cm, rib joints are instead thinner, indicating a better quality of construction. Mechanical characteristics of stones depend on the extraction site or even on the outcrop. A direct measure of mechanical parameters has not been possible, however technical literature (Cuccuru et al., 2014) suggests the following average values:

– specific weight: $\gamma = 22.5$ kN/m^3
– dynamic elastic modulus: $E_d = 17.6$ GPa

The static elastic modulus of bricks, E_b, has been obtained by the dynamic modulus through relations taken by known sources (see (Eissa and Kazi, 1988)), which directly provide Eq. (1):

$$E_b = 0.74E_d - 0.82 \cong 12\,\text{GPa} \tag{1}$$

In the same way, an estimate of the value Young's modulus for mortar (Parisi and Augenti, 2012) was obtained. In particular a value equal to $E_m = 280$ MPa has been assumed: it should be noticed that this is a conservative choice, since it results one of the smallest values obtained by experimental tests.

Then Young's modulus for (homogenized) masonry of ribs, E_r, can be obtained through Eq. (2) (Como, 2013):

$$E_r = E_m \frac{1 + \dfrac{s}{h_b}}{\dfrac{E_m}{E_b} + \dfrac{s}{h_b}} \tag{2}$$

where s and h_b represent the joint and brick thicknesses, respectively. In the present case, it has been assumed for ribs, where the quality of mortar joints is better, $s = 1$ cm and $h_b = 25$ cm respectively, which would produce, by Eq (2), $E_r = 4.598$ GPa. Similarly for the homogenized masonry vault's webs, where the quality of mortar joints is worse, the assumed

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values have been: $s = 2$ cm and $h_b = 25$ cm. As a consequence for Young's modulus a value $E_v = 2.926$ GPa has been obtained, which is in good agreement with values suggested by the Italian technical literature (NTC 2008, 2008), (Circolare n. 617, 2009).

Values obtained in this way have been successively corrected with safety coefficients provided by the Italian codes. In particular, (NTC 2008, 2008) suggests mechanical parameters for each type of masonry, and for squared stones there are the following values:

- specific weight: $\gamma = 22$ kN/m^3
- elastic modulus: $E = 2400 \div 3200$ MPa

In conclusion, based on experimental tests given by literature, suggestions of Italian codes and an *in situ* exam, the following values have been adopted:

- specific weight: $\gamma = 22$ kN/m^3
- Poisson's coefficient: $\nu = 0.2$
- Young's modulus for vault's webs masonry: $E_v = 2800$ MPa
- Young's modulus for ribs masonry: $E_r = 4200$ MPa

4 Description of the numerical analyses performed on the vault

The stellar vault of Santa Maria del Monte has been studied through limit analyses and incremental non-linear analyses in order to evaluate the ultimate load bearing capacity of this structure. Two different load configurations have been considered: uniform vertical load and point vertical load applied at the crown (Fig. 11).

Limit analyses have been performed through a new procedure recently developed by the research team of the University of Ferrara: this method, the so-called GA-NURBS limit analysis, works in the MATLAB® environment starting by the NURBS model of the vault. Therefore, the geometry has been defined by means of NURBS surfaces (Piegl and Tiller,

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1995) through the Rhinoceros® code (McNeel, 2008) (Fig. 12). The use of NURBS is particularly suited for masonry vaults, where equilibrium can be verified provided that the thrust surface is strictly enclosed within the exact outline of the structure (Heyman, 1995).

In commercial CAD packages and free-form surface modelers like Rhinoceros® description and computation of complex geometries is performed through B-splines and NURBS approximating functions. NURBS basis functions are built on B-splines basis functions, which are piecewise polynomial functions defined by *knots* (i.e. points in a parametric domain) $\Xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\}$, where p and n denote respectively the polynomial order and the total number of basis functions. Given a set of weights $w_i \in \mathbb{R}$ and the i-th B-spline basis function ($N_{i,p}$), then the NURBS basis function $R_{i,p}$ can be written as follows:

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi) \cdot w_i}{\sum_{i=1}^{n} N_{i,p}(\xi) \cdot w_i} \tag{3}$$

A NURBS surface of degree p in the u-direction and q in the v-direction is a parametric surface in the three-dimensional Euclidean space defined as:

$$S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} R_{i,j}(u,v) \boldsymbol{B}_{i,j} \tag{4}$$

where $\boldsymbol{B}_{i,j}$ form a bidirectional net of control points. A set of weights $w_{i,j}$ and two separate knot vectors in both u and v directions must be defined. Further details about mathematical formulations of NURBS surfaces can be found in (Piegl and Tiller, 1995), while more general IsoGeometric Analysis (IGA) methods have been recently presented in (Khakalo and Niiranen, 2017), in (Cuomo and Greco, 2018) and in (Yildizdag et al., 2018).

In order to perform the limit analysis, the NURBS model generated in Rhinoceros® has been imported into the MATLAB® environment through the IGES (Initial Graphic Exchange Specification) standard (Kennicott, 1996). A set consisting of 33 NURBS surfaces

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has been used: 12 surfaces defining the vault's webs, 20 the ribs and 1 the keystone (see Fig. 12). The thickness of each of these surfaces has been assigned directly in MATLAB®: the model obtained in this way is shown in Fig. 13.

A NURBS mesh is then defined on this model, where each element of the mesh still consists of a NURBS surface and, hence, can be regarded as a rigid body. Dissipation, which is evaluated according to a rigid-plastic behavior in tension, compression and shear, is allowed only along element edges: therefore, element edges represent potential fracture lines. In this way the kinematics of each element is determined by six degrees of freedom, which are the three translational and three rotational generalized velocity components $\{u_x^i, u_y^i, u_z^i, \phi_x^i, \phi_y^i, \phi_z^i\}$ of its center of mass G_i, expressed in a global reference system $O_{x,y,z}$.

Let us assume that the vault is subjected to two group of loads: *i*) loads which are independent of the collapse multiplier λ (say $\boldsymbol{F}_0$, e.g. gravity loads, which do not change their intensity during the loading process) and *ii*) loads which depend on λ (say $\lambda\boldsymbol{\Gamma}_0$, e.g. variable vertical load, whose intensity changes during the loading process, being incremented by the multiplier λ; $\boldsymbol{\Gamma}_0$ is here the so-called unit vector of actions, corresponding to a value $\lambda = 1$ of the collapse multiplier). A so-called normalization condition, classically obtained by imposing that power dissipated by loads depending on λ is unit when $\lambda = 1$, is needed to restrict the homothetic failure mechanism to one. Such normalization condition can be written as follows:

$$D_{\Gamma_0} = \boldsymbol{\Gamma}_0^T \boldsymbol{U} = 1 \tag{5}$$

where $\boldsymbol{U}$ is the vector of assembled generalized velocities for all the elements. Consequently, the dissipated external power is the following:

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$$D_{ext} = D_{F_0} + \lambda D_{\Gamma_0} = \boldsymbol{F}_0^T \boldsymbol{U} + \lambda \tag{6}$$

Equating internal, D_{int}, and external dissipation, D_{ext}, in the framework of the upper bound theorem, where it is required the minimization of λ, the objective function is therefore $\lambda = D_{int} - \boldsymbol{F}_0^T \boldsymbol{U}$ and the collapse load multiplier λ does not enter as an independent variable in the Linear Programming problem.

External constraints and boundary conditions on velocities are standard and, after suitable assemblage, lead to a set of equalities that can be written in compact notation as follows:

$$\boldsymbol{A}_{eq,geom} \boldsymbol{U} = \boldsymbol{b}_{eq,geom} \tag{7}$$

where $\boldsymbol{A}_{eq,geom}$ is the matrix of geometric constraints and $\boldsymbol{b}_{eq,geom}$ the corresponding vector of known coefficients.

Plastic compatibility constraints are imposed on interfaces, subdividing edges into N_{sd} segments and $(N_{sd}+1)$ collocation points P_i, as sketched in Fig. 14(a). A local frame of reference $(\boldsymbol{n}, \boldsymbol{s}, \boldsymbol{t})$ on each P_i is easily defined, indicating with $\boldsymbol{n}$ the unit normal vector to the interface, with $\boldsymbol{s}$ the unit tangent vector in the longitudinal direction and with $\boldsymbol{t}$ the unit tangent vector in the transversal direction. After a suitable linearization of the failure surface at P_i the following compatibility equation must hold:

$$\Delta \tilde{\boldsymbol{u}} = \left(\dot{\boldsymbol{\lambda}}^T \frac{\partial f}{\partial \boldsymbol{\sigma}} \right)^T \tag{8}$$

where $\boldsymbol{\sigma} = [\sigma_{n,n}, \tau_{n,s}, \tau_{n,t}]^T$ is the stress vector acting on P_i in the local frame of reference, $f(\boldsymbol{\sigma})$ is the linearized strength domain ($\partial f / \partial \boldsymbol{\sigma}$ is therefore a three-column matrix of coefficients of the linearization plane), $\dot{\boldsymbol{\lambda}}$ is the vector of non-negative plastic multiplier rates and $\Delta \tilde{\boldsymbol{u}}$ is a 3×1 vector of jump of velocities on P_i written in the local frame of reference.

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Internal dissipation on a single interface i of area S_i is estimated by numerical integration at collocation points as follows:

$$D_{int}^{i} = \int_{S_i} \boldsymbol{\sigma} \cdot \Delta\tilde{\boldsymbol{u}} dS = \int_{S_i} \dot{\boldsymbol{\lambda}}^T \cdot \left(\boldsymbol{\sigma}^T \frac{\partial f}{\partial \boldsymbol{\sigma}} \right)^T dS = \int_{S_i} \dot{\boldsymbol{\lambda}}^T \cdot \boldsymbol{b}_{sd} dS \tag{9}$$

where $\boldsymbol{b}_{sd}$ is the column-vector of known coefficients of the strength-domain linearization planes.

The obtained linear programming problem, which is used to estimate the collapse multiplier via an upper bound approach, is therefore the following:

$$\min\{D_{int}^{i} - \boldsymbol{F}_0^T \boldsymbol{U}\} \text{ such that } \{\boldsymbol{A}^{eq}[\boldsymbol{U}^T \dot{\boldsymbol{\lambda}}^{ass\,T}] = \boldsymbol{b}_{eq} \text{ and } \dot{\boldsymbol{\lambda}}^{ass\,T} \geq 0\} \tag{10}$$

where $\boldsymbol{A}^{eq}$ is the overall equality constraints matrix (and $\boldsymbol{b}^{eq}$ the corresponding right-hand-side vector) collecting plastic flow constraints on discontinuities, velocity boundary conditions and external power normalization condition and $\dot{\boldsymbol{\lambda}}^{ass\,T}$ is the assembled vector of plastic multiplier rates. The solution of the linear programming is found through the MATLAB® toolbox for optimization over symmetric cones SeDuMi (Self-Dual-Minimization, (Sturm, 1999)); the reader is referred to (Milani et al., 2006) for a critical discussion of the most effective tools to solve such optimization problem efficiently.

A Genetic Algorithm (GA) is here used to adjust the mesh in order to find the minimum collapse multiplier among all possible configurations and therefore to determine the actual collapse mechanism. GA is a method for solving both constrained and unconstrained optimization problems based on a natural selection process that mimics biological evolution. The algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm randomly selects individuals from the current population and uses them as parents to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution. In this way, it is possible to find the position of fracture lines associated to the minimum kinematic multiplier, which are the collapse mechanism and the collapse multiplier.

The reader is referred to (Chiozzi et al., 2017b) for further details about the GA-NURBS limit analysis and to (Chiozzi et al., 2016b), (Chiozzi et al., 2017a), (Chiozzi et al., 2018a), (Chiozzi et al., 2018b) for previous applications. It should be noticed that this is the natural evolution of the previously published (Chiozzi et al., 2016a) open-source code ArchNURBS.

The above outlined procedure has been applied in the present case to the stellar vault of Santa Maria del Monte for determining the collapse load corresponding to the two already considered cases of uniformly distributed vertical load and of point load applied at the crown. To this purpose, a Mohr-Coulomb failure criterion supplemented by a linear cap in compression (Milani and Taliercio, 2016) has been applied to interfaces between masonry elements; the three-dimensional linearized failure surface is shown in Fig. 14(b) and Fig. 14(c), whereas material parameters adopted in the analyses are reported in Table 1 (see Fig. 14(b)) for the meaning of the symbols). The collapse loads for the afore-mentioned loading conditions resulted to be 41.5 kN/m^2 and 178.8 kN respectively; the corresponding collapse mechanism are depicted in Fig. 15 and Fig. 16.

As it can be seen in Fig. 15(c), the collapse due to the uniformly distributed vertical load is mainly characterized by crushing of the masonry at the base of the vault (crushing at the base is highlighted in Fig. 15(a) by observing the position of the deformed vault in comparison with the initial constraint plane, which is depicted through dashed lines). This result descends directly from the geometry of the vault: with this load configuration, the lowered shape of the ogive and the perimeter arches prevent a collapse produced by the opening of flexural joints typical of semicircular arches. It is clear that in this situation an accurate estimate of the masonry ultimate compressive strength becomes fundamental for

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Table 1 Masonry parameters adopted for the NURBS model.

Property	Symbol	Value	Unit
Specific weight	γ	22	kN/m^3
Ultimate tensile strength	f_t	0	MPa
Cohesion	c	0.1	MPa
Friction angle	Φ	27	°
Ultimate compressive strength	f_c	2.4	MPa
Parameters defining the shape of the linearized compressive capacity	Φ_2	10	°
	ρ	0.5	—

reliably evaluating the collapse load. This is not occurring in the case of a concentrated load applied at the crown: here the opening of flexural joints on the perimeter arches is clearly visible (see Fig. 16(c)). In this case, ribs have the typical behavior of arches subjected to a pointed load applied at the midpoint, whereas vault's webs appear to follow arches in the collapse. However, for both considered load configurations, the supporting function seems to be mostly provided by ribs.

In order to confirm the results obtained with the limit analysis, a series of incremental non-linear analyses has been conducted on a FEM model of the stellar vault through the commercial software DIANA (version 10.2) (DIANA, 2015). Starting by the 3D CAD model, a mesh composed by 29665 tetrahedral finite elements has been obtained (a value of 0.2 m has been assigned as mesh-size). The use of tetrahedral elements (TE12L in DIANA, characterized by 12 degrees of freedom, Fig. 17) has allowed a more refined discretization of the complex geometry of the stellar vault in comparison with parallelepiped-shaped elements: the FEM model is shown in Fig. 18. Two different material properties have been assigned in DIANA to ribs and vault's webs in order to represent correctly the two different type of previously mentioned masonry. A Total Strain Crack Model (TSCM) has been used

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for both materials. Two different options have been considered for the tensile behavior: a bilinear elastic-softening and an elastic-plastic behavior (in the latter, elastic-plastic behavior, a very low value for ultimate tensile strength has been adopted in comparison with the value of peak tensile load in the bilinear one). Despite the fact that it is often considered unsuitable to model masonry material, an elastic-plastic tensile behavior has been taken into account in order to yield a more realistic constitutive law than the rigid-plastic behavior adopted in the limit analysis. An elastic-plastic behavior has been assigned in compression, whereas for modeling the stress-strain shear behavior, a shear retention factor β is used as a stiffness correction (by reducing the shear modulus G, which is computed for elastic isotropic conditions as $G = E/[2(1+\nu)]$) due to progressive material damaging. Masonry behavior in tension, compression and shear have been reported in Fig. 19 whereas the relevant parameter data are shown in Table 2.

DIANA software is able to model the progress of damage at each load step. With reference to the two different tensile behavior diagrams adopted, a different type of damage is related to each branch of the graphs: for clarity's sake a legend is shown in Fig. 20. A decrease of load has not been provided in this case study; however unloading phases for the single element are equally possible because of the stress redistribution due to the progressive damage of the model.

In the analysis of the vault under uniform vertical load an initial load of 1 kN/m^2 has been applied: 200 load steps have been provided with a load increment of 0.5 kN/m^2 for each step.

Ultimate load values corresponding to 39 kN/m^2 and to 48 kN/m^2 have been obtained respectively for the bilinear and the elastic-plastic behavior in tension. The corresponding load-deflection diagrams and the observed crack-patterns are shown in Fig. 21 and Fig. 22, respectively for the two different tension behaviors.

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Table 2 Homogenized masonry parameters adopted for the FEM model

Property	Symbol	Value	Unit
Young's modulus for vault's webs	E_v	2800	MPa
Young's modulus for ribs	E_r	4200	MPa
Poisson's coefficient	ν	0.2	—
Specific weight	γ	22	kN/m^3
Ultimate tensile strength for bilinear elastic-softening behavior	f_t	2.4	MPa
Ultimate tensile strain for bilinear elastic-softening behavior	ε_u	0.0001	—
Ultimate tensile strength for elastic-plastic behavior	f_t	0.05	MPa
Ultimate compressive strength	f_c	2.4	MPa
Shear retention factor	β	2.4	—

It is useful to highlight that all Figures have been drawn by assuming as the reference configuration for the vault the one due to self-weight. As a consequence, deflection values produced by the superimposed live load starts from zero, but correspond to an already stressed configuration, accounting for the effects of self-weight, acting as a dead load.

In the analysis of the vault under a concentrated vertical force applied at the crown an initial force of 1 kN has been applied. Load increments corresponding to 400 load steps have been provided, with a load increment equal to 1 kN for each step. For this load condition, ultimate load values corresponding to 144 kN and to 196 kN have been found respectively for the two different behaviors in tension. The relevant load-deflection diagrams and the resulting crack-patterns are shown in Fig. 23 and Fig. 24. Previous analyses (performed with DIANA version 9.5) corresponding to the second load condition only can be found in (Grillanda et al., 2017), where a comparison between a perfect model and a model with geometric imperfections have also been shown.

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A comparison between results obtained through the two procedures is shown in Fig. 25 and Fig. 26, respectively for uniformly distributed and point-concentrated loads. For the latter load case a higher difference between the ultimate load values can be noticed: it is the authors' opinion that this is the natural consequence of the representation of a complex collapse mechanism through few rigid elements only, and that a more accurate value could be obtained by increasing the number of NURBS surface (despite a larger number of variables and a corresponding increase of computational cost).

GA-NURBS limit analysis may be of great advantages in comparison with FEM incremental analyses. First, it allows to obtain good results with very few elements: for the stellar vault presented here a total number of 73 elements for the distributed load and 113 for the pointed load has been used. Secondly, on this elaborated case study the evaluation of the load multiplier associated to a specific configuration of fracture lines may take from 10 to 16 seconds. Despite the complexity, the possible collapse mechanism under vertical load configurations are limited, thus the convergence is usually reached after few generations even using a small population size: in this way, the whole procedure takes from 14 to 18 minutes. FEM incremental analyses on the stellar vaults took (on the same hardware platform) about 20 minutes, so the computational time of limit analysis is slightly lower. It has to be noticed that MATLAB® is not the most efficient programming environment; the use of programming languages based on pre-compiled library could reduce significantly the computing time of GA-NURBS limit analysis. Finally, the latter approach allows modifications of the final model directly in MATLAB® without resorting to the modeling software.

The great ease by which changes may be applied to the NURBS model has allowed to carry out a sensitivity analysis on ribs thickness through the presented GA-NURBS limit analysis. The aim of this sensitivity analysis is to inspect the effective contribution provided by ribs to the global strength of the stellar vault. Thickness values of ribs have been modified in MATLAB® directly and the analysis has been re-executed for each of such values: indeed, the "thickness property" of NURBS surfaces can be changed *ad libitum* and "on the fly" by the user, exactly in the same way as material properties can be modified in commercial FEM software.

For this study, only the case of a uniformly distributed vertical load has been considered, because it is more realistic in relation with the position of the vault. Indeed, the vault is part of the roofing of the church, therefore there are no structural elements that may exert point forces; the most plausible variable load condition is represented by snow load, which can be represented by a constant uniform pressure.

The ultimate load bearing capacity in terms of a uniformly distributed vertical load has been inspected by making the hypothesis that all ribs have an equal thickness corresponding to 30 cm, 20 cm, 10 cm or 0 cm (this last case corresponds obviously to the case of a rib-less stellar vault, where only vault's webs can resist to loads). For the sake of simplicity in this case the same cross-section has been assumed for the entire rib system, therefore the thickness value considered for each case has been assigned to both main and secondary ribs.

The obtained results are reported in Fig. 27 (where ribs thickness is denoted by s). Substantial differences on the detected collapse mechanisms have not been observed: because of the lowered shape of the vault, crushing of the masonry at the base of the vault remains fundamental for obtaining a collapse mechanism (in the same way as Fig. 15 shows, the dashed lines in Fig. 27 represents the initial constraint plane). The collapse multiplier instead decreases considerably, because the crushed portion of masonry varies from case to case. Fig. 28 shows the different obtained collapse loads: it can be noticed that the collapse load of the previously analyzed case (to be considered as "the real one") is about 4 times larger than the collapse load of the rib-less vault. This sensitivity analysis confirms once more, if the hypothesis of a good connection between ribs and vault's webs is fulfilled, that the support function provided by ribs is of utmost importance in the overall strength of the stellar vault.

5 Conclusions

The mechanical behavior of historical masonry stellar vaults has been investigated in this paper. In particular, the ultimate behavior under vertical loads of a complex ribbed stellar vaults, which is typical of Gothic architecture, has been studied.

The stellar vault of Santa Maria del Monte church in Cagliari (Italy) has been accurately measured through laser-scanning techniques and then reconstructed with a three-dimensional NURBS-based model.

A recently developed limit analysis approach based on an adaptive NURBS modeling coupled with a genetic algorithm has been performed on the vault. This method, particularly suited for historical masonry vaults characterized by complex geometry, has been used in order to evaluate the ultimate load of the structure and the associated collapse mechanisms for two different load conditions. Limit analysis results have then been compared with load-deflection curves and crack/damage patterns provided by finite element analyses performed with the commercial code DIANA. A good agreement in terms of both ultimate load and collapse mechanism between limit analysis and FEM-based simulations has been noticed. The reduced computational cost and the quick handling of complex models suggest the adoption of the proposed limit analysis method for future analyses on ribbed masonry vaults.

As a final test, the influence of rib thickness on the ultimate bearing capacity of the whole vault has been checked through a sensitivity analysis: the obtained results confirm the remarkable strengthening effect produced by ribs and give a useful insight on the evolution of the collapse mechanism for the whole vault. Future research directions following up the

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results contained in this paper will include the study of new monitoring techniques (as for example (Cabboi et al., 2017)), the influence of horizontal loads (Valluzzi, 2007) and settlements (Iannuzzo et al., 2018), and the development of innovative strengthening solutions (see (Garmendia et al., 2014) for applications on masonry arches).

Possible extensions of the formulation adopted for the presented problems could encompass the use of Cosserat theory, see for instance (Altenbach et al., 2010), (Eremeyev et al., 2015), (Eremeyev and Altenbach, 2017), (Miśkiewicz, 2018) for a more accurate description of the behavior of the vault's webs and ribs, and non-standard higher gradient elasticity models like those presented in (Placidi et al., 2016) and in (Andreaus et al., 2016) or, within a non-conservative setting, in (Placidi et al., 2018). Finally, inspiration for non-linear FEM analyses might be found in (Chróścielewski et al., 2018) and discrete models which are suitable for large displacement analysis are dealt with in (Turco, 2018).

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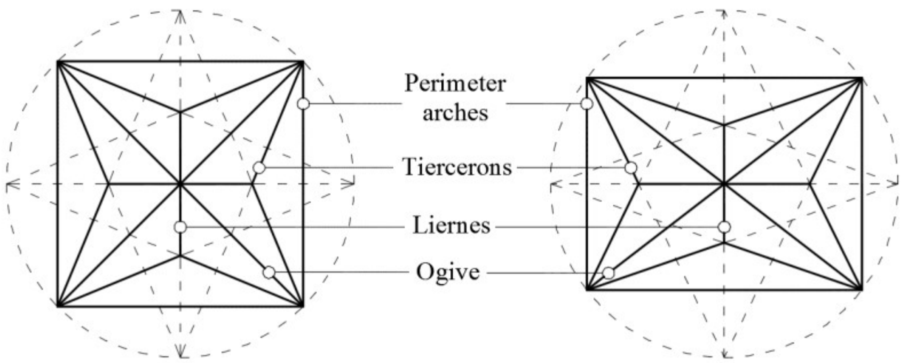
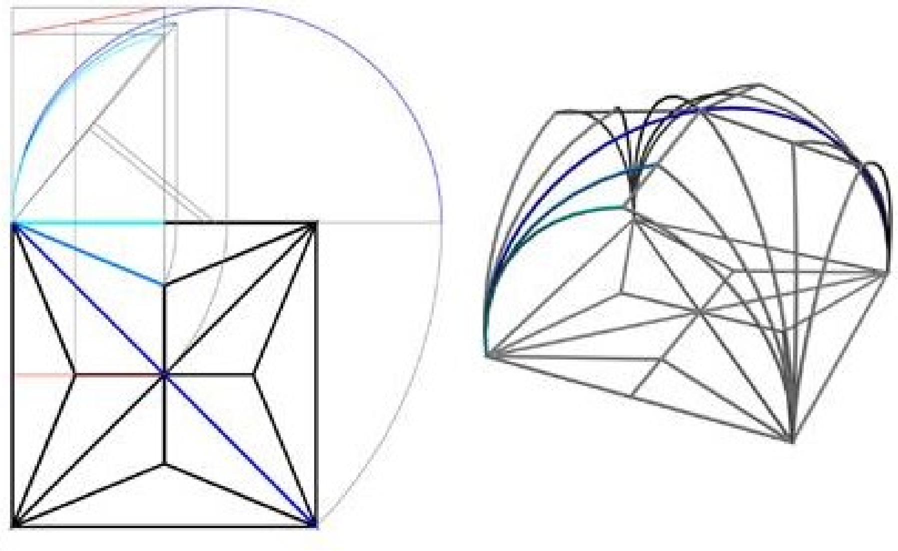
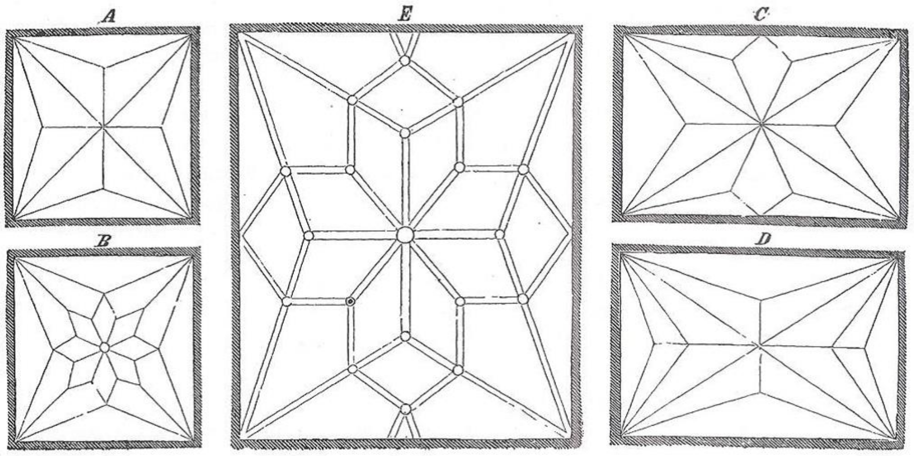


(a) Projection of the rib-system on the horizontal plane

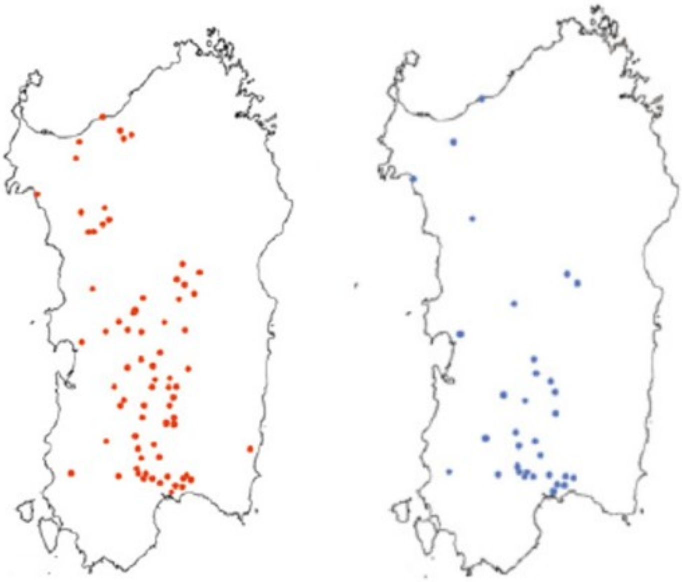


(b) Graphic method for construction



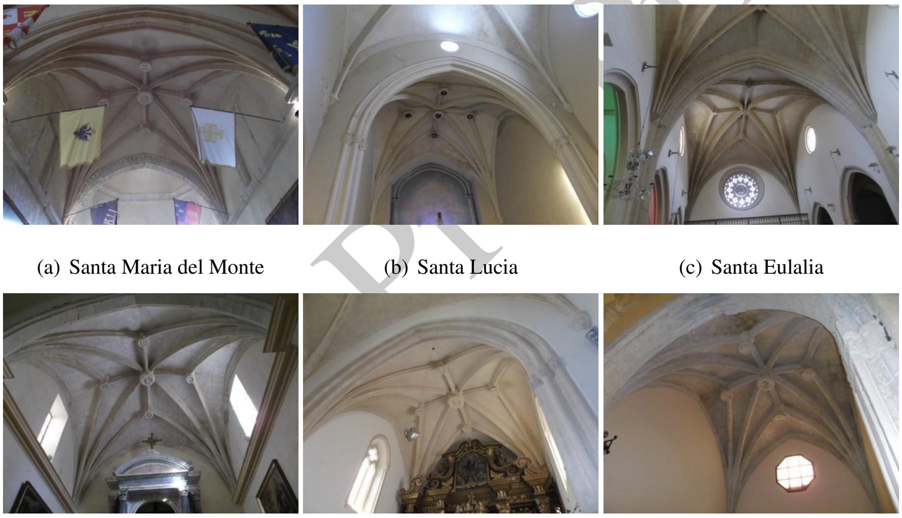
(c) Some different stellar vaults' patterns

Fig. 1 Stellar vaults design.



(a) (b)

Fig. 2 Distribution of late-Gothic vaults in Sardinia: ribbed vaults (a) and stellar vaults (b). The image has been graphically elaborated from that originally presented in (Casu, 2013).



(a) Santa Maria del Monte (b) Santa Lucia (c) Santa Eulalia

(d) Santo Sepolcro (e) La Purissima (f) San Giacomo

Fig. 3 Stellar vaults built in some churches located in Cagliari.

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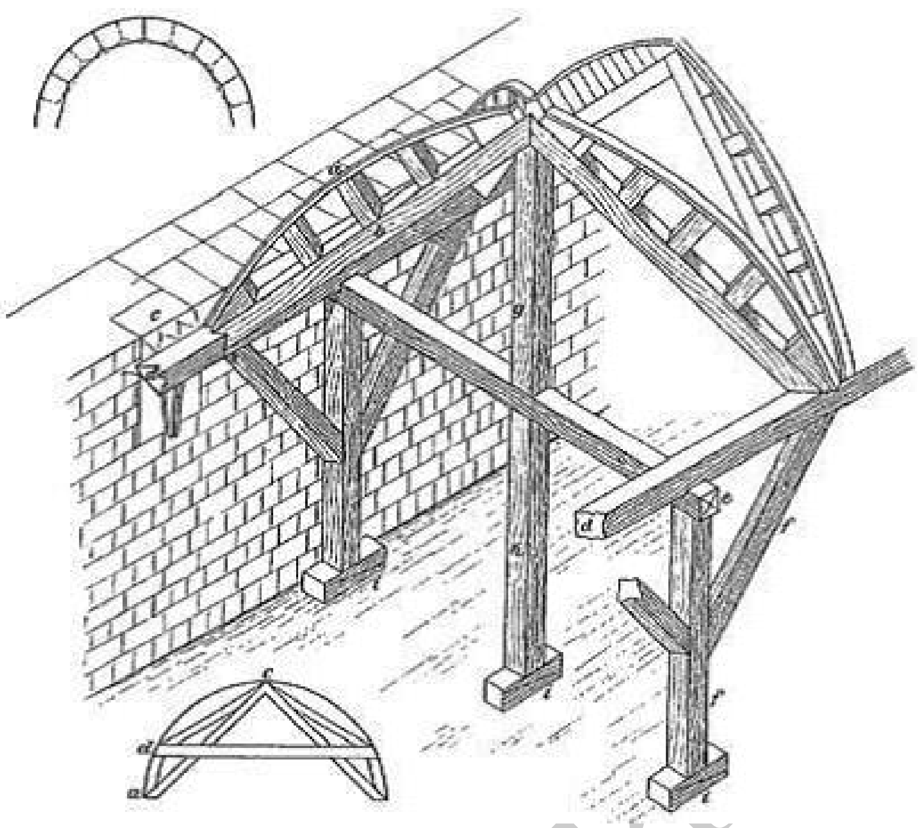

Fig. 4 Building system for Gothic ribbed vaults using centrings for the preliminary construction of ribs. Image elaborated from (Ungewitter and Mohrmann, 1890).

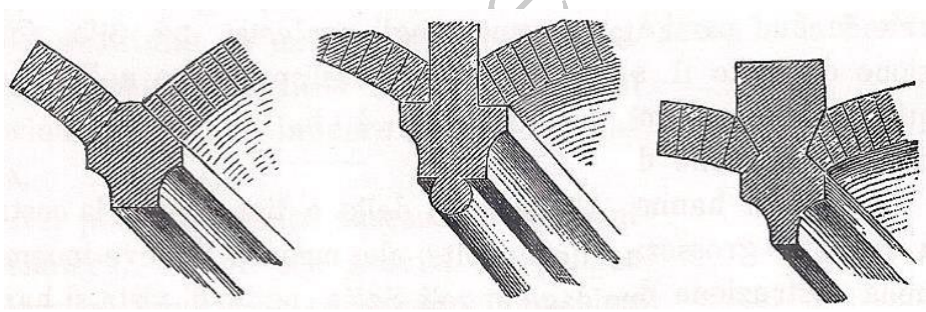

Fig. 5 Some examples of connections between diagonal ribs and vaults. Image elaborated from (Ungewitter and Mohrmann, 1890).

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Fig. 6 Facade of the church of Santa Maria del Monte.

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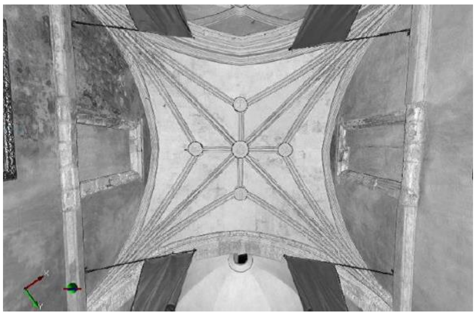
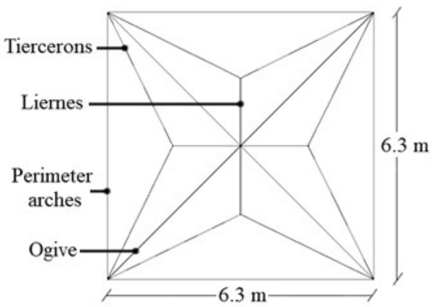
(a) Stellar vault located at the presbytery



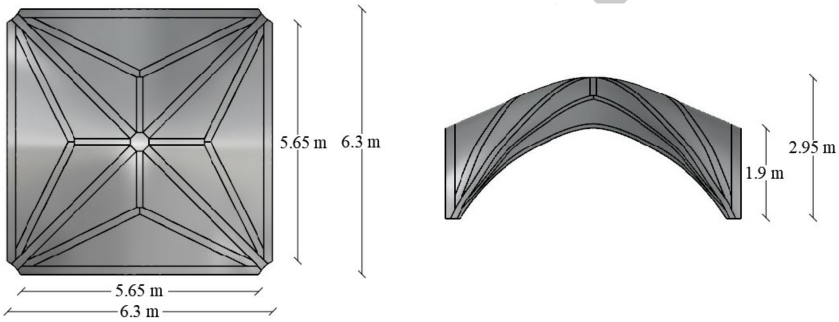
(b) Geometric outline of the vault obtained by laser scanning techniques

Fig. 7 Santa Maria del Monte stellar vault.



(a) Stellar vault projection on the horizontal plane

(b) Stellar vault NURBS model



(c) Stellar vault plan and elevation views showing essential geometric data

Fig. 8 Geometric model of the vault: essential geometric data are reported.

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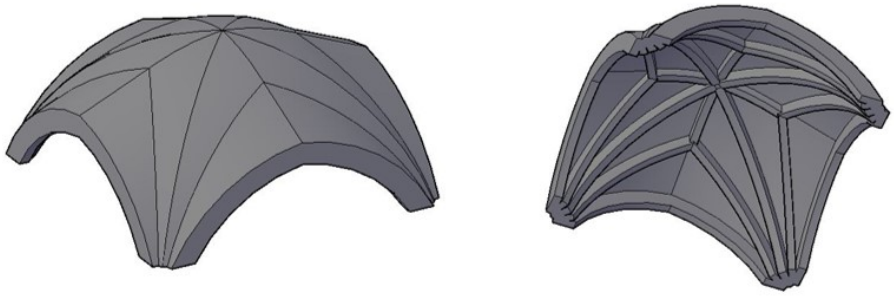


Fig. 9 3D CAD model of the vault: extrados (left) and intrados (right).

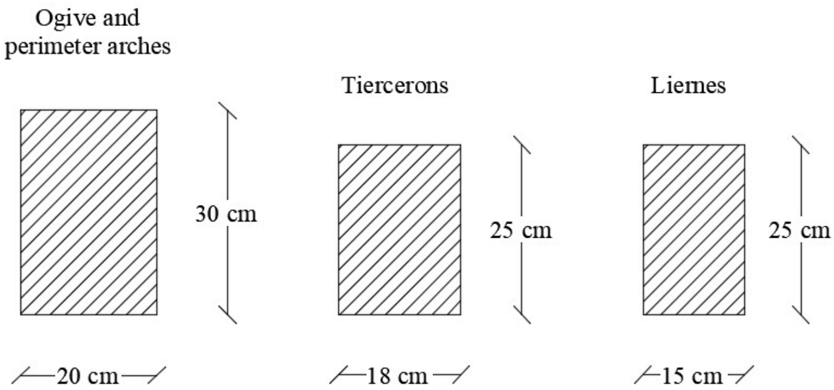
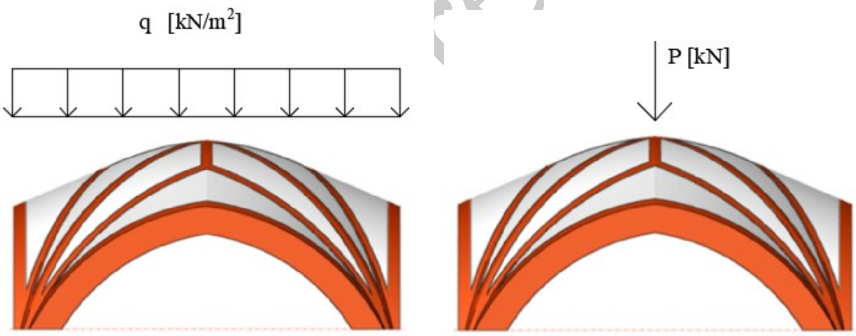


Fig. 10 Simplified cross-sections assumed for modeling ribs.



(a) Uniformly distributed vertical load (b) Concentrated force applied at the crown

Fig. 11 Considered load conditions.

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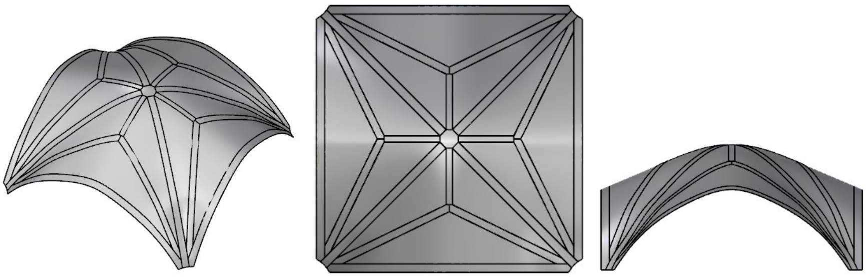
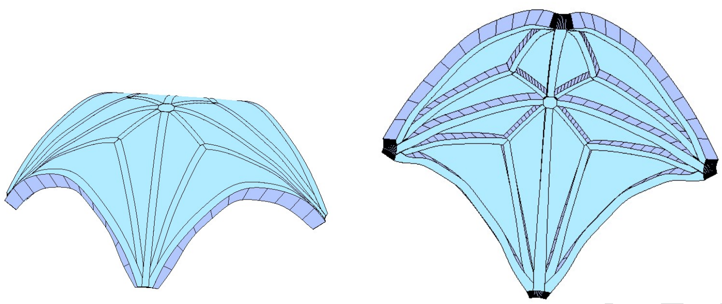
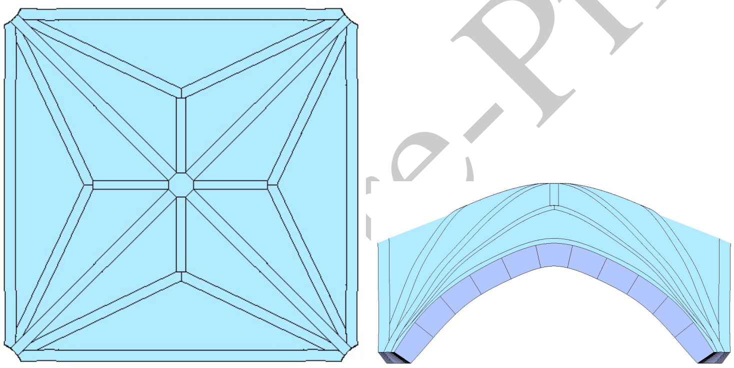


(a) Axonometric view (b) Horizontal view (c) Front view

Fig. 12 NURBS model in Rhinoceros®.



(a) Axonometric view of intrados (b) Axonometric view of extrados



(c) Horizontal view (d) Front view

Fig. 13 NURBS model in MATLAB.

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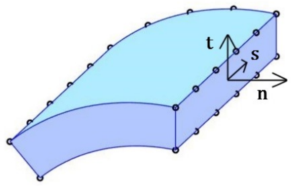
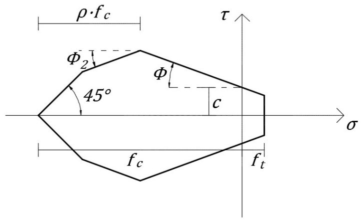
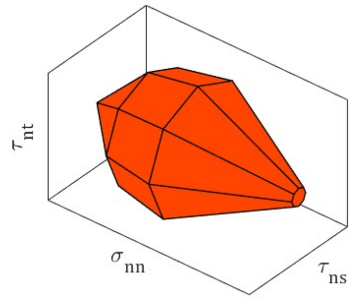


(a) Masonry-masonry interface and corresponding local reference system



(b) Parameters of the failure surface



(c) Adopted 3D linearized failure surface

Fig. 14 Masonry collapse criterion.

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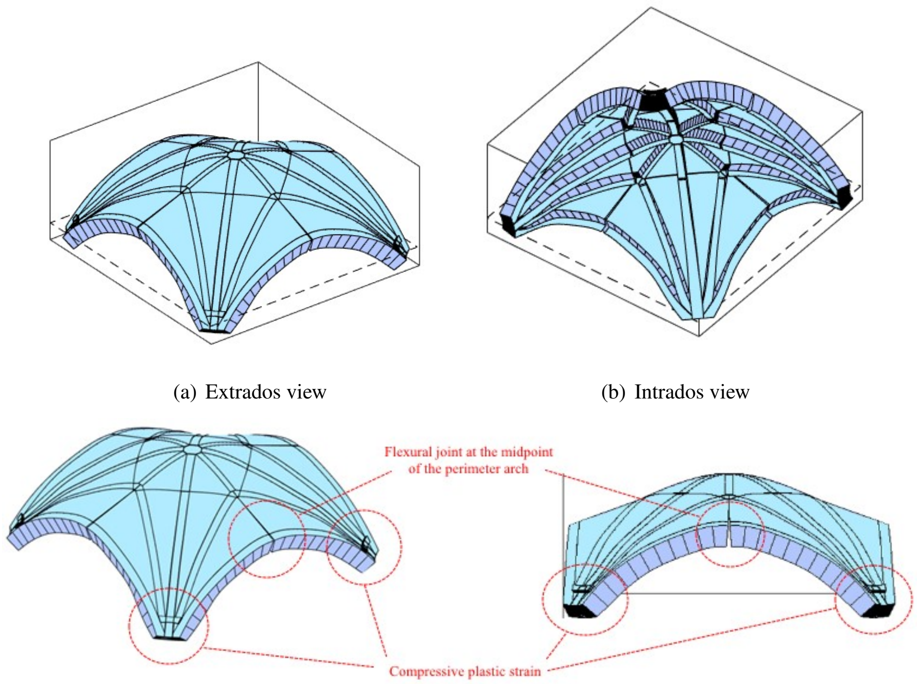

(a) Extrados view

(b) Intrados view

(c) Details of crack openings and plastic strain

Fig. 15 Collapse mechanism associated with the uniformly distributed load.

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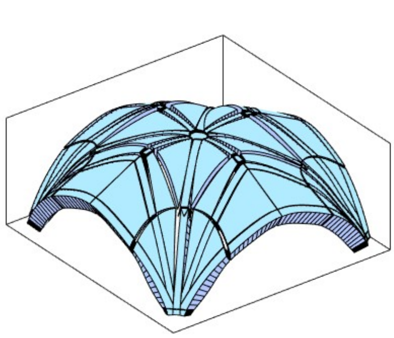
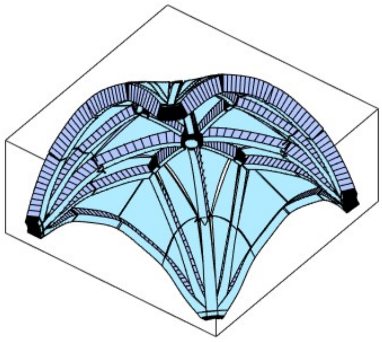
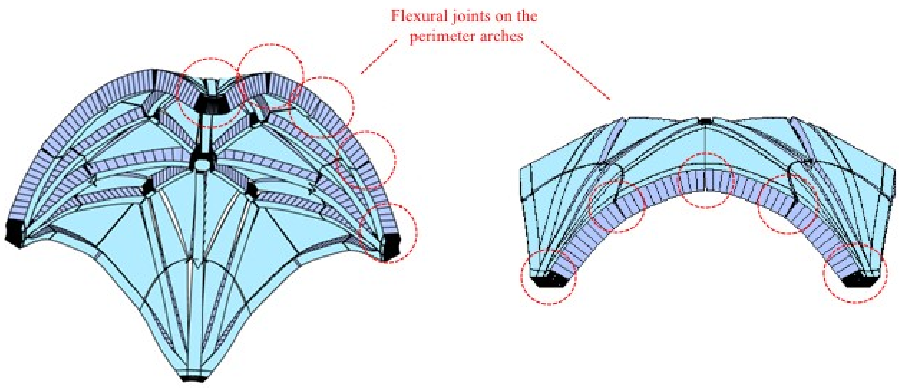


(a) Extrados view



(b) Intrados view



(c) Details of crack openings and plastic strain

Fig. 16 Collapse mechanism associated with the vertical point load applied at the crown.

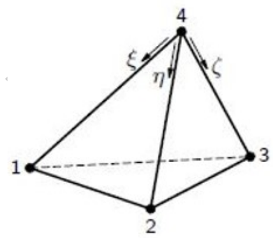


Fig. 17 Tetrahedral finite element (TE12L in DIANA) and corresponding degrees of freedom per node (DIANA, 2015).

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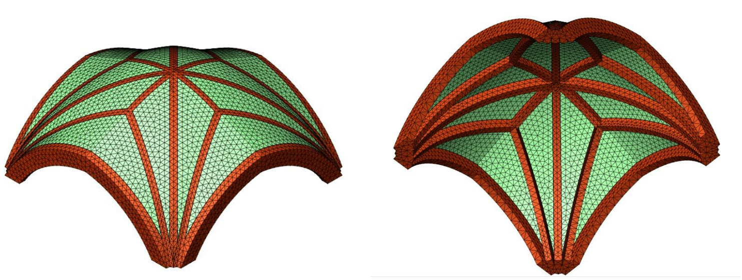
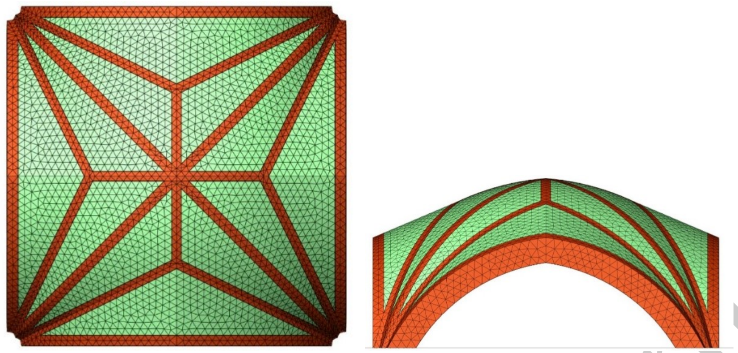


(a) Axonometric view of extrados (b) Axonometric view of intrados



(c) Horizontal projection from top view (d) Front view

Fig. 18 FEM model in DIANA.

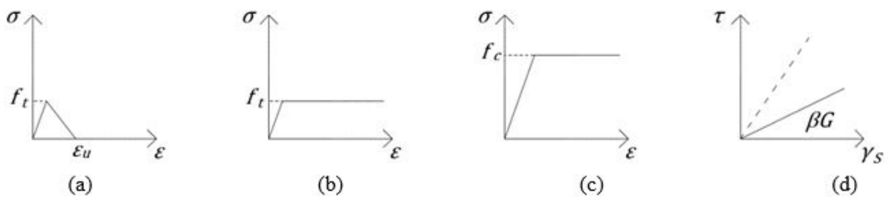


Fig. 19 Non-linear masonry behavior selected in TSCM, (a) bilinear elastic-softening tensile behavior, (b) Elastic-plastic tensile behavior, (c) Elastic-plastic compression behavior, (d) Linear shear behavior with reduced stiffness.

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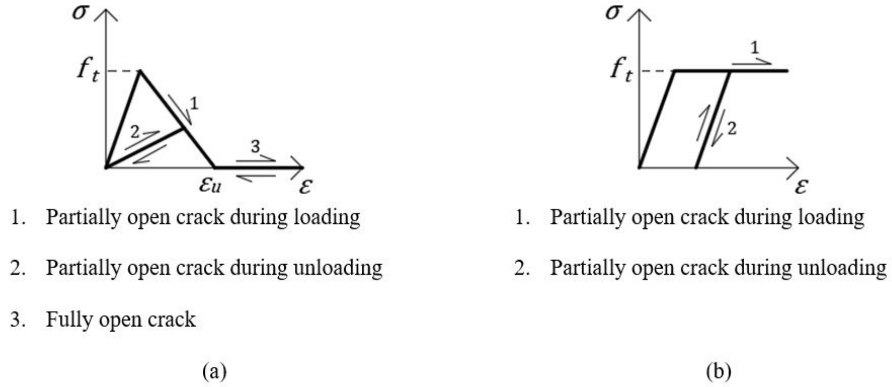

Fig. 20 Different type of damage for each branch of the two adopted tensile behaviors: (a) bilinear elastic-softening behavior, (b) elastic-plastic behavior.

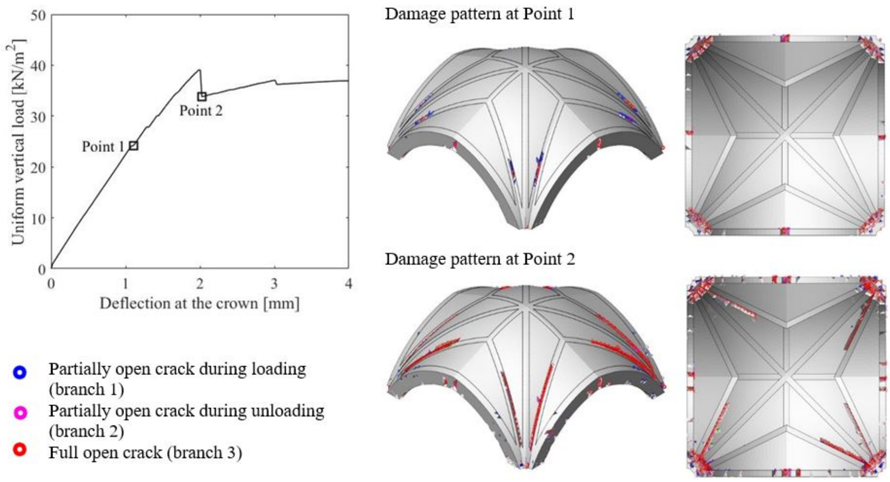

Fig. 21 Results of non-linear static analysis under uniformly distributed vertical load with a bilinear behavior in tension: load-deflection diagram and resulting crack patterns (axonometric and intrados views) at the load level marked by Point 1 and Point 2.

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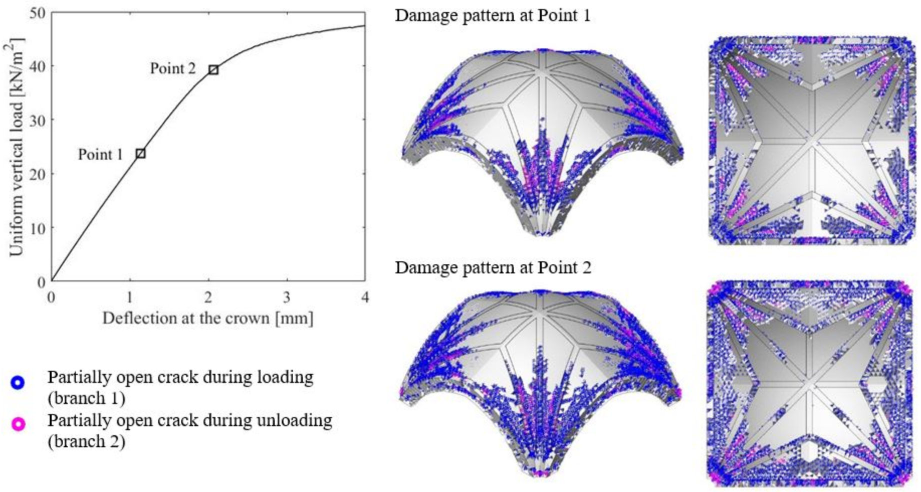

Fig. 22 Results of non-linear static analysis under uniformly distributed vertical load with an elastic-plastic behavior in tension: load-deflection diagram and resulting crack patterns (axonometric and intrados views) at the load level marked by Point 1 and Point 2.

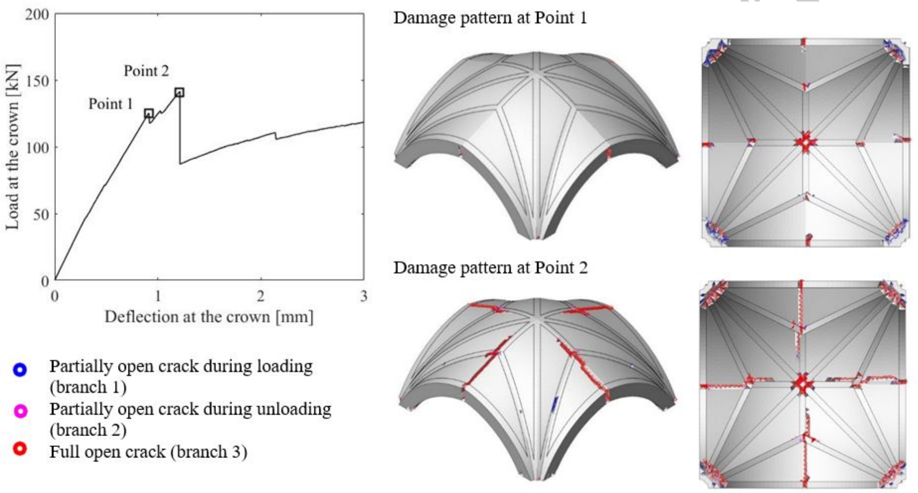

Fig. 23 Results of non-linear static analysis under concentrated load applied at the crown, with a bilinear behavior in tension: load-deflection diagram and resulting crack patterns (axonometric and intrados views) at the load level marked by Point 1 and Point 2 in the load-deflection curve.

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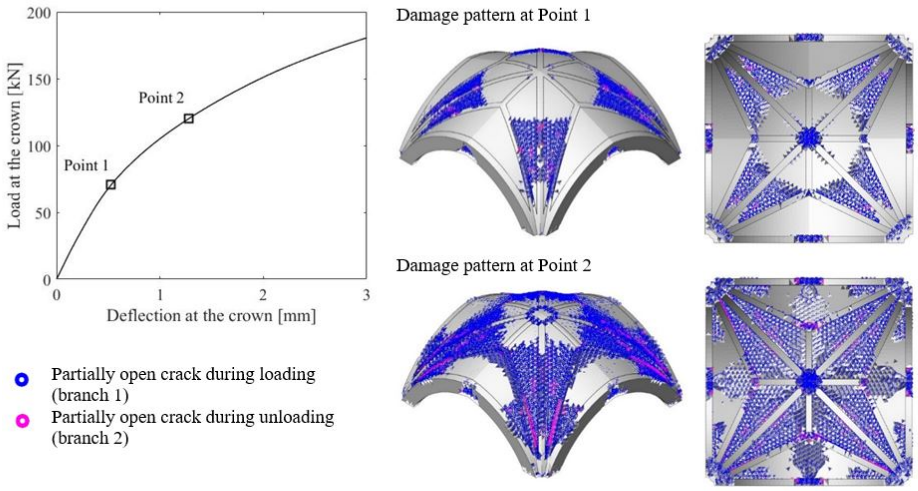

Fig. 24 Results of non-linear static analysis under concentrated load applied at the crown with an elastic-plastic behavior in tension: load-deflection diagram and resulting crack patterns (axonometric and intrados views) at the load level marked by Point 1 and Point 2.

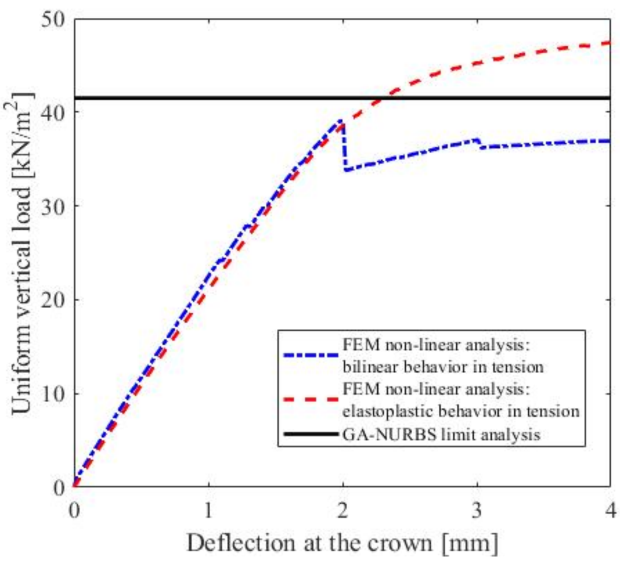

Fig. 25 Analyses with uniformly distributed vertical load: comparison of load-deflection results between FEM and limit analysis based on GA-NURBS modeling.

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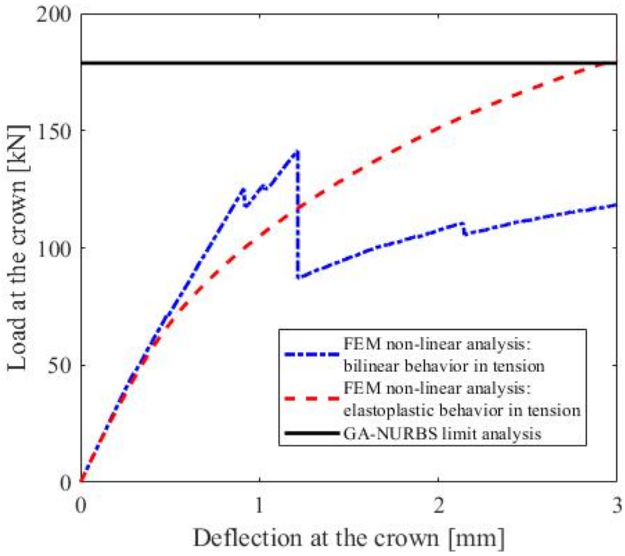
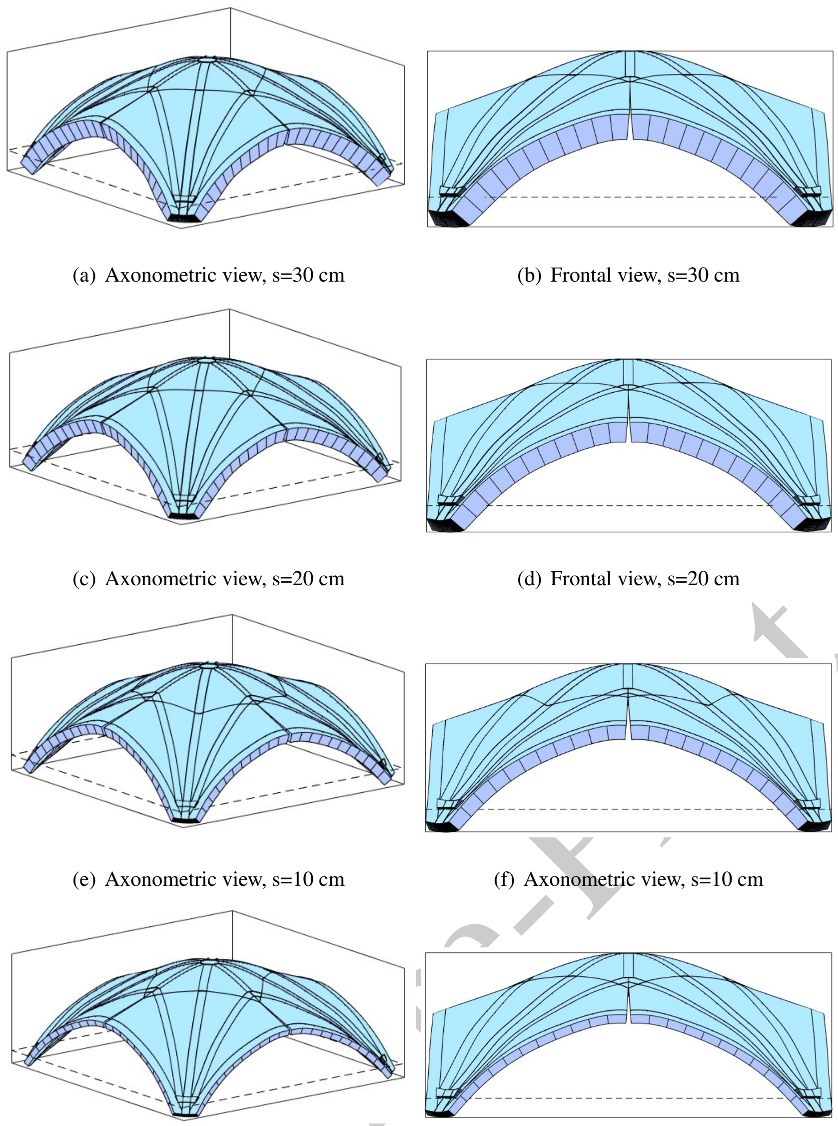


Fig. 26 Analyses with concentrated point force applied at the crown: comparison of load-deflection results between FEM and limit analysis based on GA-NURBS modeling.



(a) Axonometric view, s=30 cm (b) Frontal view, s=30 cm

(c) Axonometric view, s=20 cm (d) Frontal view, s=20 cm

(e) Axonometric view, s=10 cm (f) Axonometric view, s=10 cm

(g) Frontal view, no ribs (h) Frontal view, no ribs

Fig. 27 Detected collapse mechanisms for some values of rib thickness (axonometric view on the left and front view on the right).

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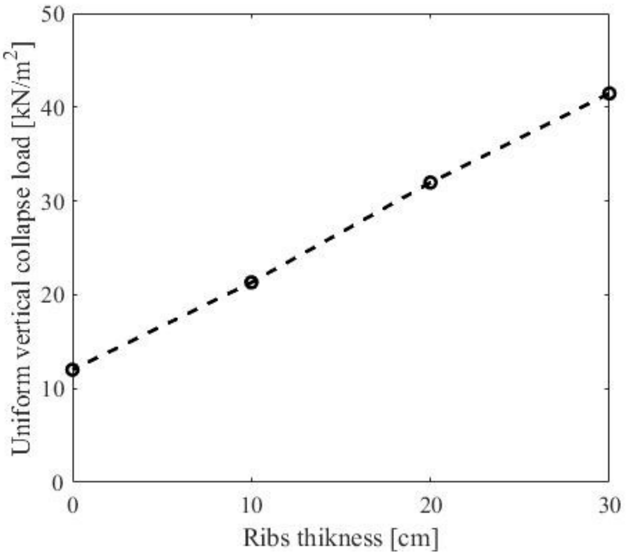

Fig. 28 Collapse load as a function of ribs thickness for the case of uniformly distributed vertical load.

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